SPIN COLLECTIVE EXCITATIONS OF ^{3}He-B

BY R. GONCZAREK

Institute of Physics, Technical University of Wrocław*

(Received March 14, 1978)

The possible spin oscillations of ^{3}He-B have been found and identified. The influence of temperature and of the Fermi-liquid interaction on the spin susceptibility has been discussed in details. All considerations are valid in the collisionless regime.

1. Introduction

In discussing collective excitations of the kind that can appear in superfluid $^3\text{He}-B$, we must not ignore the symmetry of the system and the symmetry of arising excitations. The rotational symmetry of the system allows one to classify the collective excitations, connected with the existence of the pairing forces, in terms of two-particle states, described by the spherical tensors (cf. [1]). For pure P-pairing, the admissible two-particle states are completely described by nine spherical tensors B_{JM} $0 \leqslant J \leqslant 2$, $|M| \leqslant J$ (see appendix). The symmetry properties of these two-particle states are strongly connected with the type of collective excitations. This was often overlooked by some authors, resulting in incorrect predictions.

The problem of the dispersion of the soundlike waves was discussed by many authors [1–9]. The results of these papers agree with each other and with experiment to a satisfactory degree (cf. [6] and [10]).

Quite another situation appears for collective excitations connected with fluctuations in the spin-density. The authors [3, 5, 7, 9, 11–14] are in agreement about the collective excitations with $J = 1$. On the other hand, the collective excitation with $J = 2$ are the theme of the frequent and contradictory considerations (cf. [3, 5, 7, 11, 13, 15]).

The aim of this paper is to discuss this problem once again, taking into account the symmetry of the system and the symmetry of collective excitations.

---

* Address: Instytut Fizyki, Politechnika Wrocławska, Wybrzeże Wyspiańskiego 27, 50-370 Wrocław, Poland.

2. The basic equations of the theory

The calculations presented here are based on the Larkin-Migdal-Czerwonko theory [2, 16], valid in the collisionless regime. The phase B of superfluid ^{3}He is identified with the BW state [17]. The basic equations of this theory which allow one to obtain the spin susceptibility χ_{ij} have the form

$$\overset{s}{\mathscr{T}}{}^i_j = \delta_{ij}+\langle\overset{s}{B}[(L-O)\overset{s}{\mathscr{T}}{}^i_j+(L+O)\overset{a}{\mathscr{T}}{}^i_j$$

$$+2O\overset{s}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i-2O\overset{a}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i-2M\varepsilon_{ikm}\tau^{kn}_j\hat{p}_m\hat{p}_n]\rangle, \tag{1}$$

$$\overset{a}{\mathscr{T}}{}^i_j = \langle\overset{a}{B}[(L-O)\overset{s}{\mathscr{T}}{}^i_j+(L+O)\overset{a}{\mathscr{T}}{}^i_j+2O\overset{s}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i-2O\overset{a}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i-2M\varepsilon_{ikm}\tau^{kn}_j\hat{p}_m\hat{p}_n]\rangle, \tag{2}$$

$$\tau^i_j = \langle f^{\xi}_{-1}\{[N+O+G_sG^-(1-\theta_\xi)]\tau^i_j-2O\tau^m_j\hat{p}_m\hat{p}_i-2M\varepsilon_{ikm}(\overset{s}{\mathscr{T}}{}^k_j+\overset{a}{\mathscr{T}}{}^k_j)\hat{p}_m\}\rangle, \tag{3}$$

$$\chi_{ij} = -\mu_B^2\nu\langle(L-O)\overset{s}{\mathscr{T}}{}^i_j+(L+O)\overset{a}{\mathscr{T}}{}^i_j+2O\overset{s}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i-2O\overset{a}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i-2M\varepsilon_{ikm}\tau^{kn}_j\hat{p}_m\hat{p}_n\rangle, \tag{4}$$

where $\tau^i_j \equiv \tau^{in}_j\hat{p}_n$ — the anomalous vertex function, $\overset{s}{\mathscr{T}}{}^i_j$, $\overset{a}{\mathscr{T}}{}^i_j$, $\overset{s}{B}$, $\overset{a}{B}$ — the even and odd parts, respectively, with respect to $\hat{p}$, of the normal vertex function and the antisymmetric Fermi-liquid interaction

$$B \equiv \overset{s}{B}+\overset{a}{B} = \sum_{l=0}^{\infty}(2l+1)b_lP_l(\hat{p}\hat{p}'),$$

