NONCOHERENT TWO-PHOTON AMPLIFICATION OF A LASER PULSE

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An analysis was made of noncoherent two-photon amplification of a laser pulse having a plane, cylindrical and spherical wave-front, in a medium with a monotonic distribution of population inversion. Approximative analytical and numerical solutions to the propagation equation are given. It is shown that the basic effect which accompanies pulse amplification is its time compression. A detailed analysis of this effect is given. Also the effect of a multi-photon radiation-absorption upon the pulse-duration variation in the amplifier is discussed.

1. Introduction

The possibility of generating and amplifying a coherent light with the aid of two-photon transitions was indicated in the investigations of Sorokin and Breslau [1] and Prokhorov and Selivanenko [2]. The idea formulated in [1, 2] inspired a number of investigations (see e. g. [3]) wherein the properties and fabrication conditions of a two-photon generator were analyzed. In [4–7] the possibility was studied of producing powerful, ultra-short pulses in a two-photon amplifier by making use of simple two-photon transitions.

The process of two-photon amplification, unlike the single-photon one, is non-linear for both the strong and weak signal. In the case of a weak signal we have to deal with a non-linearity due to the multi-photon interaction between the radiation and the medium. The non-linearity of the interaction between a strong signal and the medium is addi-

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tionally conditioned by the saturation effect. It is to be expected that the above effects will lead to the occurrence of a number of interesting phenomena, which on the one hand may provide valuable information on the structure and the mechanisms of the interaction between the laser radiation and the medium, and on the other hand may extend the range of application of this radiation. The literature on laser pulse propagation in a two-photon amplifier is still scarce and no comprehensive analysis of the problem is available.

The present paper deals with the evolution of a laser pulse with a plane, cylindrical and spherical wave front in a two-photon amplifier with a monotonic inversion distribution. The description is based on equations of energy balance, which are valid in the region of non-coherent interaction.

2. *Model of the medium and equations of propagation*

Let us consider the propagation of a laser radiation pulse in an isotropic medium with population inversion of two-levels, the energy difference of which is $E_2 - E_1 = 2\hbar\omega_0$ at the centre of the transition line, where ω_0 is the central frequency of the radiation spectrum.

Let us assume that the refractive index of the medium is constant, the transition-line broadening is homogeneous and the interaction between the pulse and the medium is of a non-coherent nature.

Let us assume, moreover, that in addition to the processes of emission and absorption, there occur, in the medium, linear dissipative processes and as the variation of the wave radiation front is conditioned by these processes, it can be neglected.

The approximate equations of pulse propagation with a plane, cylindrical or spherical wave front, which result from the principle of conservation of energy, have the form [8]

$$\left[\frac{1}{v}\frac{\partial}{\partial t} + \frac{1}{r^a}\frac{\partial}{\partial r} r^a - K(t, r)\right] I(t, r) = 0$$

$$\frac{\partial N(t, r)}{\partial t} + \frac{N(t, r) - N^e(r)}{T_1} + s\sigma N(t, r) I^2(t, r) = 0, \tag{1}$$

where $K(t, r) = 2\sigma N(t, r) I(t, r) - \varrho$ is a function of the amplification of the medium, I — radiation intensity[1], N — population inversion (N^e — inversion in the state of thermodynamic equilibrium), T_1 — time of inversion relaxation, σ — cross-section for two-photon forced transitions at unit intensity, ϱ — linear-loss factor, v — velocity of light in the medium, s — parameter dependent on laser action diagram (in two-level diagram $s = 2$), $a = 0, 1, 2$ for a plane-, cylindrical- and spherical wave, respectively.

In the case which is the most interesting from the practical standpoint $\tau_p \ll T_1$ (τ_p — half width of pulse) the set (1) may be reduced to the equation

$$\frac{\partial I(\tau, r)}{\partial r} = \beta(r) I^2(\tau, r) \exp\left[-s\sigma \int_{-\infty}^{\tau} I^2(\tau', r) d\tau'\right] - \left(\varrho + \frac{a}{r}\right) I(\tau, r), \tag{2}$$

[1] By radiation intensity is meant here photon flux density (in units: photon/cm^2 s).

where $\beta(r) = 2\sigma N^e(r)$, $\tau = t - \frac{r}{v}$. Making use of the equation (2) the evolution of a pulse in a two-photon amplifier may be investigated. The case of a weak signal[2] is the first to be considered.

