Vol. A53 (1978) ACTA PHYSICA POLONICA No 2

THEORY FOR PARTICLE DEPOSITION ONTO THE ROTATING DISC SURFACE*,**

By Z. Adamczyk

Research Laboratories of Catalysis and Surface Chemistry, Polish Academy of Sciences, Cracow***

and T. Dąbroś, J. Czarnecki

Institute of Chemistry, Jagellonian University, Cracow****

(*Received May 11, 1977*)

A general equation describing convective diffusion of spherical particles under an external force field has been formulated by introducing "wall effects" dyadics. The equation obtained was explicitly evaluated for the rotating disc and numerically solved on a CYBER-72 computer using our FORTRAN program based on Hamming's predictor-corrector method. The flux of particles at the disc surface was graphically presented as a function of the particle radius confined within the range of $0.05 \div 10.0$ μm.

1. *Introduction*

The deposition of fine particles from flowing suspensions onto the surfaces of macroscopic objects (often called collectors) or onto much larger particles is of great theoretical and practical importance in studies of colloid stability, filtration, detergency, froth and "carrier" flotation.

Clint et al. [1], Spielman and Friedlander [2], Ruckenstein and Prieve [3, 4], Dahneke [5], Bowen, Levine and Epstein [6] have recently developed a theoretical approximation for quantitatively determining the deposition kinetics of small particles onto solid surfaces. They have assumed a convective mass transfer in the bulk of the suspension and a first order reaction at the collector surface. Such an approximation, although useful, is not always justified and fails completely in systems with no, or very small energy barriers. Spielman et al. [7–9] have analysed collection kinetics of particles by spherical, cylindrical

---

* Dedicated to Professor Kazimierz Gumiński on the occasion of his 70-th birthday.

** This work was supported in part by the Commission of Physical Chemistry of Surface, Polish Academy of Sciences, Cracow.

*** Address: Zakład Katalizy i Fizykochemii Powierzchni, Niezapominajek 1, 30-239 Kraków, Poland.

**** Address: Instytut Chemii UJ, Krupnicza 41, 30-060 Kraków, Poland.

and rotating-disc collectors using the trajectory approach. They have taken into account real hydrodynamic, gravity, London–van der Waals, and electrical double-layer forces. However, the diffusion of the particles was not considered and consequently the results obtained by them cannot be applied for small particles (say, with radius below ~ 2 μm). Ruckenstein and Prieve [10] have solved numerically the complete convective-diffusion equation for the spherical collector, considering gravity, and London–van der Waals forces, and the position dependent diffusion coefficient. The accuracy of their calculations was not very high because of the computer memory limitations.

In the present paper a theoretical estimation of the particle flux at the rotating disc surface is presented without accepting the above mentioned simplifying assumptions. Convective diffusion in an external force field has been adopted as a model. Gravity and buoyancy, London–van der Waals and real hydrodynamic forces have been considered. Although the rotating disc has been frequently applied for studies of the heterocoagulation of colloids [1, 11, 12] quantitative theoretical calculations of the particle flux had never been done, and the experimental data were usually interpreted in terms of Levich's formula [13], which of course is a very crude approximation for particles greater than about 0.2 μm.

From the mathematical point of view, the equations describing the steady-state diffusion of particles to the rotating-disc surface are relatively easy to handle with a computer, and enable the calculations to be very accurate.

2. *Statement of the problem*

Let us consider a monodisperse suspension of spherical particles with radius a and density ϱ_p. The particles move quasi-statically at low Reynolds numbers with an instantaneous vector velocity of the centres $\boldsymbol{U}$ in a fluid velocity vector field $\boldsymbol{V}$. The velocity fields $\boldsymbol{U}$ and $\boldsymbol{V}$ are measured relative to a space coordinate system fixed on the collector surface. The particles are assumed to behave independently by excluding all particle-particle interactions and the volume of the suspension is assumed to be sufficiently large, thus the bulk concentration of the particles remains constant during the deposition. Let us assume further that the particles captured by the collector surface do not disturb the later diffusion process. This requires the particle coating not to be dense. If the instantaneous particle number concentration at a point in space is denoted by n, the mass balance of n about a volume element of the suspension may be formulated as

