Vol. A53 (1978) ACTA PHYSICA POLONICA No 6

# SPIN WAVES IN ITINERANT ELECTRON FERROMAGNETS WITH DOMAIN STRUCTURE*

By R. Świrkowicz and A. Sukiennicki

Institute of Physics, Warsaw Technical University**

(*Received October 20, 1977*)

Assuming the existence of a definite domain structure, the dispersion relation of long-wavelength spin waves in an itinerant electron ferromagnet is calculated in the random phase approximation. The obtained relation depends, in general, on the magnetization direction distribution in domain walls. The influence of the magnetocrystalline anisotropy on spin waves stiffness parameter is also derived.

Elementary excitations in ferromagnets with domain structure were up to now investigated within the framework of the phenomenological theories [1] or in Heisenberg model [2]. If we want, however, to consider the situation in metals, where electrons responsible for magnetic effects have itinerant rather than localized character, the band model approach seems to be the proper one. On the other hand, it has been recently shown by us [3–6] that the domain structure may be successfully described within the framework of the band model. Therefore, it is interesting to extend our previous considerations in such a way that the solution of the problem of spin waves in an itinerant electron ferromagnet with domain structure could be achieved. It will be the subject of this paper.

Analogically as in paper [6], the anisotropic Hamiltonian consisting of the Hubbard and pseudodipolar terms is taken as a starting point, namely

$$H = \sum_{\substack{\langle i,j\rangle \\ \sigma}} T_{ij} b^+_{i\sigma} b_{j\sigma} + I \sum_i b^+_{i\uparrow} b_{i\uparrow} b^+_{i\downarrow} b_{i\downarrow} + \sum_{\langle ij\rangle} D_{ij}[\boldsymbol{S}_i \boldsymbol{S}_j - 3r_{ij}^{-2}(\boldsymbol{r}_{ij}\boldsymbol{S}_i)(\boldsymbol{r}_{ij}\boldsymbol{S}_j)], \qquad (1)$$

where $b^+_{i\sigma}$, $b_{i\sigma}$ denote creation and annihilation operators of electrons with spin $\sigma = \uparrow$ or $\downarrow$ in the Wannier representation at the lattice point $i$, and components of the spin operator $\boldsymbol{S}_i$

* This investigation was financially supported by the Institute for Low Temperature and Structure Research of the Polish Academy of Sciences, Wrocław, Poland.

** Address: Instytut Fizyki, Politechnika Warszawska, Koszykowa 75, 00-662 Warszawa, Poland.

in the second quantization representation are:

$$S_i^x = \tfrac{1}{2}\sum_\sigma b_{i\sigma}^+ b_{i-\sigma}, \quad S_i^y = -\frac{i}{2}\sum_\sigma \hat{\sigma} b_{i\sigma}^+ b_{i-\sigma}, \quad S_i^z = \tfrac{1}{2}\sum_\sigma \hat{\sigma} b_{i\sigma}^+ b_{i\sigma},$$

where $\hat{\sigma} = +1$ for $\sigma = \uparrow$ or $-1$ for $\sigma = \downarrow$. Magnetocrystalline anisotropy is introduced here in a way which seems to be consistent with the Hubbard Hamiltonian, so the model looks like an entirely uniform one. The difficulty connected with demagnetizing effects is avoided by assuming a stray-field-free domain configuration of the Landau–Lifshitz type. To take into account the presence of the internal part of the assumed domain structure, Hamiltonian (1) is expressed in terms of new operators $c_{i\sigma}$ defined by the relation

$$b_{i\sigma} = \cos\frac{\vartheta_i}{2}\, c_{i\sigma} - \hat{\sigma}\sin\frac{\vartheta_i}{2}\, c_{i-\sigma}. \tag{2}$$

This transformation represents a rotation of the spin operator $\boldsymbol{S}_i$ about the $Y$ axis (perpendicular to the domain walls) by the angle $\vartheta$ measured with respect to the easy axis $Z$ and dependent on the variable $y$ only. Then the Hamiltonian is approximately diagonalized by means of a three-dimensional Fourier transformation.

Now, the energy of spin waves in the ferromagnet with the assumed domain structure may be calculated using the following equation of motion

$$[H, B_q^+] = \hbar\omega B_q^+, \tag{3}$$

where the creation operator $B_q^+$ of the spin wave with wave vector $\boldsymbol{q}$ is taken in the form of a linear combination of one-particle excitations [7]:

$$B_q^+ = \sum_k (\alpha_k c_{k+q\downarrow}^+ c_{k\uparrow} + \beta_k c_{k+q\uparrow}^+ c_{k\downarrow} + \gamma_k c_{k+q\uparrow}^+ c_{k\uparrow} + \delta_k c_{k+q\downarrow}^+ c_{k\downarrow}). \tag{4}$$

Equation (3) is solved within the random phase approximation. Next, it is taken into account that pseudodipolar coupling is small in comparison to the intraatomic Coulomb interaction between electrons as well as to the bandwidth. The gradual, very slow rotation of magnetization vector in Bloch walls is also exploited.