where $b_l \equiv F^a_l/(2l+1) \equiv \frac{1}{4}Z_l/(2l+1)$, comparing with the other notations used for the Landau parameters, f^{ξ}_{-1} — the pairing interaction, $\langle\ldots\rangle$ denotes the averaging over the Fermi-sphere. Kernels L, M, N, O will be defined later. (For details see [2, 14, 18].) Using the gap equation for pure P-pairing, we can rewrite Eq. (3) in the following form

$$\langle\hat{p}_n[(N+O)\tau^{im}_j\hat{p}_m-2O\tau^{km}_j\hat{p}_k\hat{p}_m\hat{p}_i]\rangle = 2\varepsilon_{ikm}\langle M(\overset{s}{\mathscr{T}}{}^k_j+\overset{a}{\mathscr{T}}{}^k_j)\hat{p}_m\hat{p}_n\rangle. \tag{5}$$

3. Spin susceptibility for $T = 0$ and $kv \ll 2\Delta$

Let us first discuss the spin susceptibility if the Fermi-liquid interaction is neglected. The kernels L, M, N, O now (i.e. when $kv \ll 2\Delta$) have the form (cf. [2] or [19])

$$L = -\tfrac{1}{2}g+(1-g)\frac{\boldsymbol{kv}}{\omega}+(1-g+\tfrac{1}{2}gh)\frac{(\boldsymbol{kv})^2}{\omega^2},$$

$$M = -\tfrac{1}{2}g\omega-\tfrac{1}{2}g\boldsymbol{kv}+\frac{gh}{2\omega}(\boldsymbol{kv})^2,$$

$$N = g(\omega^2-\tfrac{1}{2})-g\left(1+h-\frac{h}{2\omega^2}\right)(\boldsymbol{kv})^2,$$

$$O = \tfrac{1}{2}g-\frac{gh}{2\omega^2}(\boldsymbol{kv})^2, \tag{6}$$

where

$$g = \frac{\arcsin \omega}{\omega(1-\omega^2)^{1/2}} \quad \text{and} \quad h = \frac{\omega}{2g}\frac{dg}{d\omega}.$$

In this section we assume that ω and kv are measured in units of 2Δ. In the acoustic limit ($\omega, kv \ll 2\Delta$) $g = 1$ and $h = 0$ (cf. [2]). Substituting the kernels L, M, N, O given by (6) into Eq. (5) and after some calculations, we obtain the equation

$$\omega^2\tau_j^{in} - \tfrac{1}{5}k^2v^2(1+h)(\tau_j^{in} + 2\tau_j^{im}\hat{k}_m\hat{k}_n) - \tfrac{1}{5}\left(1-\tfrac{1}{7}\frac{k^2v^2}{\omega^2}h\right)(\tau_j^{in}+\tau_j^{ni}+\tau_j^{mm}\delta_{in})$$

$$+\tfrac{2}{35}\frac{k^2v^2}{\omega^2}h(\tau_j^{im}\hat{k}_m\hat{k}_n + \tau_j^{mi}\hat{k}_m\hat{k}_n + \tau_j^{mn}\hat{k}_m\hat{k}_i + \tau_j^{nm}\hat{k}_m\hat{k}_i + \tau_j^{mp}\hat{k}_m\hat{k}_p\delta_{in} + \tau_j^{mm}\hat{k}_i\hat{k}_n)$$

$$= -\omega\left(1-\tfrac{1}{5}\frac{k^2v^2}{\omega^2}h\right)\varepsilon_{imn}\hat{k}_m\hat{k}_j - \omega\left(1-\tfrac{2}{5}\frac{k^2v^2}{\omega^2}h\right)(\varepsilon_{ijm}\hat{k}_m\hat{k}_n + \varepsilon_{mjn}\hat{k}_m\hat{k}_i)$$

$$+\tfrac{1}{5}\frac{k^2v^2}{\omega}h(\varepsilon_{ijm}\hat{k}_m\hat{k}_n - \varepsilon_{mjn}\hat{k}_m\hat{k}_i). \tag{7}$$

Since our system does not distinguish any extra directions, the solution of this equation can be given only in the following form (cf. [12])

$$\tau_j^{in} = \tau_0\varepsilon_{imn}\hat{k}_m\hat{k}_j + \tau_1(\varepsilon_{ijm}\hat{k}_m\hat{k}_n + \varepsilon_{mjn}\hat{k}_m\hat{k}_i) + \tau_2(\varepsilon_{ijm}\hat{k}_m\hat{k}_n - \varepsilon_{mjn}\hat{k}_m\hat{k}_i). \tag{8}$$