3. *Evolution of weak-signal intensity in an amplifier*

For a weak signal

$$\int_{-\infty}^{\infty} I^2(\tau')d\tau' \ll (s\sigma)^{-1}$$

and the equation (2) takes the form

$$\frac{\partial I(\tau, r)}{\partial r} = \beta(r)I^2(\tau, r) - \left(\varrho + \frac{a}{r}\right) I(\tau, r). \tag{2'}$$

As is evident from (2′) the radiation amplification may occur only for intensities higher than the threshold intensity

$$I_{\text{th}} = \frac{\varrho}{\beta}. \tag{3}$$

The solution to the equation (2′) has the form

$$I(\tau, r) = I^0(\tau)\left(\frac{r_0}{r}\right)^a \exp\left[-\varrho(r-r_0)\right]\left\{1 - I^0(\tau)\int_{r_0}^{r}\left(\frac{r_0}{r'}\right)^a \beta(r')\exp\left[-\varrho(r'-r_0)\right]dr'\right\}^{-1}, \tag{4}$$

where r_0 is the radius of curvature of the wave radiation front at the input of the medium, and $I^0(\tau) = I(\tau, r = r_0)$.

Making use of (4) the evolution of the intensity of a plane wave as well as convergent and divergent wave will be investigated.

3.1. Plane wave

In the case of a pulse, with a plane wave front, which propagates in a homogeneous medium, the solution (4) becomes [4]

$$I(\tau, l) = I^0(\tau)\exp(-\varrho l)\left\{1 - \frac{\beta}{\varrho} I^0(\tau)\left[1 - \exp(-\varrho l)\right]\right\}^{-1}, \tag{5}$$

where $l = r - r_0$.

Attention should be drawn to some differences between two- and single-photon amplification of the weak signal, which result from (5).

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[2] By weak signal is meant here a laser pulse which produces no essential changes in population inversion, that is for which $N(I) \approx N^e$.

Firstly, there is a critical path of amplification

$$l_k = \frac{1}{\varrho} \ln \frac{\beta I^0}{\beta I^0 - \varrho},$$

at which the intensity becomes infinite. The existence of a critical path results from the fact that the saturation effect is left out of account. After the pulse has travelled a path approximating to l_k the amplification enters the saturation stage, which leads to the finite intensity for $l \geqslant l_k$.

Secondly, if $I^0(\tau) = I_{th}$, then $I(\tau, r) = I^0(\tau)$. In particular, if the peak intensity of the input pulse I_h^0 is equal to the threshold intensity, it remains constant in the course of propagation, whereas the fore- and the rear front of the pulse is absorbed by the medium. As a result, the greater the time compression is the higher the linear-loss factor will be.

3.2. Divergent wave

The evolution of intensity in the medium of a divergent radiation beam is conditioned by two opposing processes: the intensity increase due to the burning-out of the energy stored in the medium and the intensity decrease as a consequence of divergence. As will be shown further on, in a medium with a non-homogeneous inversion distribution in the direction of the propagation, these processes may result in the occurrence of a variety of amplification regimes. It should be emphasized that in the case of large divergences, a small increase in intensity (or even a decrease therein) may be accompanied by a large increase in the total radiation power, in view of the increase in the effective cross-section of the beam. The divergent wave, unlike the plane one, can propagate in the medium with no change in the peak intensity, at an arbitrary input intensity, if only the amplification factor for a unit intensity β varies in accordance with the expression (see equation (2′))

$$\beta(r) = \kappa \frac{r_0}{r} + \varepsilon,$$

where $\kappa = \frac{a}{r_0 I_h^0}$, $\varepsilon = \frac{\varrho}{I_h^0}$. When analyzing the effect of the inversion distribution upon the pulse evolution, it will be assumed that the dependence of the coefficient β upon r has the form

$$\beta(r) = \alpha \left(\frac{r_0}{r}\right)^c, \qquad \alpha, c = \text{const.} \tag{6}$$

Distributions of this type may be effected approximately by using a suitable configuration of the active material or pumping system. On the other hand, an analysis carried out for such a case may provide qualitative information on the pulse evolution in the media with a different, monotonic inversion distribution.

