POLARIZATION MODULATION TECHNIQUE AS A METHOD OF MEASURING THE STOKES PARAMETERS OF THE LIGHT BEAM*,**

By A. J. Wojtowicz and H. J. Łożykowski

Institute of Physics, Nicholas Copernicus University, Toruń***

(Received April 24, 1978)

The method of measuring the Stokes parameters of the light beam is described. In the method piezo-optical birefringence modulator, which modulates in the known way the state of polarization, is used. Possibilities of measuring circular and linear dichroism, birefringence, ellipticity and degree of polarization are demonstrated. Theoretical analysis is uniform for all kinds of measurements and is based on Mueller calculus. As an example, linear birefringence of achromatic and typical quarter-wave plates, circular dichroism and birefringence of vitaminum B_{12}, optical rotatory power of sugar and degree of polarization of photoluminescence 1,1, 4 4-tetraphenyl — 1,3-butadiene were measured.

1. Introduction

The purpose of this paper is to review briefly some ideas concerning measurement of the polarization parameters of the quasi-monochromatic light beam by the technique of phase modulation. A photoelectric method of measuring the Stokes parameters was proposed by Budde [1]. However, his method gives only three Stokes parameters. Method proposed by Ioshpa and Obridko [2] allows one to measure independently the four Stokes parameters. Phase modulation technique was broadly used mainly in measurements of circular dichroism [3], ellipsometry [4] and birefringence [5]. The main advantages of piezo-optical birefringence modulators, in comparison to Pockels or Kerr cells, are large angular aperture, broad wavelength range, low voltage necessary to drive modulator [6].

This paper presentes uniform and common way to measure all polarization effects through measurement of components of the Stokes vector. In Section 2 we will describe shortly the performance of photoelastic modulator. In Section 3 we will define the Stokes vector. In the next section we will show how to measure the Stokes parameters by photo-

---

* Dedicated to Professor Aleksander Jabłoński on the occasion of his 80th birthday.

** This work was supported in part by the Polish Academy of Sciences, Contract M.R.I.4.

*** Address: Instytut Fizyki UMK, Grudziądzka 5, 87-100 Toruń, Poland.

elastic modulator technique. After this we will describe experimental arrangement used in our laboratory in polarization experiments. The general formulas presented in Section 4 will be applied in Section 6 to special cases as linear and circular dichroism, birefringence and degree of polarization.

2. *Photoelastic modulator*

The detailed description of photoelastic modulator one can find in [3, 4, 6, 7]. Here we will point out only the most important things.

The optical element in the photoelastic modulator is isotropic, fused quartz block as shown in Fig. 1. When the strain is induced in this block along x axis by piezoelectric

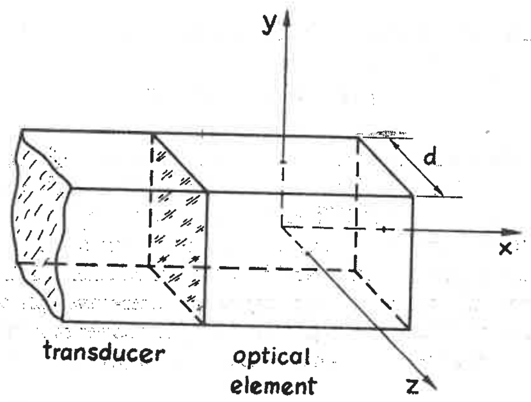

Fig. 1. Photoelastic modulator

transducer cemented to one of its ends, the block becomes uniaxial. So, when the light beam passes it along z axis there is phase difference between its x and y polarized components. The value of phase difference δ can be expressed as:

$$\delta = dS/\lambda, \tag{2.1}$$

where S is proportional to the strain, d is the thickness of the block and λ is wavelength of the light. This relation is based on the assumption about linear dependence of the index of refraction on the strain. Strain, in turn, is proportional to the voltage applied to the piezoelectric transducer. If the applied voltage is a sinusoidal function of the time we will obtain:

$$\delta = \Delta \sin \omega t, \tag{2.2}$$

where Δ is the amplitude of phase variation. We will also have

$$\Delta \sim V/\lambda, \tag{2.3}$$

with V being the voltage amplitude.

