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DEGREE OF SPIN-POLARIZATION OF PHOTOELECTRONS EMITTED FROM ALKALI ATOMS IN ONE-QUANTUM PHOTOEFFECT*

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A general formula is derived for the degree of spin-polarization of photoelectrons arising from the ground state of arbitrary alkali atoms. The spin-polarization of the photoelectrons is discussed in detail in its dependence on the polarization state of the photon, the electronic polarization of the atom, the orientation of the wave vector and that of the polarization ellipse of the photon with respect to the polarization direction of the atom, and the Fano parameter x. Both the spin-polarization of the photoelectrons emitted in a given direction and that of all photoelectrons produced, is considered. The condition ensuring 100 per cent spin-polarization of photoelectrons, generated from unpolarized or polarized alkali atoms, is determined.

1. *Introduction*

Recently, interest in the construction of sources of intense, strongly polarized electron beams has grown considerably [1]. Beams of polarized electrons are at present in use in atomic, molecular, high-energy and solid state physics in studies bearing on various finer, spin-dependent effects. A concise comparison of the now available sources of polarized electrons is performed in papers by Hughes et al. [2], Pierce and Meier [3], and Cherepkov [4].

The present work contains a consistent generation theory for polarized photoelectrons from unpolarized as well as polarized alkali atoms with a discussion of the generation efficiency in its dependence on parameters characterizing the photon, atom, and experimental geometry such as the polarization state of the photon, the degree of electronic polarization of the atom, the orientation of the wave vector and polarization ellipse of the photon with respect to the direction of electronic polarization of the atom and, lastly, the Fano parameter x. Our theory gives the degree of polarization of photoelectrons emitted

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into a well defined direction as well as that of all photoelectrons, emitted into the full body angle 4π. The calculations are carried out to the first order of perturbation calculus, in the electric dipole approximation. Hyperfine interaction of the electron angular momentum of the atom and nuclear spin, and phase shifts in partial photoelectron wave under the action of spin-orbit perturbation in P states of the continuum, are omitted.

2. *Theory*

2.1. Basic formulae of the theory; geometry of the problem

The definition formulae for the spin-polarization degree of photoelectrons ejected in the direction $\vartheta_{\vec{K}}$, $\varphi_{\vec{K}}$ and that for all photoelectrons emitted in the full body angle 4π are:

$$P_{\mathrm{el}}(\vartheta_{\vec{K}}, \varphi_{\vec{K}}) = \frac{d\sigma_{+1/2}/d\Omega_{\vec{K}} - d\sigma_{-1/2}/d\Omega_{\vec{K}}}{d\sigma_{+1/2}/d\Omega_{\vec{K}} + d\sigma_{-1/2}/d\Omega_{\vec{K}}}, \tag{1}$$

$$P_{\mathrm{el}} = \frac{\sigma_{+1/2} - \sigma_{-1/2}}{\sigma_{+1/2} + \sigma_{-1/2}}, \tag{2}$$

where $d\sigma_{\mu_{\mathrm{f}}}/d\Omega_{\vec{K}}$ and $\sigma_{\mu_{\mathrm{f}}}$ are, respectively, the differential and total cross-section for photoemission of an electron with momentum $\hbar\vec{K}$ and spin orientation $\mu_{\mathrm{f}} = \pm 1/2$, well defined with respect to the Z-axis.

Let $\sigma_{\mu_{\mathrm{f}}\mu_{\mathrm{i}}}$ denote the total cross section for a photoionisation transition from the ground state $n_0\,{}^2S_{1/2}(\mu_{\mathrm{i}})$ of an alkali atom with orientation of the optical electron spin given by the projection $\mu_{\mathrm{i}} = \pm 1/2$ to the continuum, where the spin projection is $\mu_{\mathrm{f}} = \pm 1/2$. The cross section $\sigma_{\mu_{\mathrm{f}}}$ can be obtained from $\sigma_{\mu_{\mathrm{f}}\mu_{\mathrm{i}}}$ by its averaging over spin orientations in the initial state, according to the formula:

$$\sigma_{\mu_{\mathrm{f}}} = \sum_{\mu_{\mathrm{i}} = \pm 1/2} \sigma_{\mu_{\mathrm{f}}\mu_{\mathrm{i}}} p_{\mu_{\mathrm{i}}}, \tag{3}$$

where $p_{\mu_{\mathrm{i}}}$ is the population probability of the spin state $n_0\,{}^2S_{1/2}(\mu_{\mathrm{i}})$. The probabilities $p_{\mu_{\mathrm{i}}}$ satisfy the normalization condition

$$\sum_{\mu_{\mathrm{i}} = \pm 1/2} p_{\mu_{\mathrm{i}}} = 1 \tag{4}$$

and are related with the degree of electronic polarization of the atom:

$$P_{\mathrm{at}} = p_{+1/2} - p_{-1/2} \tag{5}$$

as follows:

$$p_{\pm 1/2} = \tfrac{1}{2}(1 \pm P_{\mathrm{at}}). \tag{6}$$

The cross section $\sigma_{\mu_{\mathrm{f}}\mu_{\mathrm{i}}}$, which is a quantity of primary importance, can be calculated from the Fermi rule:

$$\frac{d\sigma_{\mu_{\mathrm{f}}\mu_{\mathrm{i}}}}{d\Omega_{\vec{K}}} = \frac{\omega}{2\pi c} K a^4 |\langle \vec{K}, \mu_{\mathrm{f}} | \vec{e}_{\pm} \cdot \vec{r} | n_0 0 \tfrac{1}{2} \tfrac{1}{2} \mu_{\mathrm{i}} \rangle|^2, \tag{7}$$