For the sake of clarity the analysis of intensity variations in the medium will be carried out in the approximation to low linear losses. Making use of (4) it can be demontrated that for intensities higher than the threshold intensity, this assumption does not influence the qualitative results obtained below, and for $I \gg I_{th}$ has only a slight influence upon the quantitative results.

With the condition (6) and $\varrho l \ll 1$ the solution (4) can be written in the form

$$I(x, \tau) = I^0(\tau) \frac{\mu(\tau) x^a}{x^\nu + \mu(\tau) - 1}, \tag{7}$$

where

$$x = \frac{r_0}{r}, \quad \nu = a + c - 1, \quad \mu(\tau) = \frac{a+c-1}{\alpha I^0(\tau) r_0}.$$

The dependence of the amplification of the divergent radiation wave $A = \frac{I}{I^0}$ upon the path travelled in the medium, for $\nu > 0$ is shown in Fig. 1. It is characteristic, in this picture, that for certain values of the parameters a, ν, μ the variation of the radiation intensity is non-monotonic. In addition the critical path of the amplification

$$l_k = r_0[(1-\mu)^{-\frac{1}{\nu}} - 1]$$

occurs only at $\mu < 1$. It will be seen also that at fixed values of ν, μ the character of the radiation-intensity variation depends upon the type of divergence. By way of example: at $\mu < 1$ and $1 < \frac{\nu}{\mu} < 2$ the intensity of the wave with cylindrical divergence ($a = 1$) increases monotonically, whereas the intensity of the wave with spherical divergence ($a = 2$) varies in the medium non-monotonically in the interval $(0, l_k)$.

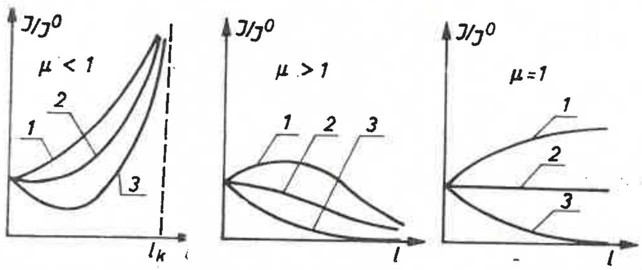

Fig. 1. Dependence of intensity of a weak signal upon the path travelled in a two-photon amplifier at $\nu > 0$. *1* — $\nu > \mu a$; *2* — $\nu = \mu a$; *3* — $\nu < \mu a$

If $\nu < 0$, which for $a \geq 1$ means that the inversion increases in the direction of propagation ($c < 0$) then, at the arbitrary values of the parameters occurring in (7), there exists a critical path of amplification

$$l_k = r_0[(1+|\mu|)^{\frac{1}{|\nu|}} - 1].$$

At the same time, for $\frac{\nu}{\mu} \geq a$, the radiation intensity increases monotonically, while in the opposite case the amplification has its minimum at the point

$$x = \left[\frac{a+|\nu|}{a(1+|\mu|)}\right]^{\frac{1}{|\nu|}}.$$

Fig. 2 depicts the evolution of the Gaussian pulse with a spherical wave front in a homogeneous ($c = 0$) two-photon amplifier. The graphs were plotted on the strength of the solution (4) at

$$r_0 = 5 \text{ cm}, \quad \varrho = 0.001 \text{ cm}^{-1}, \quad \mu_h = 0.9, \quad \left(\mu_h = \frac{a+c-1}{\alpha I_h^0 r_0}\right).$$

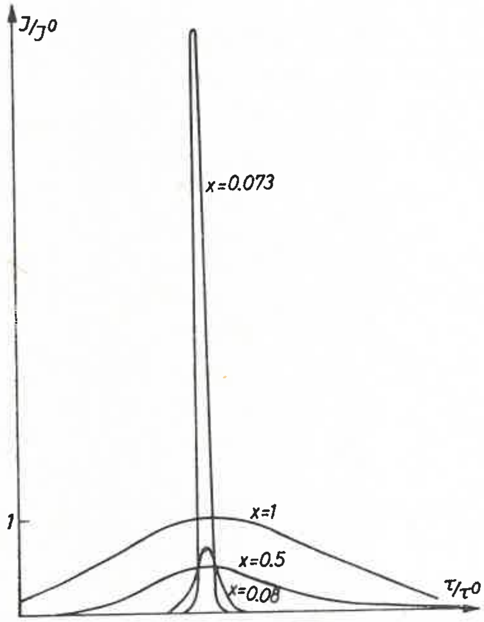

Fig. 2. Evolution of a weak signal with the spherical wave front in a homogeneous two-photon amplifier

As is evident from the diagram, the relatively small variations in the peak intensity of a divergent-radiation pulse are accompanied by its strong time compression. This effect will be discussed further on in detail.