The experimental check of the linear relationship between V and λ is shown in Fig. 2 for few values of Δ. Fig. 2 demonstrates also the validity of an assumption about linear relationship between Δ and V/λ. This latter relation is important because it can be used to find voltage which should be applied for given wavelength to obtain a fixed value of Δ. We can conclude that photoelastic modulator can be treated as variable retarder plate.

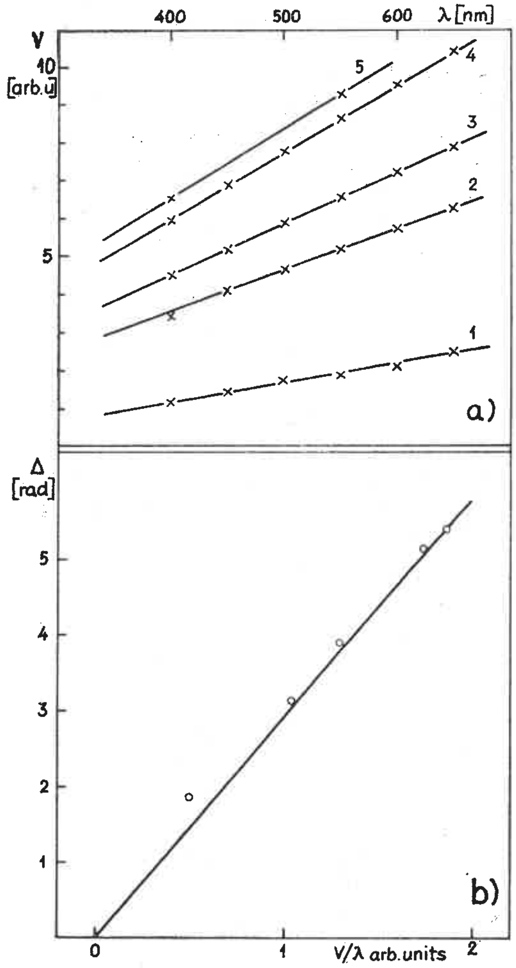

Fig. 2. (a) Dependence of V on λ for few values of Δ. Δ were chosen as specific points (extremum or zero) of Bessel functions. $1 \to \Delta = 1.85$, $2 \to \Delta = 3.1$, $3 \to \Delta = 3.83$, $4 \to \Delta = 5.1$, $5 \to \Delta = 5.34$. (b) Dependence of Δ on V/λ. Experimental points were obtained from (a).

3. The Stokes vector

If a plane wave emitted from a steady source is quasi-monochromatic then it can have the properties of a perfectly polarized monochromatic wave, however it is also possible that it will be completely unpolarized.

The Stokes parameters are defined in the following way [8–11]:

$$
\begin{aligned}
I &= \langle a_x^2\rangle+\langle a_y^2\rangle,\\
M &= \langle a_x^2\rangle-\langle a_y^2\rangle,\\
C &= \langle 2a_xa_y\cos(\theta_x-\theta_y)\rangle,\\
S &= \langle 2a_xa_y\sin(\theta_x-\theta_y)\rangle,
\end{aligned}
\tag{3.1}
$$

where brackets indicate time averaging, a_x, a_y, θ_x, θ_y are amplitudes and phases of x and y components of the electric field travelling in the z direction. Parameters defined by equations (3.1) constitute together the so-called "Stokes vector". They are related to the parameters describing polarization ellipse.

The degree of polarization ϱ is defined as

$$\varrho = \frac{1}{I}\sqrt{M^2+C^2+S^2}\,. \tag{3.2}$$

4. *The measurement of the Stokes parameters of the light beam by photoelastic modulator technique*

Detection system contains four elements: quarter-wave plate, photoelastic modulator, polarizer and detector. Such system is capable of measuring the Stokes parameters of the light beam [2].