Comparing the symmetry properties of these three tensors with the symmetry properties of the spherical tensors (A4) we see that τ_0 is connected with collective excitations of the type $J = 1$, $M = 0$, τ_1 with $J = 1$, $M = \pm 1$, and τ_2 with $J = 2$, $M = \pm 1$. The breaking of the symmetry of the system, by distinguishing the direction $\hat{k}$, causes excitations with the same M not to be independent, they are mutually mixed. Hence, both τ_1 and τ_2 are connected with excitations to the states $J = 1$, $M = \pm 1$, and $J = 2$, $M = \pm 1$. However, as long as $kv \ll 2\Delta$, this effects does not influence the dispersion laws, and they have the form (cf. [3, 5, 7])

$$\omega^2 = \tfrac{1}{5}k^2v^2; \quad J = 1, \quad M = 0$$

$$\omega^2 = \tfrac{2}{5}k^2v^2; \quad J = 1, \quad M = \pm 1$$

$$\omega^2 = \tfrac{2}{5} + \tfrac{2}{5}\frac{k^2v^2}{\omega^2}[\omega^2 + h(\omega^2 - \tfrac{3}{7})]; \quad J = 2, \quad M = \pm 1.$$

On the other hand, there are no more tensors for which the solutions of Eq. (7) are not equal to zero. In order to obtain some other excitations we have to distinguish a new direction (cf. [1, 10, 11, 13, 15, 20, 22]).

Substituting the solutions of Eq. (7) into expression (4) we obtain

$$\chi_{ij} = \tfrac{2}{3}\mu_B^2 \nu \left\{ \frac{k^2v^2(\frac{3}{10}g-\frac{1}{2})\hat{k}_i\hat{k}_j}{\omega^2-\frac{1}{5}k^2v^2(1+h)} \right.$$

$$\left. + \frac{k^2v^2(\omega^2-\frac{2}{5})(\frac{1}{10}g-\frac{1}{2})(\delta_{ij}-\hat{k}_i\hat{k}_j)}{\omega^2(\omega^2-\frac{2}{5})-\frac{2}{5}k^2v^2[\omega^2+h(\omega^2-\frac{3}{7})+(\omega^2-\frac{2}{5})(1+h)]} \right\}. \tag{9}$$

As we see, for the system with the BW state ($J = 0$), except for one longitudinal ($J = 1$, $M = 0$) and two transversal ($J = 1$, $M = \pm 1$) gapless modes, two transversal modes ($J = 2$, $M = \pm 1$) with the gap can be excited. It is remarkable that the acoustic limit is equivalent to the appearance of collective excitations with $J = 1$ (cf. [12]).

The pairing interaction can be expressed by means of the spherical tensors (cf.(A2)). Such a procedure makes it easier to identify the collective excitations.

4. *The inclusion of the Fermi-liquid interaction*

In order to include the Fermi-liquid interaction we have to replace the right-hand side of Eq. (7) by the expression:

$$-3\omega\varepsilon_{ikm}\langle \overset{s}{\mathscr{T}}{}^k_j\hat{p}_m\hat{p}_n\rangle - 3kv\varepsilon_{ikm}\langle \overset{a}{\mathscr{T}}{}^k_j\hat{p}_m\hat{p}_n\hat{p}_r\rangle\hat{k}_r + 3\frac{k^2v^2}{\omega}h\varepsilon_{ikm}\langle \overset{s}{\mathscr{T}}{}^k_j\hat{p}_m\hat{p}_n\hat{p}_q\hat{p}_r\rangle\hat{k}_q\hat{k}_r. \tag{10}$$

Using Eqs. (1) and (2) we can express all these averages by means of some combinations of the anomalous vertex function, the unit vector $\hat{k}$ and the Levi-Civita tensor. The obtained equation is similar to Eq. (7), but now very complicated coefficients appear. However, this fact does not change the character of the solutions, allthough effects strongly dependent on the Landau parameter can appear. Hence, the Fermi-liquid interaction introduces quantitative changes only.

Since our calculations are performed with an accuracy up to k^2v^2, only averages of the following types can appear

$$\varepsilon_{l_1l_2l_3}\langle \overset{s}{\mathscr{T}}{}^{l_4}_j\hat{p}_{m_1}\cdots\hat{p}_{m_r}\rangle\hat{k}_{l_5}\hat{k}_{l_6}, \tag{11}$$

$$\varepsilon_{l_1l_2l_3}\langle \overset{a}{\mathscr{T}}{}^{l_4}_j\hat{p}_{m_1}\cdots\hat{p}_{m_r}\rangle\hat{k}_{l_5}, \tag{12}$$

where two of the indices $\{l, m\}$ are free, and others are paired (summational convention). If we insist that $m_s \neq m_t$ if $s \neq t$, then expression (11) vanishes if $r > 4$, and expression (12) if $r > 3$. Now, using the averaging formulae given in appendix B, the expressions (11) and (12) can be obtained from Eqs. (1) and (2). Since the kernels L, M, N, O are analytic functions of ω and kv, the spin susceptibility, in its final form, depends only on five Landau parameters $b_0, \ldots, b_4$. The calculations described above are very complicated and they have not yet been carried out to completion. The complete solution of this problem exists in the acoustic limit, and was given by Czerwonko [12] (cf. also [9]).