$$\frac{\partial n}{\partial t} + \nabla \cdot \boldsymbol{j} = Q(\boldsymbol{R}, n, t), \tag{1}$$

where $\boldsymbol{j}$ is the particle flux vector, and the term $Q(\boldsymbol{R}, n, t)$ denotes the rate of particle creation or disappearance, and $\boldsymbol{R}$ is the position vector. The total flux $\boldsymbol{j}$ consists of a diffusion flux due to the random Brownian motion of particles and a convective flux due to the action of any hydrodynamic or external force and torque acting on the particles. Thus,

$$\boldsymbol{j} = -\mathscr{D} \cdot \nabla n + \boldsymbol{U} n, \tag{2}$$

where $\mathscr{D}$ is the translational diffusivity diadic of a spherical particle in proximity of the bounding surface. The velocity vector of the particle centre $\boldsymbol{U}$ can be determined from the relations

$$-\eta(\mathscr{K}_t \cdot \boldsymbol{U} + \mathscr{K}_c^{\dagger} \cdot \boldsymbol{\omega}) = \boldsymbol{F}_r, \tag{3}$$

$$-\eta(\mathscr{K}_c \cdot \boldsymbol{U} + \mathscr{K}_r \cdot \boldsymbol{\omega}) = \boldsymbol{T}_r, \tag{4}$$

which are analogical to those given by Brenner [14–15]. In the above relations $\mathscr{K}_t$, $\mathscr{K}_r$, $\mathscr{K}_c$ are respectively, the translational, rotational, and coupling dyadics of a spherical particle in proximity of boundaries, η is the viscosity of the fluid, and $\boldsymbol{\omega}$ is the angular velocity of the particle; the affix † denotes the transposition operation. $\boldsymbol{F}_r$ and $\boldsymbol{T}_r$ are respectively, the hydrodynamic resistance force and torque acting upon the particle. The $\mathscr{D}$, $\mathscr{K}_t$, $\mathscr{K}_r$ and $\mathscr{K}_c$ dyadics are functions of the space coordinates only.

The net force and torque acting on the particle must, of course, be equal to zero, thus we can write $\boldsymbol{F}_r + \boldsymbol{F}_p = \boldsymbol{0}$, and $\boldsymbol{T}_r + \boldsymbol{T}_p = \boldsymbol{0}$ where $\boldsymbol{F}_p$ or $\boldsymbol{T}_p$ consist of the external force or torque $\boldsymbol{F}_{ex}$, $\boldsymbol{T}_{ex}$ and the hydrodynamic force $\boldsymbol{F}_h$ or torque $\boldsymbol{T}_h$ experienced by a particle taken to be held fixed in the fluid velocity field $\boldsymbol{V}$. Considering the above and eliminating $\boldsymbol{\omega}$, equations (3) and (4) yield

$$\boldsymbol{U} = -\eta\boldsymbol{F}_p \cdot \mathscr{K}_r \cdot (\mathscr{K}_t \cdot \mathscr{K}_r - \mathscr{K}_c^{\dagger} \cdot \mathscr{K}_c)^{-1} - \eta\boldsymbol{T}_p \cdot \mathscr{K}_c^{\dagger} \cdot (\mathscr{K}_t \cdot \mathscr{K}_r - \mathscr{K}_c^{\dagger} \cdot \mathscr{K}_c)^{-1}, \tag{5}$$

From the generalized Stokes-Einstein equation given by Brenner [15–16] the hydrodynamic resistance dyadics $\mathscr{K}_t$, $\mathscr{K}_r$, $\mathscr{K}_c$ are related to the diffusivity dyadic $\mathscr{D}$ by the following equation:

$$\mathscr{D} = \frac{kT}{\eta} \mathscr{K}_r \cdot (\mathscr{K}_t \cdot \mathscr{K}_r - \mathscr{K}_c^{\dagger} \cdot \mathscr{K}_c)^{-1} = kT\mathfrak{M}, \tag{6}$$

where $\mathfrak{M}$ is the mobility dyadic, and T is the absolute temperature. Substituting Eqs (2), (5), and (6) into Eq. (1) we finally obtain:

$$\frac{\partial n}{\partial t} + \eta\nabla \cdot [-kT\mathfrak{M} \cdot \nabla n + (\mathscr{K}_r\mathfrak{M} \cdot \boldsymbol{F}_p - \mathscr{K}_c^{\dagger} \cdot \mathscr{K}_r^{-1} \cdot \mathfrak{M} \cdot \boldsymbol{T}_p)n] = Q(\boldsymbol{R}, n, t), \tag{7}$$

In deriving Eq. (7) all inertia effects were neglected, therefore it can be applied for sufficiently small translational Reynolds numbers of the particle only. The above equation with appropriate boundary conditions describes the convective diffusion of spherical particles under an arbitrary vector field of forces to an arbitrary collector surface.