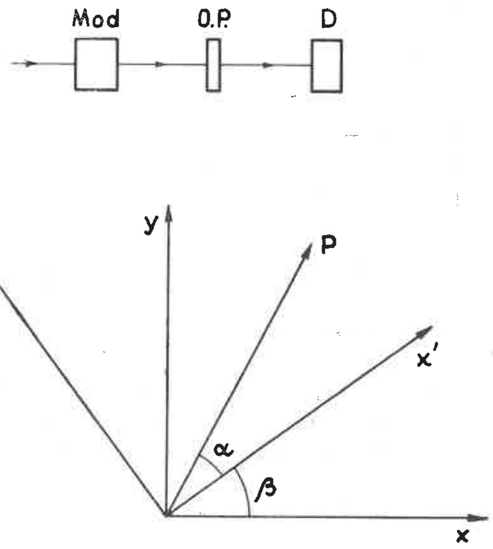

Fig. 3. The block diagram of the detection system and relative orientations of optical elements. Mod — photoelastic modulator, OP — output polarizer, D — detector, XY — modulator frame as in Fig. 1, $X'Y'$ — Stokes vector frame, P — axis of the output polarizer

Orientations of these elements are shown in Fig. 3. The Stokes vector components are defined in $X'Y'$ frame. Modulator axes are X and Y (as in Fig. 1). To simplify the equations we have chosen $\alpha = 90°$ and $\beta = 45°$. Then we will obtain

$$I_c = I; \quad I_1 = 1.04\, S \sin \omega t; \quad I_2 = 0.86\, M \cos 2\omega t. \tag{4.1}$$

Sometimes it is necessary to use quarter-wave plate. Quarter-wave plate is placed before modulator. An angle θ between axis of quarter-wave plate and X axis can take two values. For $\theta = 0$ we have

$$I_c = I; \quad I_1 = 1.04\, C \sin \omega t; \quad I_2 = 0.86\, M \cos 2\omega t, \tag{4.2}$$

and for $\theta = 45°$.

$$I_c' = I; \quad I_1' = 1.04 M \sin \omega t; \quad I_2' = 0.86 S \cos 2\omega t, \tag{4.3}$$

where I_c, I_1, I_2, I_2', I_1' are constant, ω — frequency, 2ω — frequency terms in the signal measured by detector. These equations were obtained under condition that $\Delta = 2.4$, then 1.04 and 0.86 are values of the appropriate Bessel functions of the first kind multiplied by 2. Four properly chosen equations from (4.2) and (4.3) will give all the Stokes parameters of the light beam. Sometimes it is also possible to use equations (4.1).

5. *Experimental arrangement*

In this section we will describe shortly an experimental system used in our laboratory. As the source of the light we use mercury (200 W), xenon (100 W) or halogen lamp (500 W). A light from the lamp passes filter, monochromator M_1 (Zeiss SPM-2), mechanical chop-

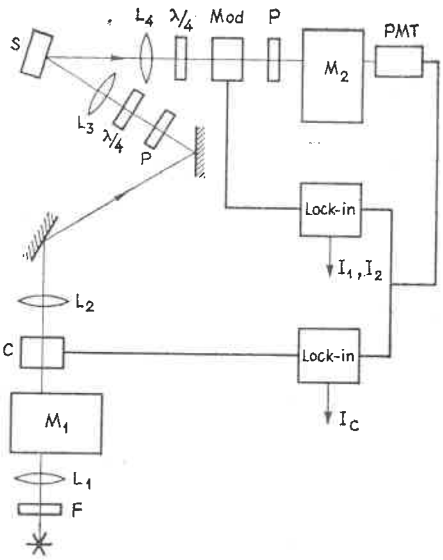

Fig. 4. Experimental system used to measure components of the Stokes vector in the reflection geometry

per C (1 kHz) and through polarizer and eventually quarter-wave plate falls on the sample S. The light, reflected or emitted, is gathered by lens and goes through quarter-wave plate, photoelastic modulator Mod (Morvue model PEM-3), polarizer, monochromator M_2 (Zeiss GDM-1000) to the photomultiplier PMT (RCA C 31034). Signal from phototube contains 1 kHz component which is detected by first lock-in amplifier (Unipan 232) supplied with reference voltage by chopper C (1 kHz signal is equal to I_c). High frequency components ($\omega = 50$ kHz, $2\omega = 100$ kHz) are detected by second lock-in (PARC 128 A). 50 kHz signal is equal to I_1 and 100 kHz signal is equal to I_2.