ω denoting the circular frequency and c the velocity of light, $K = |\vec{K}|$ with $\vec{K}$ — the wave vector of the photoelectron infinitely remote from the nucleus, $a = 5.29 \times 10^{-9}$ cm, $\vec{e}_\pm$ the polarization vector of the photon normalized to unity, $\vec{r}$ the radius vector of the optical electron. Above, $|n_0\, 0\, \frac{1}{2}\, \frac{1}{2}\, \mu_i\rangle$ is the notation of the ground state $n_0\,{}^2S_{1/2}(\mu_i)$ in the representation $|nlsjM_j\rangle$, whereas $|\vec{K}, \mu_f\rangle$ is the state of the photoelectron, with the momentum $\hbar\vec{K}$ and spin orientation μ_f. Mathematically, the state $|\vec{K}, \mu_f\rangle$ of the photoelectron in which the orbital and spin momenta are uncoupled can be constructed from states with well defined total angular momentum j_f and projection M_f as follows [5], [6]:

$$|\vec{K}, \mu_f\rangle = (8\pi^3/Ka)^{1/2} \sum_{l_f=0}^{\infty} \sum_{j_f, M_f} i^{l_f} e^{i\delta l_f'} Y^*_{l_f, M_f-\mu_f}(\vartheta_{\vec{K}}, \varphi_{\vec{K}}) C^{j_f, M_f}_{l_f, M_f-\mu_f, 1/2, \mu_f} |K l_f\, \tfrac{1}{2}\, j_f M_f\rangle, \quad (8)$$

where $\vartheta_{\vec{K}}$ and $\varphi_{\vec{K}}$ are, respectively, the polar angle and azimuth of the wave vector $\vec{K}$ of the photoelectron, δ_{l_f} the shift in phase of the l_f-th partial wave by assumption independent of j_f, and $C^{j_3 m_3}_{j_1 m_1 j_2 m_2}$ the Clebsch–Gordan coefficient in the notation of Varshalovich et al. [7]. According to the selection rules for one-photon electric dipole transitions, for ionisation of an alkali atom from the ground state, in the final state $l_f = 1$ and $j_f = 3/2, 1/2$.

Irrespective of the photon propagation direction, the unit polarization vector $\vec{e}_\pm$ (in the meaning $|\vec{e}_\pm|^2 = 1$) can be represented quite generally as:

$$\vec{e}_\pm = (1+\kappa^2)^{-1/2}(\vec{e}_1 \pm i\kappa\vec{e}_2 e^{i\alpha}), \quad (9)$$

where $\vec{e}_1$ and $\vec{e}_2$ are two unit vectors at right angles to each other and to the wave vecto of the photon, and directed along the larger ($\vec{e}_1$) and smaller ($\vec{e}_2$) semi-axis of the ligh polarization ellipse, respectively. For polarized light the phase α is zero irrespective of the state of polarization, whereas for unpolarized light it is a random function of time taking all values from the interval $(0, 2\pi)$ with equal probability. Thus, for unpolarized light, isotropic averaging over α will have to be performed in the final formulae. The quantity κ is the ellipticity parameter, defined as the ratio of the small and large semi-axis of the polarization ellipse of light ($0 \leqslant \kappa \leqslant 1$). The limit $\kappa = 1$ corresponds to circular polarization, when $\alpha = 0$, or to unpolarized light, when α is random; $\kappa = 0$ signifies that the light is linearly polarized in the direction of $\vec{e}_1$. For $\kappa \neq 0$ and, simultaneously, $\alpha = 0$, the upper sign "+" at the imaginary unit corresponds to the right sense of polarization and, respectively, the lower sign "−" to the left sense. The helicity of the photon is defined here in agreement with the natural angular momentum convention.

We express the polarization vector $\vec{e}_\pm$ of the photon in the basis $\vec{i}, \vec{j}, \vec{k}$ of the Cartesian system of coordinates XYZ (Fig. 1). Let us begin by assuming that the beam of alkali atoms is polarized along Z. Moreover, we assume the wave vector $\vec{K}_{ph}$ of the photon to subtend an angle δ with the Z-axis, and the X-axis to lie in the plane of the quantization direction of the electron angular momentum and the propagation direction of the photons. Thus, the three vectors $\vec{K}$, $\vec{K}_{ph}$ and $\vec{i}$ lie in one plane. Another plane is defined by the three vectors $\vec{e}_1$, $\vec{j}$, $\vec{e}_2$. The two planes intersect on a straight line, defined by the vector $\vec{u}$, subtending the angle δ with the X-axis. Let ϕ denote the angle between the principal axis of the polarization ellipse (versor $\vec{e}_1$) and the Y-axis. With this geometry, the unit vectors

$\vec{e}_1$ and $\vec{e}_2$ describing the orientation of the polarization ellipse of the photon are given as follows in terms of the basis vectors $\vec{i}$, $\vec{j}$, $\vec{k}$:

$$\vec{e}_1 = \sin\phi \cos\delta\, \vec{i} + \cos\phi\, \vec{j} - \sin\phi \sin\delta\, \vec{k},$$
$$\vec{e}_2 = -\cos\phi \cos\delta\, \vec{i} + \sin\phi\, \vec{j} + \cos\phi \sin\delta\, \vec{\kappa}. \tag{10}$$

