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VALIDITY OF NEWTON'S FORMULA IN A FOURIER TRANSFORMING CONFIGURATION

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The parabolic mirror, which has been used as a Fourier-transforming element produces the Fourier-transform of the input objects placed in front of it. The information carrier signal is spherical in nature as it is originated from a point source of light. The gaussian or paraxial approach has been adopted to relate the amplitude distributions at the object plane and the Fourier-transform plane. This relation comes out as the Fourier-transform relationship. The exact Fourier-transform has been obtained from this general expression by imposing certain conditions. Thus the optical parameters which favour this relationship, obey Newton's formula.

1. Introduction

Newton's formula is applicable for focussing optical elements, e.g. lenses and mirrors. Husain-Abidi and Krile [1] have discussed the advantages of paraboloidal mirror segment as a Fourier-transforming element over the utility of Fourier-transform lenses. They have predicted the phase transformation property of the parabolic mirror theoretically and supported the results by experiments. The plane wave front as an information carrier signal has been utilized in their investigations. Kasana et al. [2] treated a more general case by considering the spherical illumination as an information carrier signal and established some theoretical results and facts regarding the Fourier-transforming properties of the parabolic mirror.

In this paper, the general Fourier-transform relationship has been treated to have the exact Fourier-transform. This expression is obtained by omitting the phase term

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involved in the expression. Thus this condition gives a modified relation which is known as Newton's formula. This is a logical consequence of the derivations established for predicting the amplitude distributions at various planes.

2. *Optical configuration*

As shown in Fig. 1, *Pr* is the point source of monochromatic light which propagates the spherical waves. The input transparency function $S(x_1, y_1)$ is placed in plane P_1 at distance u from *Pr* and at distance F from the mirror vertex. $P_2(x_2, y_2)$ is the mirror plane. $P_3(x_3, y_3)$ is the observation plane, where the Fourier-transform of the object-function $S(x_1, y_1)$ occurs. The separation between the planes $P_1(x_1, y_1)$ and $P_3(x_3, y_3)$ is equal

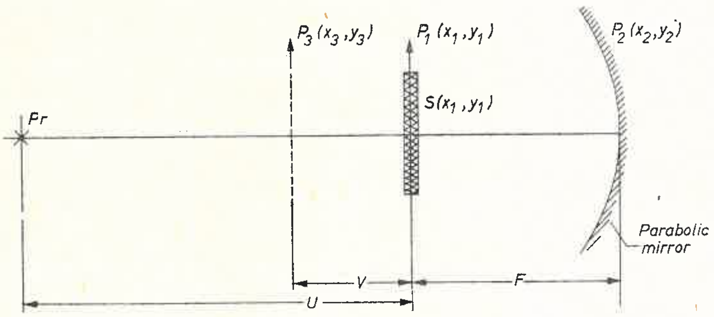

Fig. 1. General Fourier-transforming configuration

to V. For simplicity the observation plane $P_3(x_3, y_3)$ has been taken parallel to the object plane $P_1(x_1, y_1)$ and normal to the optical axis otherwise it may be obtained somewhere by tilting the mirror segment. However, the results will not show any marked deviation.

3. *Theory*

The disturbance at any point in plane $P_1(x_1, y_1)$ due to spherical illumination originating from the point source *Pr* is given by

$$U(r) = \exp(ikr)/ikr, \tag{1}$$

where r is the distance of the point of interest from *Pr*. If this point exists in a plane at a distance u from *Pr* along the optical axis, the disturbance at this point can be predicted by using the paraxial approximation and

$$U(r) = \exp[ik(x_1^2+y_1^2)/2u]. \tag{2}$$

The constant phase factor has been omitted here.

Now the amplitude distribution just behind the object function in the object plane can be written as

$$U_1(x_1, y_1) = S(x_1, y_1)\exp[ik(x_1^2+y_1^2)/2u]. \tag{3}$$

The amplitude distribution near the mirror surface can be calculated by using the Fresnel diffraction formula [3]

$$U_2(x_2, y_2) = \exp[ik(x_2^2+y_2^2)/2F] \iint_{-\infty}^{\infty} \exp \frac{ik}{2}(1/u+1/F)(x_1^2+y_1^2)$$

$$\times S(x_1, y_1) \exp[-ik(x_1x_2+y_1y_2)/F]dx_1dy_1. \tag{4}$$

When this disturbance strikes the mirror surface, it introduces a multiplicative phase transformation.