3.3. Convergent wave

In the case of convergent radiation propagating in the amplifier, the intensity increase is conditioned, on one hand, by burning-out of the energy of the medium, and on the other, by the convergence of radiation. Although from the viewpoint of obtaining a large total pulse power, this case is of less interest than the case of a divergent wave, since the intensity increase is accompanied by a decrease in the cross-section of the beam, the application of a convergent wave may be advisable in the first phase of amplification, in a medium of a high threshold intensity.

If the threshold intensity is lower than the intensity causing the breakdown of the medium, then with the aid of a convergent wave, it is relatively simple to obtain the amplification threshold. After the pulse has obtained the threshold intensity, the convergent wave can be transformed into a divergent one by means of the optical system, and the second amplification wave can be generated, the intensity increase of which may be accompanied by a large increase in the total power. In order to effect this type of a two stage amplification, long-focus convergent lenses may be utilized.

The intensity variation of a convergent wave, in approximation to low linear losses, can be described by the expression (7), if r is replaced by $-r$, and the assumption is made that r varies from r_0 to 0. It is easy to demonstrate that in the case of $\nu > 0$, for the arbitrary numerical values of ν, μ, a there is a critical path of amplification

$$l_k = |r_0| \left[1-(1+|\mu|)^{-\frac{1}{\nu}}\right],$$

whereas at $\nu < 0$, the critical path of amplification occurs only when $|\mu| < 1$, and is

$$l_k = |r_0| \left[1-(1-|\mu|)^{\frac{1}{|\nu|}}\right].$$

However, unlike the divergent wave, the intensity of the convergent wave, at $r \to 0$, increases infinitely (within the framework of the model assumed) also in the case where the critical amplification-path does not occur.

A detailed analysis of the intensity variation of convergent radiation can be carried out, on the strength of the modified formula (7), in the same way as for a divergent wave.

4. *Weak-signal compression in an amplifier*

An analysis of the pulse-duration variation in a two-photon amplifier will be carried out by using the function of the time compression introduced into [8] and defined by the relation

$$T \equiv -\frac{1}{\tau_p}\frac{d\tau_p}{dr}. \tag{8}$$

In accordance with [8], the relation between the function of the time compression T and the function of the amplification of the medium K, in the case where K does not depend explicitly upon time, has the form

$$T = \frac{\delta_1(r)+\delta_2(r)}{2}\left[K(r, I_h)-K(r, \tfrac{1}{2} I_h)\right], \tag{9}$$

where I_h is the pulse peak-intensity, and δ_1, δ_2 are the slope coefficients of the fore and rear pulse-front, defined by the relations

$$\left.\frac{\partial I(\tau, r)}{\partial \tau}\right|_{\tau_1} = \frac{1}{\delta_1(r)}\frac{I_h(r)}{\tau_p(r)}; \quad \left.\frac{\partial I(\tau, r)}{\partial \tau}\right|_{\tau_2} = -\frac{1}{\delta_2(r)}\frac{I_h(r)}{\tau_p(r)}; \quad \delta_1, \delta_2 > 0,$$

(τ_1, τ_2 — points at fore and rear front, respectively, at half-height). In the case where the function $K(I)$ is monotonic, the coefficients $\delta_1(r), \delta_2(r)$ are, in general, slow-variable functions in comparison with $I_h(r)$ and $\tau_p(r)$ for typical pulses (Gauss-, Lorentz-, exponential etc.) with values approximating to unity.

Making use of (9) and keeping in mind that for a weak signal

$$K = \beta(r)I(\tau, r) - \varrho \tag{10}$$

we obtain

$$T = \frac{\delta_1 + \delta_2}{4} \beta(r) I_h(r). \tag{11}$$

As is evident from (11), the rate of time compression of the weak signal increases proportionally to its peak intensity.