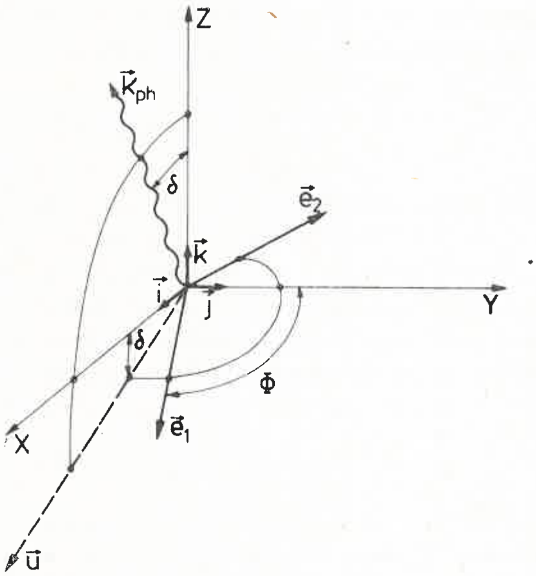


Fig. 1. Coordinate system for calculation of the matrix element $\langle \vec{K}, \mu_f | \vec{e}_\pm \cdot \vec{r} |\, n_0 0 \tfrac{1}{2} \tfrac{1}{2} \mu_i \rangle$

Accordingly, the scalar product $\vec{e}_\pm \cdot \vec{r}$ can be expressed in the form:

$$\vec{e}_\pm \cdot \vec{r} = r \sum_{\gamma = -1, 0, +1} \beta_\gamma Y_{1\gamma}(\vartheta, \varphi), \tag{11}$$

where ϑ and φ are the spherical angular coordinates of the radius vector $\vec{r}$ of the electron in the XYZ system, and

$$\beta_\gamma = -[2\pi/3(1+\kappa^2)]^{1/2} \{[\gamma \cos\delta + \sqrt{2}\,(1-\gamma^2) \sin\delta]\,(\sin\phi \mp i\kappa \cos\phi e^{i\alpha})$$
$$- i|\gamma|\,(\cos\phi \pm i\kappa \sin\phi e^{i\alpha})\}. \tag{12}$$

The coefficients β_γ convey complete information on the polarization state of the light beam as well as the orientation of the wave vector $\vec{K}_{\text{ph}}$ of the photon and its polarization ellipse with respect to the electron polarization direction of the atom. Let us now assume, for the case of unpolarized atoms, that the quantization axis Z is defined by the propagation direction of the beam of photons, so that $\delta = 0$. In this situation, the angle ϕ is arbitrary since, if $\delta = 0$, it is meaningless to speak of a plane defined by the quantization direction of electron angular momentum and the photon propagation direction, since these two directions coincide. It is convenient to assume $\phi = \pi/2$ i.e. that in the case of unpolarized atoms the X axis of Cartesian coordinates is defined by the larger semi-axis of the light polarization ellipse ($\vec{e}_1 = \vec{i}$) and the Y-axis — by the smaller semi-axis ($\vec{e}_2 = \vec{j}$). Thus, the transition in the formulae for the β_γ and product $\vec{e}_\pm \cdot \vec{r}$ from the case of polarized to that of unpolarized atoms is simply given by the substitution $\delta = 0, \phi = \pi/2$. For polarized atoms, the polar angle $\vartheta_{\vec{K}}$ of the photoelectron wave vector $\vec{K}$ is measured from the electron polarization direction of the atom and the azimuth $\varphi_{\vec{K}}$ — from the plane of atom polarization and photon propagation. For unpolarized atoms, $\vartheta_{\vec{K}}$ is measured from the photon propagation direction, and $\varphi_{\vec{K}}$ — from the larger semi-axis of the light polarization ellipse.

On insertion of the expansion (11) and function $|\vec{K}, \mu_f\rangle$ given by Eq. (8), into the Fermi rule (7), one is faced with the task of calculating the matrix element of the operator $rY_{1\gamma}(\vartheta, \varphi)$ in the representation $|nlsjM_j\rangle$. By spherical tensor algebra, one obtains [7]:

$$\langle Kl_f \tfrac{1}{2} j_f M_f | rY_{1\gamma} | n_0 0 \tfrac{1}{2} \tfrac{1}{2} \mu_i \rangle = \delta_{l_f,1} \delta_{j_f,1/2\;3/2} \delta_{M_f,\mu_i+\gamma}$$

$$\times C^{j_f,M_f}_{1/2,\mu_i,1,\gamma} [(-1)^{l_f+j_f+3/2}/3\sqrt{4\pi}] \Delta R [x + 3(j_f - 7/6)], \tag{13}$$

where x is the so called Fano parameter [8], defined as follows:

$$x = (2R_{3/2} + R_{1/2})/\Delta R, \quad \Delta R = R_{3/2} - R_{1/2}, \tag{14}$$

with

$$R_{j_f} = \int_0^\infty R(\vec{K}\,^2P_{j_f}) r R(n_0^2 S_{1/2}) r^2 dr. \tag{15}$$

Here, $R(n_0\,^2S_{1/2})$ and $R(\vec{K}\,^2P_{j_f})$ are radial wave functions of the electron in its initial and final state, respectively. According to Seaton [9], the integral $R_{1/2}$ slightly differs from $R_{3/2}$ due to spin-orbit interaction in P states of continuous spectrum of all alkali atoms, heavier than lithium. The limit $\Delta R = 0$, identical with $x = \infty$, corresponds to an absence of spin-orbit perturbation in the continuum. However, calculations by Weisheit and Dalgarno [10], Chang and Kelly [11], and Norcross [12], as well as measurements by Baum et al. [13] for K, Rb and Cs, have shown that for certain energies of the photon the Fano parameter x can take values as low as 0, +1, −1, pointing to an essential influence of spin--orbit effects in the continuum on the photoionization of alkali metal vapours.