Reflection geometry shown in Fig. 4 can be easily changed to transmission geometry. An experimental arrangement shown in this figure was used in both geometries to measure different optical quantities as described in Section 6.

6. *Theoretical analysis and experimental results of measurements of some optical quantities*

6.1. Linear dichroism and birefringence

Let us assume that we use transmission geometry in the measurement and the incident beam is perpendicular to the optic axis of the sample. The sample is so oriented that an angle between its optic axis and X' axis of the Stokes vector frame is 90° as shown in Fig. 5. The Mueller matrix of sample showing linear dichroism is:

$$[\mathrm{LD}] = \frac{1}{2}\begin{vmatrix} k_\parallel^2+k_\perp^2, & k_\parallel^2-k_\perp^2, & 0, & 0 \\ k_\parallel^2-k_\perp^2, & k_\parallel^2+k_\perp^2, & 0, & 0 \\ 0, & 0, & 2k_\parallel k_\perp, & 0 \\ 0, & 0, & 0, & 2k_\parallel k_\perp \end{vmatrix}. \tag{6.1.1}$$

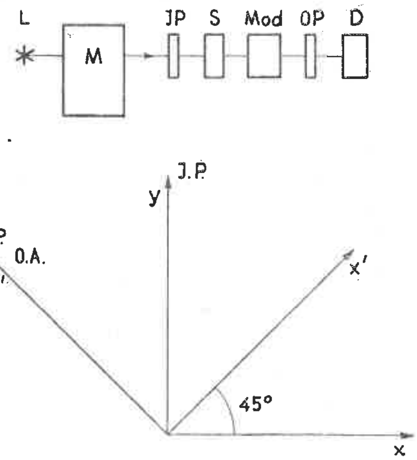

Fig. 5. The block diagram of the experimental system for measuring linear dichroism and birefringence. L — source of the light, M — monochromator, IP — input polarizer (input polarizer axis), S — sample (OA — sample optic axis), Mod — photoelastic modulator, OP — output polarizer (output polarizer axis), D — detector, XY — modulator frame, $X'Y'$ — Stokes vector frame

Coefficients $k_{\parallel}^2$, $k_{\perp}^2$ are defined by the intensities

$$J_{\parallel} = k_{\parallel}^2 J_{0\parallel}; \quad J_{\perp} = k_{\perp}^2 J_{0\perp}, \tag{6.1.2}$$

where $J_{0\parallel}$, $J_{0\perp}$ and $J_{\parallel}$, $J_{\perp}$ are intensities of the incident and transmitted beams polarized parallelly and perpendicularly to the optic axis of the investigated sample. If the sample shows linear dichroism and birefringence the Stokes vector of the light transmitted through it will be

$$\tfrac{1}{2}(k_{\parallel}^2 + k_{\perp}^2, k_{\parallel}^2 - k_{\perp}^2, 2k_{\parallel}k_{\perp} \cos\delta, -2k_{\parallel}k_{\perp} \sin\delta). \tag{6.1.3}$$

Using equations (4.1) we have

$$I = I_c; \quad M = I_2/0.86; \quad S = I_1/1.04 \tag{6.1.4}$$

and

$$\alpha = M/I; \quad \sin\delta = S/\sqrt{I+M}\sqrt{I-M}, \tag{6.1.5}$$

where $\alpha = (k_{\parallel}^2 - k_{\perp}^2)/(k_{\parallel}^2 + k_{\perp}^2)$ and $\delta = 2\pi d(n_{\parallel} - n_{\perp})/\lambda$. $n_{\parallel}$ and $n_{\perp}$ are refraction indices for light polarized parallelly and perpendicularly to the optic axis of the sample. When $k_{\parallel}^2 + k_{\perp}^2 \gg k_{\parallel}^2 - k_{\perp}^2$ we can use simpler expressions, namely

$$\alpha = M/I; \quad \sin\delta = S/I. \tag{6.1.6}$$

One can see that this method is good and accurate for the measurement of small birefringence.