The general dependence of the pulse compression $C_\tau = \tau_p^0/\tau_p$ upon its peak intensity, valid in a homogeneous medium, at small rediation divergences, has the form [8]

$$C_\tau = \exp\left[\int\limits_{I_h{}^0}^{I_h} \frac{T(I_h)}{I_h K(I_h)}\, dI_h\right], \tag{12}$$

where $\tau_p^0 = \tau_p(r_0)$, $I_h^0 = I_h(r_0)$. From (10), (11) and (12) we have

$$C_\tau = \left|\frac{I_h - I_{th}}{I_h^0 - I_{th}}\right|^{\bar{\delta}/2}, \tag{13}$$

where $\bar{\delta}$ is the mean value of the function $\frac{1}{2}[\delta_1(r) + \delta_2(r)]$ in the variation interval under consideration r. It will be seen that the nearer I_h^0 is to the threshold intensity, the greater is the shortening of the pulse per unit increase in the peak intensity, the pulse compression occurring for both $I_h^0 > I_{th}$ and $I_h^0 < I_{th}$. In the latter case appreciable compressions can be obtained only when I_h is near enough I_{th}, since the limit compression (at $l \to \infty$) is

$$C_\tau^{lim} = \left|\frac{I_{th}}{I_{th} - I_h^0}\right|^{\bar{\delta}/2}.$$

At $I_h^0 > I_{th}$, the pulse compression for $l = l_k$ becomes infinite. Making use of (13) and (5) we obtain

$$C_\tau = \left[1 - \frac{I_h^0}{I_{th}}(1 - e^{-\varrho l})\right]^{-\bar{\delta}/2}. \tag{14}$$

For $\bar{\delta} = 1$, an analogous expression was obtained in [4] in a different way. If $I_h^0 = I_{th}$, then in accordance with (14) the pulse shortens, in the medium, approximately exponentially

$$C_\tau = \exp\left(\tfrac{1}{2}\bar{\delta}\varrho l\right).$$

Next, the effect will be considered of the pulse time-compression in the case of a divergent and convergent radiation propagating in a medium with the inversion distribution of the type (6). Utilizing (11), (6)—(8) we obtain

$$\frac{d\tau_p}{dx}+\tfrac{1}{4}\left[\delta_1(x)+\delta_2(x)\right]\frac{\nu x^{\nu-1}}{x^\nu+\mu_h-1}\tau_p=0, \tag{15}$$

where $\mu_h=\dfrac{a+c-1}{\alpha I_h^0 r_0}$. The solution of equation (15) can be written in the form

$$\tau_p(x)=\tau_p^0\left|\frac{x^\nu+\mu_h-1}{\mu_h}\right|^{\bar{\delta}/2}. \tag{16}$$

Fig. 3 shows the dependence of the pulse compression C_τ, defined by formula (16), upon the amplification path, for a divergent wave, at $\nu>0$. As is seen from the graphs,

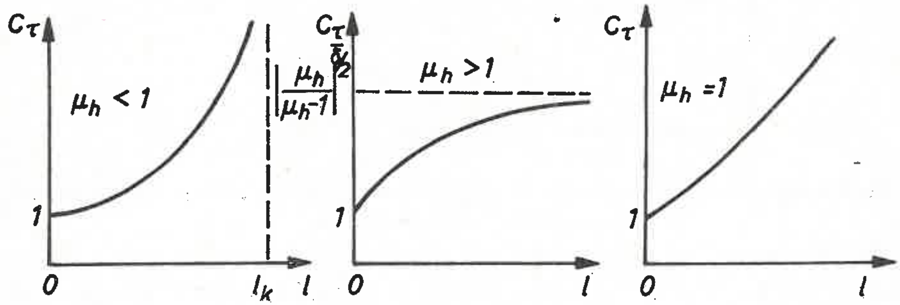

Fig. 3. Dependence of time compression of a weak signal upon the path travelled in a two-photon amplifier at $\nu>0$ — divergent wave

at $\mu_h>1$ obtaining high pulse compressions requires a stabilization of the peak power of the input pulse, since olny for μ_h close to one the limit compression

$$C_\tau^{\lim}=\left(\frac{\mu_h}{\mu_h-1}\right)^{\bar{\delta}/2}$$

assumes high values. The duration of a pulse with stabilized peak power may be varied by varying the amplification path. This will be clearly seen if we put $\mu_h=1$ in (16). Then

$$C_\tau=\left(1+\frac{l}{r_0}\right)^{\nu\bar{\delta}/2}$$

and by varying l the wanted compressions may be obtained.