2.2. Cross sections for photoemission of the electron with well defined spin orientation and generalized Seaton formula

By using the relations of previous subsection, we obtain following expression for $d\sigma_{\mu_f}/d\Omega_{\vec{K}}$ — the averaged over electron spin orientations in the initial state $n_0\,^2S_{1/2}$ cross section for the photoelectron emitted in the direction $\vartheta_{\vec{K}}, \varphi_{\vec{K}}$ to have the spin orientation μ_f:

$$d\sigma_{\mu_f}/d\Omega_{\vec{K}} = \frac{2\pi^2}{9} \frac{a^3}{\lambda} \frac{(\Delta R)^2}{1+\kappa^2} \{a_1 + \tfrac{1}{2}(1+\xi(\mu_f)P_{at})a_2 \pm \tfrac{1}{2}\xi(\mu_f)(1-\xi(\mu_f)P_{at})a_3$$

$$+ [b_1 \mp \xi(\mu_f)a_3 + \tfrac{1}{4}(1+\xi(\mu_f)P_{at})(b_2 + b_3 \pm \xi(\mu_f)b_4)] \sin^2 \vartheta_{\vec{K}}$$

$$+ [c_1 \pm \xi(\mu_f)c_2 + \tfrac{1}{4}(1+\xi(\mu_f)P_{at})(c_3 \pm \xi(\mu_f)c_4)] \sin 2\vartheta_{\vec{K}}\}, \tag{16}$$

where

$$\xi(\mu_f) = \begin{cases} +1 & \text{for} \quad \mu_f = +1/2 \\ -1 & \text{for} \quad \mu_f = -1/2, \end{cases} \tag{17}$$

and moreover:

$$a_1 = 1+\kappa^2-[1-(1-\kappa^2)\cos^2\phi]\sin^2\delta, \tag{18}$$

$$a_2 = (1+x^2)[1-(1-\kappa^2)\cos^2\phi]\sin^2\delta-(1+\kappa^2), \tag{19}$$

$$a_3 = 2\kappa\cos\alpha\cos\delta, \tag{20}$$

$$b_1 = 2[1-(1-\kappa^2)\cos^2\phi]\sin^2\delta-(1+\kappa^2), \tag{21}$$

$$b_2 = (x^2-1)\{(1-\kappa^2)[\sin 2\phi\sin 2\varphi_{\vec{K}}\cos\delta-\cos 2\phi\cos 2\varphi_{\vec{K}}] - [1-(1-\kappa^2)\cos^2\phi]\sin^2\delta\cos 2\varphi_{\vec{K}}\}, \tag{22}$$

$$b_3 = (1+\kappa^2)(3+x^2)-(3x^2+5)[1-(1-\kappa^2)\cos^2\phi]\sin^2\delta, \tag{23}$$

$$b_4 = 4(1+x)\kappa\cos\alpha\cos\delta, \tag{24}$$

$$c_1 = \sin\delta\{[1-(1-\kappa^2)\cos^2\phi]\cos\delta\cos\varphi_{\vec{K}}+\tfrac{1}{2}(1-\kappa^2)\sin 2\phi\sin\varphi_{\vec{K}}\}, \tag{25}$$

$$c_2 = \kappa\sin\delta\cos\alpha\cos\varphi_{\vec{K}}, \tag{26}$$

$$c_3 = -2(1+x^2)\sin\delta\{[1-(1-\kappa^2)\cos^2\phi]\cos\delta\cos\varphi_{\vec{K}}+\tfrac{1}{2}(1-\kappa^2)\sin 2\phi\sin\varphi_{\vec{K}}\}, \tag{27}$$

$$c_4 = -2(1+x)\kappa\sin\delta\cos\alpha\cos\varphi_{\vec{K}}. \tag{28}$$

Eq. (16) is identical for $\mu_f = +1/2$ and $-1/2$ for the two following cases only: (*i*) unpolarized atoms and unpolarized photons ($P_{at} = 0$, $\kappa = 1$, $\langle\cos\alpha\rangle_\alpha = 0$), (*ii*) unpolarized atoms and linearly polarized photons ($P_{at} = 0$, $\kappa = 0$, $\alpha = 0$). Except for the aforesaid two cases, the angular distributions of photoelectrons with spin oriented parallel to the Z-axis ($\mu_f = +1/2$) differ from those for photoelectrons with spin oriented antiparallel to the Z-axis ($\mu_f = -1/2$). We shall consider the differences between angular distributions of photoelectrons with spins oppositely directed by performing an analysis of the following two examples: (*i*) unpolarized atoms and right-handed circularly polarized photons, (*ii*) polarized atoms and unpolarized photons in parallel geometry ($\delta = 0$). For these two cases, the basic expression (16) reduces to:

(*i*) unpolarized atoms + right-handed circularly polarized photons

$$\frac{d\sigma^c_{+1/2}}{d\Omega_{\vec{K}}} = \frac{3\sigma_S}{8\pi}\left[\frac{2}{x^2+2}+\frac{(x-1)(x+3)}{2(x^2+2)}\sin^2\vartheta_{\vec{K}}\right], \tag{29}$$

$$\frac{d\sigma^c_{-1/2}}{d\Omega_{\vec{K}}} = \frac{3\sigma_S}{8\pi}\frac{(x-1)^2}{2(x^2+2)}\sin^2\vartheta_{\vec{K}}, \tag{30}$$