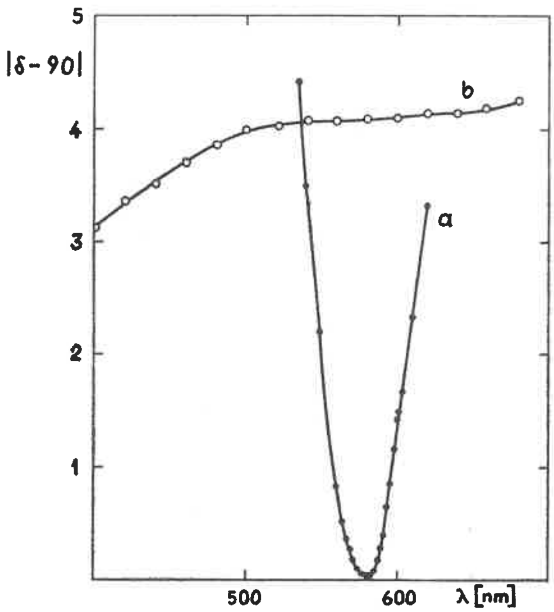

Fig. 6. Phase shift between fast and slow axis polarized components of the light for normal (a) and achromatic (b) quarter-wave plates

In comparison to the method of Wardzyński [12] our method is not much worse as far as the precision is concerned and is certainly more convenient. When the phase shift δ is about $\pi/2$ (calibration of quarter-wave plates, Fig. 6) it is better to use geometry with OA parallel to Y axis and JP parallel to X' axis.

6.2. Circular dichroism and birefringence

Let us assume that we use transmission geometry. The Mueller matrix of the sample showing circular dichroism is:

$$[\mathrm{CD}] = \frac{1}{2}\begin{vmatrix} k_1^2+k_2^2, & 0, & 0, & k_1^2-k_2^2 \\ 0, & 2k_1k_2, & 0, & 0 \\ 0, & 0, & 2k_1k_2, & 0 \\ k_1^2-k_2^2, & 0, & 0, & k_1^2+k_2^2 \end{vmatrix}, \tag{6.2.1}$$

where coefficients k_1^2, k_2^2 are defined by relations

$$J_L = k_1^2 J_{0L}, \quad J_R = k_2^2 J_{0R}. \tag{6.2.2}$$

J_{0L}, J_{0R}, J_L, J_R are the intensities of the incident and transmitted light beams left and right circularly polarized. The Mueller matrix of the sample showing circular birefringence is simply rotation matrix $[R_\varphi]$, where rotation angle φ can be written as

$$\varphi = (\pi d/\lambda)(n_L - n_R), \tag{6.2.3}$$

where n_L, n_R are refraction indices for left and right circularly polarized light.

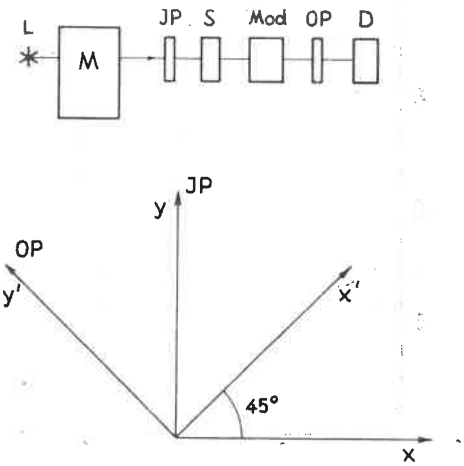

Fig. 7. The block diagram of the experimental system used for measuring circular dichroism and birefringence

The Stokes vector of the incident light should be (1, 0, 1, 0). This can be realized in the configuration shown in Fig. 7. The Stokes vector of the light transmitted through the sample will be

$$\tfrac{1}{2}(k_1^2+k_2^2,\ 2k_1k_2\sin 2\varphi,\ 2k_1k_2\cos 2\varphi,\ k_1^2-k_2^2). \tag{6.2.4}$$