Then, the energy of long-wavelength spin waves in a crystal of simple hexagonal structure is obtained in the following form

$$\hbar\omega = 2K\mu(\tfrac{2}{3} - \overline{\sin^2\vartheta}) + \frac{1}{2N\mu}\sum_k \Bigg\{ (\boldsymbol{q}\nabla)^2 \left[ \varepsilon_k + \tfrac{1}{8}\frac{\partial^2 \varepsilon_k}{\partial k_y^2} \overline{\left(\frac{d\vartheta}{dy}\right)^2} \right]$$
$$- \frac{2}{I\mu}\hat{\sigma}\left[ \boldsymbol{q}\nabla \left( \varepsilon_k + \tfrac{1}{8}\frac{\partial^2 \varepsilon_k}{\partial k_y^2} \overline{\left(\frac{d\vartheta}{dy}\right)^2} \right) \right]^2 \Bigg\} n_{\sigma k} - \tfrac{3}{8}(1 + c_{100}) D_1 \mu a^2 (1 - \tfrac{9}{4}\overline{\sin^2\vartheta})(q_x^2 + q_y^2)$$
$$+ \tfrac{1}{2}(1 + c_{001}) D_2 \mu c^2 (1 - \tfrac{3}{2}\overline{\sin^2\vartheta}) q_z^2, \tag{5}$$

where $K$ is the uniaxial anisotropy constant found in paper [6], $\mu$ denotes the mean spontaneous magnetization per atom, $\overline{\sin^2 \vartheta} = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} \sin^2\vartheta \, dy$, and $\vartheta(y)$ is the distribution function of magnetization directions in Bloch walls, which was also derived in paper [6]. Here, $\Delta$ denotes the domain width, $\varepsilon_k$ is the Bloch energy, $a$ and $c$ are lattice constants and $D_1$, $D_2$ are pseudodipolar coupling constants. The coefficients $c_h$ are defined as follows:

$$c_h = \frac{1}{N^2\mu^2} \sum_{k,k',\sigma} e^{i(k'-k)r_h} n_{k\sigma} n_{k'\sigma}, \tag{6}$$

where $n_{k\sigma}$ denotes the distribution function of occupation numbers for the Hartree–Fock one-electron states.

According to Eq. (5), the obtained dispersion relation is in fact quadratic and the energy gap for $q = 0$ is connected with anisotropy. However, it should be emphasized that the influence of the pseudodipolar coupling is not limited only to the shifting of the energy spectrum by a value proportional to the anisotropy constant (a result well-known from phenomenological theories [8] as well as from the Heisenberg model [9]) but it causes also an additional modification of spin wave energy. Namely, the coefficient of $q^2$, itself, depends on the pseudodipolar coupling parameters and is of anisotropic character. A similar variation of the spin wave stiffness parameter in the band model, caused by magnetocrystalline anisotropy was obtained earlier in paper [10].

Moreover, analysis of Eq. (5) shows that the domain structure also influences the spin wave energy. The coefficient of $q^2$ depends on the type of domain structure through the distribution function $\vartheta(y)$ of magnetization directions. This dependence is rather weak, of course; it is related to the fact that the rotation of magnetization vector in Bloch walls is very slow.

Considering the obtained result it should be mentioned that no localized spin waves connected with domain walls were found. First of all, it is a result of our choice of the Hamiltonian which does not include any demagnetizing effects. Secondly, it could be eventually connected with the approximate method of diagonalization.

As a conclusion we can state that the spin wave energy in a ferromagnet with domains depends in general on the domain structure i.e. on the distribution of magnetization directions in domains considered. This fact has been shown here within the framework of the band model. It is valid, therefore, for metals in which electrons responsible for magnetism have an itinerant character.

## REFERENCES

[1] M. Fartzdinov, E. A. Turov, *Fiz. Met. Metalloved.* **29**, 458 (1970).
[2] J. M. Winter, *Phys. Rev.* **124**, 452 (1961).
[3] R. Świrkowicz, A. Sukiennicki, *Acta Phys. Pol.* **A46**, 667 (1974).
[4] R. Świrkowicz, *Acta Phys. Pol.* **A50**, 675 (1976).

[5] R. Świrkowicz, A. Sukiennicki, *Physica* **86–88B**, 1349 (1977).
[6] R. Świrkowicz, A. Sukiennicki, *Acta Phys. Pol.* **A52**, 253 (1977).
[7] A. Sukiennicki, R. Świrkowicz, L. Adamowicz, *J. Phys. C* **5**, L216 (1972).
[8] C. Herring, C. Kittel, *Phys. Rev.* **81**, 5 (1951).
[9] S. H. Charap, P. R. Weiss, *Phys. Rev.* **116**, 6 (1959).
[10] W. Jaworski, J. Morkowski, *J. Phys. C* **9**, 2767 (1976).