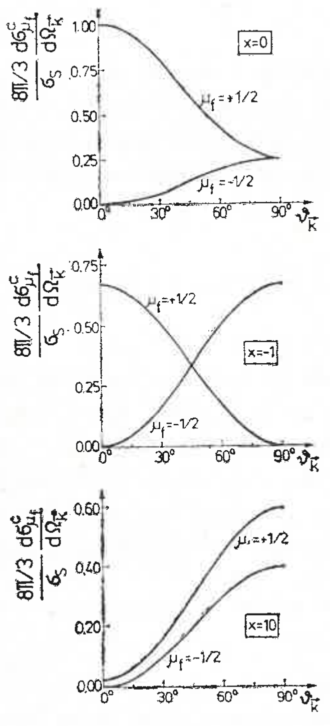
(*ii*) polarized atoms + unpolarized photons in the parallel geometry

$$\left[\frac{d\sigma^{un}_{\pm 1/2}}{d\Omega_{\vec{K}}}\right]_{\parallel} = \frac{3\sigma_S}{8\pi}\left\{\frac{1\mp P_{at}}{x^2+2}+\frac{1}{2(x^2+2)}[x^2-1\pm(x^2+3)P_{at}]\sin^2\vartheta_{\vec{K}}\right\}. \tag{31}$$

In these particular expressions $\vartheta_{\vec{K}}$ is the angle between the photon propagation and photoelectron emission directions, and σ_S is the SEATON photoionization cross section [9] for unpolarized alkali atoms, equal to:

$$\sigma_S = \left(\frac{2\pi}{3}\right)^3 \frac{a^3}{\lambda} (\Delta R)^2 (x^2+2). \tag{32}$$

On the right-hand side of equation (31), the upper sign corresponds to $\mu_f = +1/2$, whereas the lower sign corresponds to $\mu_f = -1/2$. Figs. 2 and 3 illustrate the relations (29), (30)



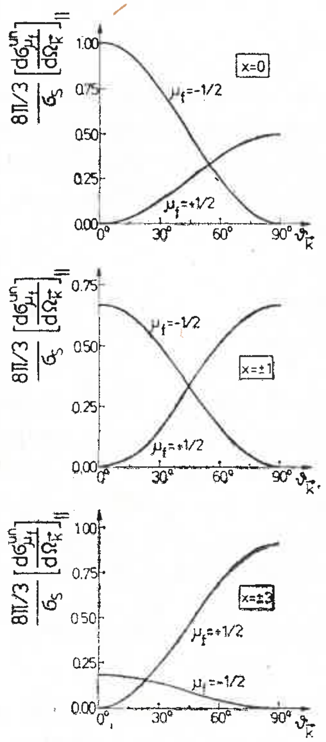


Fig. 2 Fig. 3

Fig. 2. Angular distributions of photoelectrons with spin orientation well defined with respect to the photon propagation direction, for the case of unpolarized atoms + right-handed circularly polarized photons

Fig. 3. Angular distributions of photoelectrons with spin orientation well defined with respect to the photon propagation direction, for the case of completely polarized atoms ($P_{at} = 1$) + unpolarized photons, in the parallel geometry ($\delta = 0$)

and (31) for some values of the Fano x parameter. Fig. 3 was made assuming complete polarization of the ionized atoms ($P_{at} = 1$). Figs 2 and 3 show that for some values of the Fano x parameter the differences between the angular distributions for photoelectrons with spins oppositely directed with respect to the photon propagation direction, are significant. From Fig. 2 it is obvious that, for unpolarized atoms, these differences vanish when $x \to \infty$, whereas from Fig. 3 we conclude that for completely polarized atoms these differences become greater when x tends to infinity.

The total cross sections for photoemission of an electron with well defined spin orientation are as follows:

$$\sigma_{\mu_f} = \left(\frac{2\pi}{3}\right)^3 \frac{a^3}{\lambda} (\Delta R)^2 \{1 + \tfrac{1}{2}(1 + \xi(\mu_f) P_{at}) x^2 \pm \xi(\mu_f) [1 - \xi(\mu_f) P_{at}$$

$$+ 2(1 + \xi(\mu_f) P_{at}) x] (\kappa \cos\alpha / 1 + \kappa^2) \cos\delta - \xi(\mu_f) [1 - (1 - \kappa^2) \cos^2\phi] (P_{at}/1 + \kappa^2) \sin^2\delta\}. \tag{33}$$

If the experimenter refrains from performing a polarization analysis of the photoelectrons emitted in the photoionization experiment, the quantity of interest to him is the cross section σ, i.e. the photoionization cross section averaged over spin orientations of the electron in the initial state $n_0\,^2S_{1/2}$ and summed over spin orientations of the electron in the final state:

$$\sigma = \sum_{\mu_f} \sigma_{\mu_f} = \left(\frac{2\pi}{3}\right)^3 \frac{a^3}{\lambda} (\Delta R)^2 \{x^2 + 2 \pm (2\kappa \cos\alpha / 1 + \kappa^2)(2x - 1) P_{at} \cos\delta\}. \tag{34}$$

From the last equation it follows that in the one-quantum atomic photoeffect the total cross section σ can depend on the polarization state of the light albeit only if the ionized alkali atoms are polarized and simultaneously spin-orbit perturbation in the P states of the continuous spectrum is non-negligible i.e. if $\Delta R = R_{3/2} - R_{1/2} \neq 0$. Our formula (34) is a generalisation of Seaton's formula (32), extended to include the case of polarized alkali atoms ionized by light optionally polarized and in optional experimental geometry.