Using equations (4.1) we have

$$I = I_c; \quad M = I_2/0.86; \quad S = I_1/1.04,$$

and

$$\alpha = S/I; \quad \sin 2\varphi = M/\sqrt{I+S}\,\sqrt{I-S}, \tag{6.2.5}$$

where

$$\alpha = (k_1^2-k_2^2)/(k_1^2+k_2^2).$$

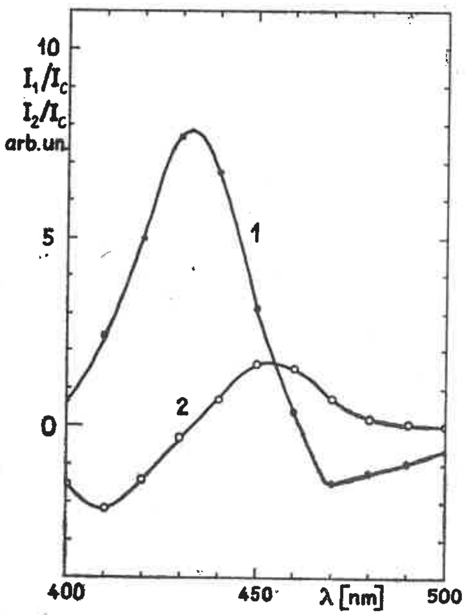

Fig. 8

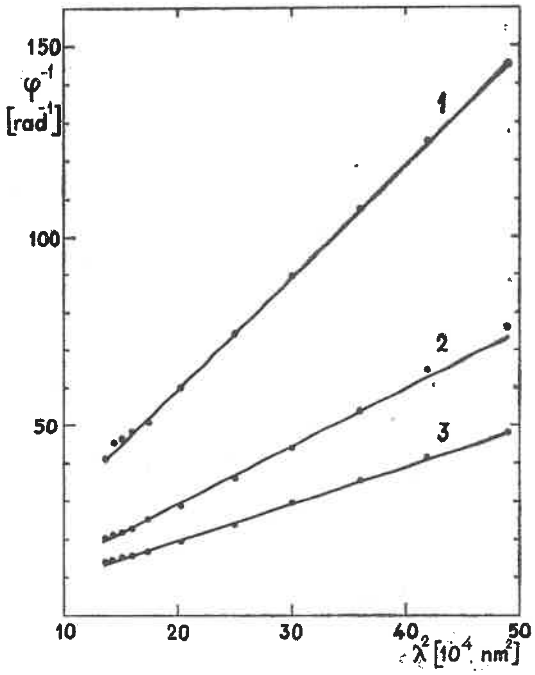

Fig. 9

Fig. 8. Optical rotatory dispersion and dichroism of vitaminum B_{12}, *1* — dichroism I_1/I_c, *2* — birefringence I_2/I_c

Fig. 9. Optical rotatory dispersion of sugar. *1* — concentration 0.1 mol/l, *2* — concentration 0.2 mol/l, *3* — concentration 0.3 mol/l

When $k_1^2+k_2^2 \gg k_1^2-k_2^2$ we can use simpler equations

$$\alpha = S/I; \quad \sin 2\varphi = M/I. \tag{6.2.6}$$

Using this method optical rotatory dispersion and circular dichroism for vitaminum B_{12} were measured. Also optical rotatory dispersion for sugar was measured and validity of

Drude equation for this substance was checked as shown in Fig. 8 and 9. It is worth noting that angles of rotation of the plane of polarization as 20′ were measured with good accuracy and reproducibility.