If the input pulse has no stabilized peak power (or the stabilization is inadequate) then, in order to obtain a high compression, it is more convenient to implement the case $\mu_h<1$.

If, at a divergent wave, $\nu<0$, then, for arbitrary μ_h, there occurs a critical amplification-path l_k and, at $l\to l_k$, $C_\tau\to\infty$.

The case of a convergent wave, for $\nu < 0$, is represented in Fig. 4. The analogy can be seen for the case $\nu > 0$, at a divergent wave. Time compression variation of a convergent wave, at $\nu > 0$, occurs in turn in an analogous way as the compression variation of the divergent wave at $\nu < 0$.

Although the dependence of the pulse time-compression upon the amplification path is to a large extent analogous, for both the convergent and divergent wave, in practice, in order to obtain high compression, it makes some difference which type of wave will be used. As a large increase in the peak intensity may lead to the occurrence of non-linear effects broadening the pulse or to the breakdown of the medium, in practice it is advisable for a high pulse-compression to occur for a small increase of the peak intensity.

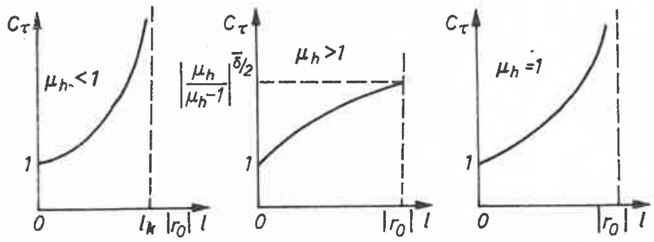

Fig. 4. Dependence of time compression of a weak signal upon the path travelled in a two-photon amplifier at $\nu < 0$ — convergent wave

Utilizing (7) and (16) it is possible to obtain a relation between the peak-intensity amplification $A_h = I_h/I_h^0$ and the pulse time-compression

$$C_\tau = \left(\frac{A_h}{x^a}\right)^{\bar{\delta}/2}. \tag{17}$$

It will be seen therefrom that, at equal amplification values A_h, the time compression for a divergent wave ($x < 1$) may be appreciably higher than for a convergent ($x > 1$) or plane ($a = 0$) wave. Moreover, from formula (17) it follows that the highest compressions may be obtained for pulses of divergent radiation with the spherical wave front.

The foregoing considerations concern the pulse with monotonic fronts. In case where the pulse is of a markedly irregular structure, as, for example, the pulse of a generator with self mode-locking, the pulse compression may occur still more effectively than in the case of a pulse with monotonic fronts. It is possible, in this case, to discern between to mechanisms leading to the formation of an ultra-short pulse, that is: cut-out of the most intensive sub-pulse out of the input pulse as a result of a marked intensity discrimination; shortening of the singled-out sub-pulse in the process of a non-linear amplification.

It is noteworthy that pulse compression occurs likewise in the region of coherent interaction [6], which, in the media of small dispersion and phase modulation, offers a distinct possibility of forming, in a two-photon amplifier, a single pulse of duration $\lesssim 10^{-13}$s.

5. *Evolution of a strong signal in an amplifier*

A detailed analysis of non-linear amplification of a strong signal in a two-photon amplifier will be carried out in a separate paper, and that is why attention will be payed here to certain aspects of the problem only. Exact, quantitative information on the evolution of the pulse in the saturation region can be obtained by using the numerical solutions of equation (2). Essential, qualitative information on the amplification process can be acquired by the analysis of the function of amplification of the medium. Our considerations will be confined to the case of radiation with small divergence that is, where $K \gg a/r$.

The function of amplification of the medium, at $\tau_p \ll T_1$, may be written in the form

$$K = \beta f(\tau) I_h \exp\left[-s\sigma\chi(\tau) I_h^2\right] - \varrho, \tag{18}$$

where $\chi(\tau) = \int\limits_{-\infty}^{\tau} f^2(\tau')d\tau'$, $f(\tau)$ — function determining the shape of the pulse, and satisfying the condition: $f(\tau_h) = 1$ while τ_h is the value τ corresponding to the position of the pulse maximum.