5. *Non-zero temperatures*

For non-zero temperatures the kernels L, M, N, O become more complicated (cf. [4, 21, 23]). They are not analytic functions of ω and kv (except for the cases $\omega = 0$, cf. [18], or $kv = 0$). This forces us to impose restrictions on the Fermi-liquid interaction (cf. [14]). Some methods for such calculations were given in [4, 7, 8, 11, 13–15]. Though these calculations are now very complicated, there are not any reasons for any qualitative change in the solution of our problem (cf. [14], where the spin susceptibility in the acoustic limit with two non-vanishing Landau parameters was obtained). In our paper we confine ourselves only to the derivation of the gap equation for the collective excitations $J = 2$, which can be obtained for $kv = 0$.

Now, the kernels L, M, N, O have the form (cf. [21, 23])

$$-L = O = \tfrac{1}{2} F, \quad M = -\frac{\omega}{4\Delta} F, \quad N = \left(\frac{\omega^2}{4\Delta^2} - \tfrac{1}{2}\right) F, \tag{13}$$

where

$$F = \int_{\Delta}^{\infty} dE \frac{\operatorname{th}(E/2T)}{(E^2 - \Delta^2)^{1/2}} \frac{4\Delta^2}{4E^2 - \omega^2},$$

we find the following gap equation: (cf. [13])

$$\omega^2(1 + \tfrac{2}{5} b_2 F) = \tfrac{8}{5} \Delta^2 (1 + b_2 F). \tag{14}$$

The modes with $J = 2$ are connected with the spin oscillations of D-wave symmetry, thus, the frequency of these modes is modified by the Landau parameter b_2.

6. *Conclusions*

In discussing Eq. (9) we can suspect that the system can be excited by means of the monochromatic and polarized electromagnetic wave. This, of course, need not be right since, in our predictions, we went beyond the accuracy of the applied theory. Eq. (14) is very helpful to estimate the frequency of such mode.

The authors would like to thank Professor J. Czerwonko for comments and helpful discussions in reference to these topics.

APPENDIX A

Spherical tensors

In the case considered (angular momentum $L = 1$ and spin $S = 1$) the spherical tensors have the form (cf. [3, 7])

$$B_{JM} = B_{JM}^{kn} \sigma^k \hat{p}_n i\sigma^y,$$

where $0 \leqslant J \leqslant 2$, $|M| \leqslant J$.

$$B_{00}^{kn} = \frac{1}{\sqrt{3}} \delta_{kn},$$

$$B_{10}^{kn} = \frac{1}{\sqrt{2}} (\delta_{ky}\delta_{nx} - \delta_{kx}\delta_{ny}),$$

$$B_{11}^{kn} = \tfrac{1}{2} (\delta_{ky}\delta_{nz} - \delta_{kz}\delta_{ny} - i\delta_{kz}\delta_{nx} + i\delta_{kx}\delta_{nz}),$$

$$B_{20}^{kn} = -\frac{1}{\sqrt{6}} (\delta_{kn} - 3\delta_{kz}\delta_{nz}),$$

$$B_{21}^{kn} = \tfrac{1}{2} (-\delta_{kx}\delta_{nz} - \delta_{kz}\delta_{nx} + i\delta_{ky}\delta_{nz} + i\delta_{kz}\delta_{ny}),$$

$$B_{22}^{kn} = \tfrac{1}{2} (\delta_{kx}\delta_{nx} - \delta_{ky}\delta_{ny} - i\delta_{kx}\delta_{ny} - i\delta_{ky}\delta_{nx}),$$