2.3. Polarization degree of a photoelectron beam propagating in the direction $\vartheta_{\vec{K}}$, $\varphi_{\vec{K}}$

A direct consequence of the nonidentity of the cross sections $d\sigma_{+1/2}/d\Omega_{\vec{K}}$ and $d\sigma_{-1/2}/d\Omega_{\vec{K}}$ as well as $\sigma_{+1/2}$ and $\sigma_{-1/2}$ is that both the photoelectron beam emitted in a direction defined by the angles $\vartheta_{\vec{K}}$ and $\varphi_{\vec{K}}$ and the ensemble of all produced photoelectrons are spin polarized. According to definition formula (1), the spin-polarization degree of a photoelectron beam emitted in a given direction is

$$P_{el}(\vartheta_{\vec{K}}, \varphi_{\vec{K}}) = \frac{A_1 + B_1(\varphi_{\vec{K}}) \sin^2\vartheta_{\vec{K}} + C_1(\varphi_{\vec{K}}) \sin 2\vartheta_{\vec{K}}}{A_2 + B_2(\varphi_{\vec{K}}) \sin^2\vartheta_{\vec{K}} + C_2(\varphi_{\vec{K}}) \sin 2\vartheta_{\vec{K}}}, \tag{35}$$

where

$$A_1 = \pm 4\kappa \cos\alpha \cos\delta + 2P_{at}\{(1 + x^2)[1 - (1 - \kappa^2)\cos^2\phi] \sin^2\delta - (1 + \kappa^2)\}, \tag{36}$$

$$B_1(\varphi_{\vec{K}}) = \pm 4(x - 1)\kappa \cos\alpha \cos\delta + P_{at}\{(1 + \kappa^2)(3 + x^2)$$

$$-(3x^2+5)\,[1-(1-\kappa^2)\cos^2\phi]\sin^2\delta+(x^2-1)\,[(1-\kappa^2)\,(\sin 2\phi\sin 2\varphi_{\vec{K}}\cos\delta$$

$$-\cos 2\phi\cos 2\varphi_{\vec{K}})-(1-(1-\kappa^2)\cos^2\phi)\cos 2\varphi_{\vec{K}}\sin^2\delta]\}, \qquad (37)$$

$$C_1(\varphi_{\vec{K}}) = -2\sin\delta\{\pm(x-1)\kappa\cos\alpha\cos\varphi_{\vec{K}}$$

$$+(1+x^2)P_{\rm at}[(1-(1-\kappa^2)\cos^2\phi)\cos\varphi_{\vec{K}}\cos\delta+\tfrac{1}{2}(1-\kappa^2)\sin 2\phi\sin\varphi_{\vec{K}}]\}, \qquad (38)$$

$$A_2 = 2(1+\kappa^2)+2(x^2-1)\,[1-(1-\kappa^2)\cos^2\phi]\sin^2\delta\mp 4P_{\rm at}\kappa\cos\alpha\cos\delta, \qquad (39)$$

$$B_2(\varphi_{\vec{K}}) = (x^2-1)\,\{1+\kappa^2+(1-\kappa^2)\,[\sin 2\phi\sin 2\varphi_{\vec{K}}\cos\delta-\cos 2\phi\cos 2\varphi_{\vec{K}}]$$

$$-[1-(1-\kappa^2)\cos^2\phi]\,(3+\cos 2\varphi_{\vec{K}})\sin^2\delta\}\pm 4(1+x)P_{\rm at}\kappa\cos\alpha\cos\delta, \qquad (40)$$

$$C_2(\varphi_{\vec{K}}) = -2\sin\delta\{(x^2-1)\,[(1-(1-\kappa^2)\cos^2\phi)\cos\varphi_{\vec{K}}\cos\delta+\tfrac{1}{2}(1-\kappa^2)\sin 2\phi\sin\varphi_{\vec{K}}]$$

$$\pm(1+x)P_{\rm at}\kappa\cos\alpha\cos\varphi_{\vec{K}}\}. \qquad (41)$$

In the limit $x \to \infty$, corresponding to an absence of spin–orbit perturbation in P states of the continuum, we have

$$\lim_{x\to\infty} P_{\rm el}(\vartheta_{\vec{K}}, \varphi_{\vec{K}}) = P_{\rm at}, \qquad (42)$$

irrespective of the polarization state of the light and the emission direction of the photoelectrons. In this case the spin-polarization degree of the electron beam emitted in an arbitrary direction coincides with that of all photoelectrons generated. From Eq. (42) it follows that in the limit $x \to \infty$ it is impossible to attain a spin-polarization degree of photoelectrons greater than the electronic polarization degree of the ionized atoms. In fact, Eq. (42) is the mathematical formulation of an old standing idea of Fues and Hellman [14] that polarized electrons could be obtained by photoionization of polarized alkali atoms. Their prediction was shown to be correct by Hughes et al. and Alguard et al. [2] not very long ago.