It will be seen from (18) that the pulse intensity increase is possible if its peak intensity at the input of the medium satisfies the condition

$$I_h^0 > I_{th} \exp\left(s\sigma\chi_0 I_h^{02}\right), \tag{19}$$

where: $\chi_0 = \int\limits_{-\infty}^{\tau_h{}^0} f_0^2(\tau')d\tau'$ and $\tau_h^0, f_0(\tau)$ concern the pulse entering the medium. It the condition (19) is satisfied, then, as $l \to \infty$ the pulse peak-intensity approaches infinity (within the framework of the assumed model). As the energy of the asymptotic pulse should be

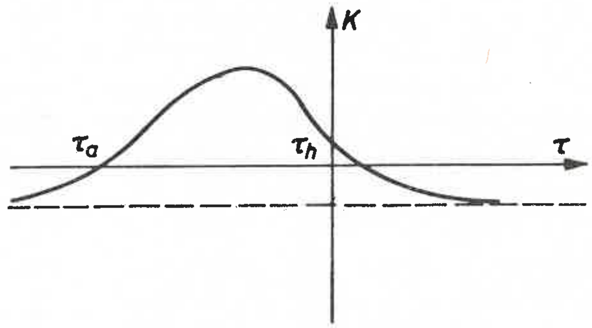

Fig. 5. Dependence of amplification function of a strong signal in a two-photon medium upon local time

finite (not greater than the energy accumulated in the medium and linked with the existence of population inversion) the pulse duration will approach zero.

One of the major differences in the amplification process of a weak and a strong signal is the asymmetry of the amplification of the fore and the rear front of the latter, due to the saturation effect. This will be seen from Fig. 5, which depicts the qualitative dependence of the amplification function upon the local time τ for the symmetric pulse with monotonic fronts. The stronger amplification of the fore front of the pulse than that of its rear front leads to a shifting of the pulse maximum along the fore front. From the viewpoint of an observer positioned inside a motionless system, the pulse maximum shifts at a velocity higher than the group velocity of light v in the medium. However, in contrast to the case of a single-photon amplification the shifting of the pulse maximum along the fore front cannot be unlimited. This is linked with the existence of the point τ_a (see Fig. 5) at the fore front, below which the amplification function assumes negative

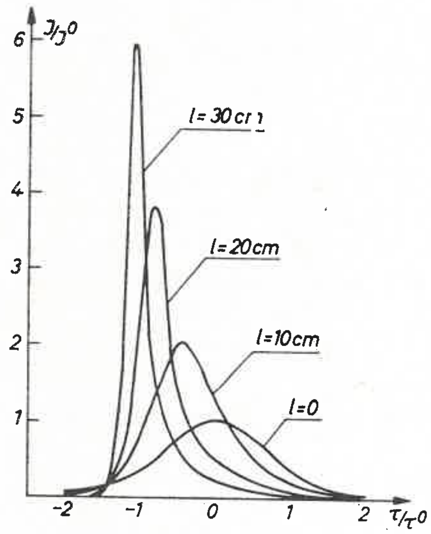

Fig. 6. Evolution of a strong signal with plane wave front in a homogeneous two-photon amplifier. $\sigma = 10^{-46}$ cm^{4}s, $s = 2$, $\tau_p^0 = 10^{-11}$ s, $I_h^0 = 5\times10^{28}$ cm^{-2}s^{-1}, $\beta = 10^{-29}$ cms, $\varrho = 0.06$ cm^{-1}

values (that is there occurs an absorption). This point forms as if it were a barrier which makes it impossible for the maximum to shift beyond the value τ_a. It can be demonstrated that τ_a is limited from below by the value τ_{th} (that is $\tau_a > \tau_{th}$) given by the equality

$$I_0(\tau_{th}) = I_{th}.$$

For a Gaussian input pulse with the maximum at the point $\tau = 0$, the half-width τ_p^0 and the peak intensity I_h^0 we have, for instance:

$$\tau_{th} = \tfrac{1}{2}\,\tau_p^0 \left[\frac{\ln\left(\dfrac{I_h^0}{I_{th}}\right)}{\ln 2}\right]^{1/2}.$$

It is the existence of the point τ_a which limits the shifting of the pulse maximum along the fore front, that, independently of the shape of the pulse entering the medium, makes the compression of this pulse possible[3]. As is well-known [9], at a single-photon amplification, only the pulses with a sharp enough fore front undergo compression.