and

$$B_{JM}^{*kn} = (-1)^M B_{J-M}^{kn}. \tag{A1}$$

These quantities have the following properties

$$B_{JM}^{im} B_{JM}^{*jn} = \delta_{ij}\delta_{mn}, \tag{A2}$$

$$B_{JM}^{im} B_{J'M'}^{*im} = \delta_{JJ'}\delta_{MM'}, \tag{A3}$$

$$B_{00}^{im} = B_{00}^{mi}; \qquad B_{00}^{ii} = \sqrt{3},$$

$$B_{1M}^{im} = -B_{1M}^{mi}: \qquad B_{1M}^{ii} = 0,$$

$$B_{2M}^{im} = B_{2M}^{mi}; \qquad B_{2M}^{ii} = 0. \tag{A4}$$

APPENDIX B

Averaging formulae

$$\langle B \rangle = b_0, \tag{B1}$$

$$\langle B\hat{p}_i' \rangle = b_1 \hat{p}_i, \tag{B2}$$

$$\langle B\hat{p}_i'\hat{p}_j' \rangle = \tfrac{1}{3}(b_0 - b_2)\delta_{ij} + b_2 \hat{p}_i \hat{p}_j, \tag{B3}$$

$$\langle B\hat{p}_i'\hat{p}_j'\hat{p}_k' \rangle = \tfrac{1}{5}(b_1 - b_3)(\hat{p}_i\delta_{jk} + \hat{p}_j\delta_{ik} + \hat{p}_k\delta_{ij}) + b_3\hat{p}_i\hat{p}_j\hat{p}_k, \tag{B4}$$

$$\langle B\hat{p}_i'\hat{p}_j'\hat{p}_k'\hat{p}_n' \rangle = (\tfrac{1}{15} b_0 - \tfrac{2}{21} b_2 + \tfrac{1}{35} b_4)(\delta_{ij}\delta_{kn} + \delta_{ik}\delta_{jn} + \delta_{in}\delta_{jk})$$
$$+ \tfrac{1}{7}(b_2 - b_4)(\hat{p}_i\hat{p}_j\delta_{kn} + \hat{p}_i\hat{p}_k\delta_{jn} + \hat{p}_i\hat{p}_n\delta_{jk} + \hat{p}_j\hat{p}_k\delta_{in} + \hat{p}_j\hat{p}_n\delta_{ik} + \hat{p}_k\hat{p}_n\delta_{ij}) + b_4\hat{p}_i\hat{p}_j\hat{p}_k\hat{p}_n. \tag{B5}$$

REFERENCES

[1] A. J. Leggett, *Rev. Mod. Phys.* **47**, 331 (1975).
[2] J. Czerwonko, *Acta Phys. Pol.* **32**, 335 (1967).
[3] Yu. A. Vdovin, in *Application of the Methods of Quantum Field Theory of the Many-Body Problems*, Atomizdat, Moscow 1963 (in Russian).
[4] K. Maki, *J. Low Temp. Phys.* **16**, 465 (1974).
[5] K. Nagai, *Prog. Theor. Phys.* **54**, 1 (1975).
[6] P. Wölfle, *Phys. Rev.* **14B**, 89 (1976).
[7] K. Maki, *J. Low Temp. Phys.* **24**, 755 (1976).
[8] K. Maki, H. Ebisawa, *J. Low. Temp. Phys.* **26**, 627 (1977).
[9] L. Jacak, *J. Phys. C* **10**, 4701 (1977).
[10] J. C. Wheatley, *Rev. Mod. Phys.* **47**, 415 (1975).
[11] L. Tewordt, D. Fay, P. Dörre, D. Einzel, *J. Low Temp. Phys.* **21**, 645 (1975).
[12] J. Czerwonko, *Zh. Eksp. Teor. Fiz.* **71**, 1099 (1976).
[13] K. Nagai, *J. Low Temp. Phys.* **28**, 139 (1977).
[14] R. Gonczarek, *Acta Phys. Pol.* **A54**, 141 (1978).
[15] L. Tewordt, D. Einzel, *Phys. Lett.* **56A**, 97 (1976).
[16] A. I. Larkin, A. B. Migdal, *Zh. Eksp. Teor. Fiz.* **44**, 1703 (1963).
[17] P. W. Anderson, F. Brinkman, *Phys. Rev. Lett.* **30**, 1108 (1973).
[18] R. Gonczarek, L. Jacak, *Acta Phys. Pol.* **A53**, 41 (1978).
[19] V. G. Vaks, V. M. Galitskii, A. L. Larkin, *Zh. Eksp. Teor. Fiz.* **41**, 1956 (1961).
[20] A. J. Leggett, *Ann. Phys.* (New York) **85**, 11 (1974).
[21] K. Maki, H. Ebisawa, *J. Low Temp. Phys.* **15**, 213 (1974).
[22] P. Wölfle, *Phys. Rev. Lett.* **37**, 1279 (1976).
[23] A. J. Leggett, *Phys. Rev.* **147**, 119 (1966).