6.3. Degree of polarization of luminescence

Independent measurement of all the four Stokes parameters makes it possible to evaluate degree of polarization from equation (3.2). It is worth noting that in such a case it is not necessary to assume that luminescence which is orthogonally polarized to the exciting light is not coherent with luminescence polarized in the same manner. It is possible with this technique to find the exact degree of polarization and polarization ellipse parameters for the completely polarized component without any assumptions. Let us consider first the case when exciting light is linearly polarized with the Stokes vector (1, 1, 0, 0). Then if the polarized component of the luminescence is also (1, 1, 0, 0) the Stokes vector of

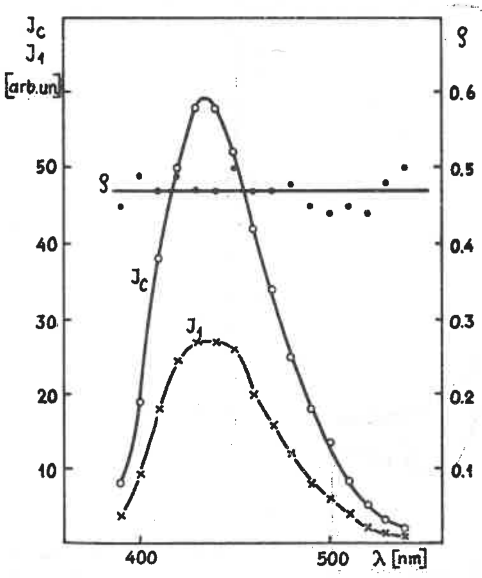

Fig. 10. Degree of polarization of photoluminescence for 1,1,4,4-tetraphenyl-1,3-butadiene. Exciting light is 365 nm linearly polarized mercury line. Excitation was perpendicular to the line of observation of luminescence. Polarized part of luminescence was also linearly polarized

the whole luminescence light will be (1, ϱ, 0, 0). In this case it is enough to measure I_c and I_2 to obtain degree of polarization. But when the polarization of polarized component is not linear or there is rotation included we have to use quarter-wave plate and equations (4.2) and (4.3) to obtain ϱ from (3.2). The second interesting case is when exciting light is circularly polarized. All optical pumping and orientation experiments [13, 14] are of this type. In such experiment exciting light is (1, 0, 0, 1). Also in this case when polarized part of luminescence is (1, 0, 0, 1) the whole light will have vector (1, 0, 0, ϱ). Again we have to measure only I_c and I_1 to obtain ϱ. Example of degree of polarization measurement is shown in Fig. 10. Experiments of this type are now performed in our laboratory and will be published later.

7. *Discussion*

Photoelastic modulator technique presents a wide variety of different possibilities. The accuracy of the method is high, practically limitations are determined not by the method but by such factors as intensity of the light source, resolving power of the monochromators, sensitivity, linear range and noise performance of the photomultiplier and electronic equipment used to amplify and select signals.

The authors appreciate A. Szyjkowski for assistance in measurements and F. Firszt for the calculations and construction of the achromatic quarter-wave plates.

REFERENCES

[1] W. Budde, *Appl. Optics* **1**, 201 (1962).
[2] B. A. Isopha, V. N. Obridko, *Opt. Spektrosk.* **25**, 119 (1963).
[3] L. F. Mollenauer, D. Downie, H. Engstrom, W. B. Grant, *Appl. Optics* **8**, 661 (1969).
[4] S. N. Jasperson, S. E. Schnatterly, *Rev. Sci. Instrum.* **40**, 761 (1969).
[5] F. A. Modine, R. W. Major, E. Sonder, *Appl. Optics* **14**, 757 (1975).
[6] J. C. Kemp. *J. Opt. Soc. Am.* **59**, 950 (1969).
[7] R. K. Bauer, A. Kowalczyk, A. Balter, H. Cherek, W. Rogulski, *Optica Applicata* **5**, 43 (1975).
[8] P. Clarcke, I. F. Grainger, *Polarized Light and Optical Measurements*, Pergamon Press, London 1971.
[9] W. A. Shurcliff, *Polarized Light: Production and Use*, Harvard University Press, Cambridge, Mass. 1962.
[10] J. M. Stone, *Radiation and Optics*, Mc Graw Hill Book Company, New York 1963.
[11] M. Born, E. Wolf, *Principles of Optics*, Pergamon Press, New York 1959.
[12] W. Wardzyński, *Acta Phys. Pol.* **A39**, 21 (1971).
[13] R. R. Parsons, *Can. J. Phys.* **49**, 1850 (1971).
[14] A. Bonnot, R. Planel, C. Benoit á la Guillaume, *Phys. Rev.* **B9**, 690 (1974).