From the above qualitative considerations it will be seen that in a two-photon amplifier not only the weak pulses undergo compression but also the strong pulses, causing the saturation of amplification. A quantitative corroboration of this conclusion is shown in Fig. 6 which illustrates the evolution of a Gaussian pulse $\left(f(\tau) = \exp\left[-\left(\frac{\tau}{\tau_0}\right)^2\right]\right)$ in the saturation region. The graphs were plotted on the strength of numerical solutions to equation (2). An analogous numerical analysis performed for a Lorentz pulse (that is a pulse which undergoes broadening in a single-photon amplifier) shown that such a pulse undergoes compression in a two-photon amplifier.

Throughout the region of l-variation, as assumed in calculations (from 0 to 100 cm), the peak intensity of the two types of pulses increases monotonically, while their half--witdh decreases monotonically.

6. *Effect of multi-photon absorption upon pulse-duration variation*

A two-photon amplification of a laser pulse occurs at relatively high radiation intensities, and this may result in the occurrence of non-linear losses which reduce the rate of pulse compression. One of the main factors which cause the reduction of the compression rate is the multi-photon radiation-absorption. We shall analyze the latter effect upon the duration variation of a weak signal.

The amplification function of the medium, which takes into account the multi-photon losses, has the form [8]

$$K = \beta I - \beta_n I^{n-1} - \varrho, \qquad n > 1,$$

where β_n is the multi-photon-absorption coefficient at unit intensity. For the function of time compression we obtain, in this case, the expression

$$T = \frac{\delta_1 + \delta_2}{2}\left[\tfrac{1}{2}\beta - (1 - 2^{1-n})\beta_n I_h^{n-2}\right] I_h. \tag{20}$$

It will be seen from (20) that the multi-photon losses lead to a decrease in the pulse--compression rate.

For $n = 2$, from (20) we have

$$T = \frac{\delta_1 + \delta_2}{4}(\beta - \beta_2) I_h,$$

which yields $T > 0$ at $\beta > \beta_2$. Consequently, a sufficient condition for the pulse com-

[3] In the case where $I_h^0 \gg I_{th}$ and the pulse maximum lies in the region of intense saturation, the pulse with a gentle fore front may, in the initial phase undergo broadening. However, when the maximum is near enough the value τ_{th}, the pulse compression follows.

pression to take place, at the occurrence of two-photon absorption, is to have the two-photon amplification factor β higher than the two-photon absorption coefficient β_2, what, in turn, is necessary for the amplification to be possible at all.

For $n > 2$, the condition for pulse compression ($T > 0$) is

$$I_h < \left[\frac{1}{2(1-2^{1-n})}\,\frac{\beta}{\beta_n}\right]^{1/(n-2)}. \tag{21}$$

At peak pulse-intensities higher than those defined by the right-hand side of the inequality (21) there occurs pulse broadening: $T < 0$. In practice such a situation is possible only at very high intensities.

7. *Summary*

The preceeding analysis indicates a number of features of non-coherent two-photon amplification which differ basically from those of single-photon amplification. The most important of them are:

— two-photon amplification may occur only for intensities higher than the threshold one;

— propagation of a pulse in a two-photon amplifier is accompanied by its time compression in the case of both weak and strong signals;

— compression rate of a weak signal is proportional to its peak intensity;

— pulse compression is possible independently of its shape at the input medium;

— in practice the highest compressions can be obtained in the case of the divergent radiation with the spherical wave front;

— under optimum conditions a compression of $\sim 10^2$ can be obtained in two-photon amplifier. In the case of a picosecond input pulse, in a medium of a broad amplification line and small dispersion, this offers the possibility of obtaining single, powerful pulses of a duration of $\lesssim 10^{-13}$ s. A hypothetical mechanism of forming an ultra-short pulse consists in separating out of an input pulse the most intense sub-pulse, and in its compression in the process of a non-linear amplification;

— one of the factors reducing the compression rate in a two-photon amplifier may be the multi-photon radiation absorption.

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