Let now consider two particular, practically important cases of our basic Eq. (35). The case of parallel geometry, realized by $\delta = 0$ and $\phi = \pi/2$, is of the greatest significance [2], [15] from the viewpoint of the construction of photoionization sources of polarized electrons. With this geometry, the orientation of photoelectron spin is referred to the photon propagation direction, both for polarized and unpolarized atoms. The angles $\vartheta_{\vec{K}}$ and $\varphi_{\vec{K}}$ are then measured, respectively, with respect to the photon propagation direction and the direction defined by the major semi-axis of the light polarization ellipse, irrespective of whether the atom is polarized or unpolarized. If, in addition, light is unpolarized ($\kappa = 1$, $\langle\cos\alpha\rangle_\alpha = 0$), then from (35) one obtains

$$[P^{\rm un}_{\rm el}(\vartheta_{\vec{K}})]_{\parallel} = P_{\rm at}R(\vartheta_{\vec{K}}), \qquad (43)$$

where

$$R(\vartheta_{\vec{K}}) = \frac{(x^2+3)\sin^2\vartheta_{\vec{K}}-2}{(x^2+3)\sin^2\vartheta_{\vec{K}}+2\cos 2\vartheta_{\vec{K}}} \qquad (44)$$

and is comprised in the interval $-1 \leqslant R(\vartheta_{\vec{K}}) \leqslant +1$. With regard to its range of variability, the quantity $R(\vartheta_{\vec{K}})$ can be dealt with as a depolarization parameter owing to which the modulus of the right side of Eq. (43) is always less than P_{at}, except in some particular cases ((*a*) finite x and $\vartheta_{\vec{K}} = 0, \pi/2, \pi$, (*b*) $x \to \infty$). The depolarization is caused by spin-orbit perturbation of the radial wave function of the electron in P states of the continuum. On the other hand the same spin-orbit perturbation which is a disadvantageous depolarizing factor in the case of polarized atoms + unpolarized photons is the main factor thanks to which the generation of polarized electrons from unpolarized atoms subjected to ionization by light circularly polarized ($\alpha = 0$, $\kappa = 1$) is possible, according to relation

$$P^c_{el}(\vartheta_{\vec{K}}) = \pm 2 \frac{1+(x-1)\sin^2\vartheta_{\vec{K}}}{2+(x^2-1)\sin^2\vartheta_{\vec{K}}}. \tag{45}$$

Figs 4 and 5 show, respectively, the dependence of the spin-polarization degree of photoelectrons P^c_{el} and that of the depolarization parameter R on the emission angle $\vartheta_{\vec{K}}$. It follows from Fig. 5 that in the case of weak spin-orbit perturbation in the continuum (large x) the

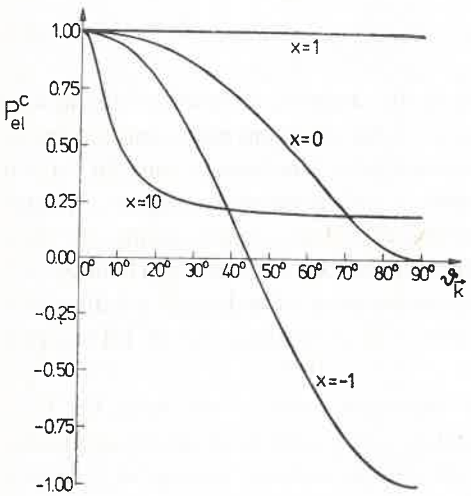


Fig. 4

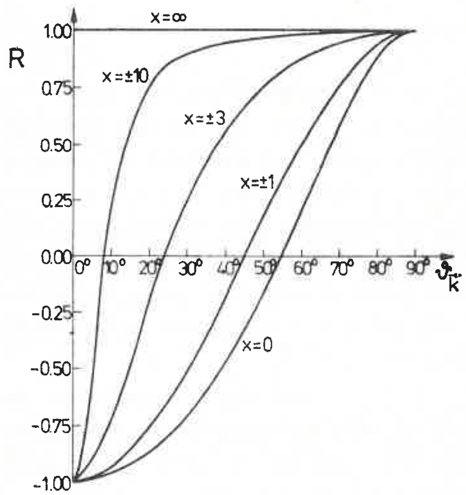


Fig. 5

Fig. 4. Polarization degree of photoelectrons in its dependence on the emission angle $\vartheta_{\vec{K}}$, measured with respect to the photon propagation direction, for the case of unpolarized atoms + right-handed circularly polarized photons

Fig. 5. Depolarization parameter in its dependence on the emission angle $\vartheta_{\vec{K}}$, measured with respect to the photon propagation direction, for the case of polarized atoms + unpolarized photons in parallel geometry

depolarization is essential for small angles $\vartheta_{\vec{K}} \neq 0$ only. It is noteworthy that electrons ejected at right angles to the photon propagation direction undergo no depolarization.

We now deal with the photoelectron emission in the photon propagation direction, defined by $\vartheta_{\vec{K}} = \delta$ and $\varphi_{\vec{K}} = 0$, in conformity with Fig. 1. We have to stress that this direction of photoemission is possible only if $\Delta R \neq 0$ (finite x). For this direction and photon beam polarized circularly (c), linearly (l), or unpolarized (un), Eq. (35) takes, respectively, the following forms:

$$P_{el}^{c}(\delta, 0) = \pm\cos\delta, \quad (46)$$

$$P_{el}^{un}(\delta, 0) = -P_{at}\cos^2\delta, \quad (47)$$

$$P_{el}^{l}(\delta, 0) = (2\sin^2\delta\cos^2\phi - 1)P_{at}. \quad (48)$$

If, in addition, the atoms are unpolarized ($P_{at} = 0$, $\delta = 0$, $\phi = \pi/2$) then from Eq. (46) it follows that the photoelectrons emitted from the ground state of heavy alkali atoms in the direction of the wave vector of a circularly polarized photon beam are completely spin-polarized. Our conclusion, though here derived along quite different lines, is consistent with that drawn previously by Heinzmann et al. [16]. Moreover, it follows from Eqs (46) and (47), that if the circularly polarized or unpolarized photon beam is at right angles to the direction of electronic polarization of the atom, the photoelectron beam emitted along the propagation direction of the photons is entirely unpolarized. We note that Eqs (46) and (47) do not depend on the angle ϕ due to the complete rotational symmetry about the wave vector $\vec{K}_{ph}$. The dependence on ϕ occurs in Eq. (48), which holds for linearly polarized light. In particular, if $\phi = \pi/2$, corresponding to linear polarization in the plane $Z\vec{K}_{ph}$, the relation $P_{el}^{l}(\delta, 0) = -P_{at}$ is satisfied, irrespective of the angle δ between the Z-axis and $\vec{K}_{ph}$ vector. Whereas if $\phi = 0$, corresponding to linear polarization in the plane perpendicular to the plane $Z\vec{K}_{ph}$, $P_{el}^{l}(\delta, 0) = -P_{at}\cos(2\delta)$ and is equal to zero for $\delta = 45°$. This simple example points to an essential influence of the orientation of the electric vector of light (in general, the orientation of the polarization ellipse) on the spin-polarization degree of photoelectrons originating in the ground state of polarized alkali atoms under the action of linearly or elliptically polarized light.

2.4. Polarization degree of all photoelectrons produced and the 100% spin-polarization condition

The expression determining the polarization degree of all photoelectrons, emitted into the full body angle 4π is derived from Eq. (35) by separate integration of its numerator and denominator over the angles $\vartheta_{\vec{K}}$ and $\varphi_{\vec{K}}$. The result is:

$$P_{el} = \frac{\{(1+\kappa^2)x^2 - 2[1-(1-\kappa^2)\cos^2\phi]\sin^2\delta\}P_{at} \pm 2(1+2x)\kappa\cos\alpha\cos\delta}{(1+\kappa^2)(2+x^2) \pm 2(2x-1)P_{at}\kappa\cos\alpha\cos\delta}. \quad (49)$$

To our knowledge, Eq. (49) is at present the most general equation describing the spin-polarization degree of an ensemble of photoelectrons, generated from the ground state of alkali atoms, in its dependence on parameters such as: κ, α, the helicity of the photon, P_{at}, x, δ and ϕ characterizing the interacting objects, the atom and the photon, and the geometry of experiment. From Eq. (49) it follows that if simultaneously $\delta = 0$, $\alpha = 0$, $\kappa = 1$ and $x = 1$ then $P_{el} = \pm 1$, irrespective of the value of P_{at}. Hence, we maintain that independently of the electronic polarization degree of the atoms, complete polarization of all photoelectrons generated is achieved only if the atoms are ionized in parallel geometry by circularly polarized light of a suitable wavelength, ensuring that the equality $x = 1$ shall be fulfilled. This statement is a generalization of the previously derived Fano condition [8] which, however, was true for unpolarized atoms only. The condition of 100% spin-polarization, in form given above, is not correct for two cases, namely: (*i*) $\delta = 0$, $P_{at} = +1$ and right-handed circularly polarized light, (*ii*) $\delta = 0$, $P_{at} = -1$ and left-handed circularly polarized light. In the cases (*i*) and (*ii*) the sole open photoionization channel is $n_0\,{}^2S_{1/2} \to K^2P_{3/2}$, according to the selection rule for magnetic quantum numbers, and entailing $x = 2$. Although x differs from 1 in the cases (*i*) and (*ii*), spin-polarization of photoelectrons is complete. One sees, that, in the cases (*i*) and (*ii*), the generalized Seaton's formula (34) does not depend on the radial integral of $R_{1/2}$, corresponding to the channel $n_0\,{}^2S_{1/2} \to \vec{K}^2P_{1/2}$.

3. *Final remarks*

It is easily convinced that, for unpolarized atoms, complete spin-polarization of photoelectrons is closely related with the isotropy of their angular distributions. It is noteworthy that the inverse conclusion (the isotropy of angular distributions of the photoelectrons ejected from the ground state of unpolarized alkali atoms entails the complete spin-polarization of the photoelectrons) is not correct, because the angular distributions are isotropic for the condition $x = -1$ as well which, however, does not guarantee complete polarization. It is curious that for $x = -1$ the angular distributions of photoelectrons emitted from the ground state of polarized alkali atoms are also isotropic, irrespective of the parameter δ and ϕ.

It can be proved that for atoms unpolarized, or polarized but subjected to ionization in parallel geometry, a magical direction, i.e. one defined by $\vartheta_{\vec{K}} = 54.73°$ and $\varphi_{\vec{K}} = 45°$ exists, for which the relation $d\sigma_{\mu_f}/d\Omega_{\vec{K}} = \sigma_{\mu_f}/4\pi$ is valid. With regards to this relation, the following identity holds

$$[P_{el}(54.73°, 45°)]_{||} = [P_{el}]_{||}. \tag{50}$$

The identity (50) deserves especial attention because it allows us to replace the measurement of the spin-polarization degree for all photoelectrons produced by that of the polarization degree of the beam of photoelectrons emitted in the magical direction. It should be kept in mind that, in the case of elliptical polarization of the light, the magical direction is that subtending the same angle of 54.73° ($\cos^2 54.73° = 1/3$) with the photon propagation direction $||Z$ and the major as well as minor semi-axes of the photon polarization ellipse. In the important cases of circularly polarized and unpolarized light, the magical direction is fully defined by the angle $\vartheta_{\vec{K}} = 54.73°$ only, thanks to complete rotational symmetry about the light propagation direction.

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