$$T(x_2, y_2, F) = \exp[-ik(x_2^2+y_2^2)/2F]. \tag{5}$$

The constant phase term has been neglected here. Hence the amplitude distribution over the mirror surface is

$$U_2^1(x_2, y_2) = T(x_2, y_2, F)U_2(x_2, y_2). \tag{6}$$

Using the equations (4), (5) and (6)

$$U_2^1(x_2, y_2) = \iint_{-\infty}^{\infty} \exp[ik(1/u+1/F)(x_1^2+y_1^2)/2]$$

$$\times S(x_1, y_1) \exp[-ik(x_1x_2+y_1y_2)/F]dx_1dy_1. \tag{7}$$

Similarly the amplitude distribution in plane $P_3(x_3, y_3)$ can be calculated

$$U_3(x_3, y_3) = \exp\left[\frac{ik}{2}(x_3^2+y_3^2)/(F+V)\right]$$

$$\times \iint_{-\infty}^{\infty} S(x_1, y_1) \exp\left[\frac{ik}{2}(1/u+1/F)(x_1^2+y_1^2)\right] dx_1dy_1$$

$$\times \iint_{-\infty}^{\infty} \exp[-ik\{x_2(x_1/F+x_3/(F+V))+y_2(y_1/F+y_3/(F+V))\}]$$

$$\times \exp[ik(x_2^2+y_2^2)/2(F+V)]dx_2dy_2. \tag{8}$$

Let

$$x_3/(F+V)+x_1/F = \lambda p, \quad y_3/(F+V)+y_1/F = \lambda q. \tag{9}$$

Now considering the second integral of Eq. (8) we have

$$\iint_{-\infty}^{\infty} \exp\left[\frac{ik}{2}(x_2^2+y_2^2)/(F+V)\right] \exp[-2\pi i(px_2+qy_2)]dx_2dy_2 \tag{10}$$

$$= \text{Fourier transform of } \exp[ik(x_2^2+y_2^2)/2(F+V)] \text{ at } (p, q)$$

$$= \exp[-\pi i\lambda(F+V)(p^2+q^2)]. \tag{11}$$

Putting the value of p and q

$$= \exp\left[\frac{ik}{2}(F+V)\{(x_1/F+x_3/F+V)^2+(y_1/F+y_3/F+V)^2\}\right]. \tag{12}$$

Thus by using Eqs (8) and (12), the amplitude distribution in the observation plane $P_3(x_3, y_3)$ is

$$U_3(x_3, y_3) = \iint\limits_{-\infty}^{\infty} \exp\left[\frac{ik}{2}\left(1/u+1/F-\frac{F+V}{F^2}\right)(x_1^2+y_1^2)\right]$$

$$\times S(x_1, y_1) \exp\left[\frac{-ik}{F}(x_1x_3+y_1y_3)\right] dx_1dy_1. \tag{13}$$

This equation shows that the functions $S(x_1, y_1)$ and $U_3(x_3, y_3)$ are the Fourier-transforms of each other having the phase factor and a constant magnification equal to (λF).

To have the exact Fourier-transform relationship, the phase curvature involved in Eq. (13) must vanish. Hence

$$1/u+1/F-(F+V)/F^2 = 0, \tag{14}$$

by solving it

$$uV = F^2. \tag{15}$$

This Eq. (15) is nothing but Newton's formula. If this Eq. (15) holds good, the exact Fourier-transform can be written as follows;

$$U_3(x_3, y_3) = \iint\limits_{-\infty}^{\infty} S(x_1, y_1) \exp\left[-2\pi i(p^1x_1+q^1y_1)\right] dx_1dy_1, \tag{16}$$

where p^1 and q^1 are the reduced spatial frequency co-ordinates

$$p^1 = x_3/\lambda F, \quad q^1 = y_3/\lambda F. \tag{17}$$

Thus the spatial frequency spectrum will be free from phase curvature only if Newton's formula is valid. This favours the correctness of the treatment.

4. *Conclusions*

Newton's formula is a consequence of the treatment applied to a Fourier-transforming configuration to have the exact Fourier-transform. The spherical wave front as an information carrier signal has been considered. This is also the essential condition to achieve the exact Fourier-transform relationship between the amplitude distributions in the input object plane and the observation plane.

The signal waves are reflected by the mirror. The reflected waves are modified by the phase-transformation function $T(x_2, y_2, F)$ of the mirror. These waves display the information of the object function in the form of Fourier-transforms. Hence the filter for reflected waves may be used to pass or block the information existing in Fourier-transforms.

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REFERENCES

[1] A. S. Husain-Abidi, T. F. Krile, *Opt. Commun.* **3**, 409 (1971).
[2] R. S. Kasana, G. P. Bhatnagar, V. S. Dubey, *Atti Fond. Giorgio Ronchi Contr. Ist. Naz. Ottica* **31**, 909 (1976).
[3] J. W. Goodman, *Introduction to Fourier-Optics*, McGrow Hill Book Co. 1968, p. 60.