3. *Rotating disc*

Let us assume that the collector is a disc of radius R_d rotating with constant angular velocity ω_d in a liquid, and that the disc is sufficiently large, so the edge effects can be ignored. The ω_d-vector lies parallel to the z-axis of the cylindrical coordinate system, having its origin at the disc centre (cf. Fig. 1). The explicit evaluation of Eq. (7) for the rotating disc collector requires the analytical expressions for $\mathfrak{M}$, $\mathscr{K}_r$, $\mathscr{K}_c^{\dagger}$ and $\boldsymbol{T}_p$ to be known. According to Brenner [17] the motion of a spherical particle in the vicinity of a solid plane boundary may be decomposed into two separate motions with translation directions perpendicular and parallel to the boundary. This is possible because all equations govering particle motion are linear, and the boundary conditions are additive. Thus, the resistance dyadic take the following forms [18]:

$$\mathscr{K}_t = \boldsymbol{i}_1\boldsymbol{i}_1 K_{t,\perp} + (\mathscr{I} - \boldsymbol{i}_1\boldsymbol{i}_1)K_{t,\parallel}, \tag{8}$$

$$\mathscr{K}_r = \boldsymbol{i}_1\boldsymbol{i}_1 K_{r,\perp} + (\mathscr{I} - \boldsymbol{i}_1\boldsymbol{i}_1)K_{r,\parallel}, \tag{9}$$

$$\mathscr{K}_c = \mathscr{E} \cdot \boldsymbol{i}_1 K_{c,\parallel}, \tag{10}$$

where $\boldsymbol{i}_1$ is the unit vector directed perpendicular to the wall pointing into the liquid, $\mathscr{I}$ is the unit dyadic and $\mathscr{E}$ is the unit isotropic triadic (alternating triadic cf. Happel, Brenner [19]), $K_{t,\perp}$, $K_{t,\parallel}$, $K_{r,\perp}$, $K_{r,\parallel}$ and $K_{c,\parallel}$ are the scalar resistance coefficients, each of them depends only upon a/h (h is the separation between the particle and collector surfaces). They are always positive numbers and possess the following properties: $K_{t,\perp}/6\pi a$, $K_{t,\parallel}/6\pi a$, $K_{r,\perp}/8\pi a^3$, $K_{r,\parallel}/8\pi a^3$, tend asymptotically to unity as $a/h \to 0$ and $K_{c,\parallel}$ tends to zero in this limit. The tabulated values of the resistance coefficients as functions of h/a are to be found in Refs [20–22]. Considering Eqs (8)–(10), the mobility dyadic takes the form

$$\mathfrak{M} = \boldsymbol{i}_1\boldsymbol{i}_1(K_{t,\perp} \cdot K_{r,\perp})^{-1} + (\mathscr{I} - \boldsymbol{i}_1\boldsymbol{i}_1)(K_{t,\parallel} \cdot K_{r,\parallel} - K_{c,\parallel}^2)^{-1}, \tag{11}$$

Assuming that the $\boldsymbol{i}_1$ unit vector coincides with the $\boldsymbol{i}_z$ unit vector and that the particle concentration is a function of the z-coordinate only, Eq. (7), by considering relations (9)–(11), can be formulated explicitly as follows (in cylindrical coordinates r, θ, z)

$$\begin{aligned}\frac{\partial n}{\partial t} &+ \eta \frac{1}{r}\frac{\partial}{\partial r}\{r[K_{r,\parallel}(K_{t,\parallel} - K_{c,\parallel}^2)^{-1}F_{p,r} - K_{c,\parallel}(K_{t,\parallel}K_{r,\parallel} - K_{c,\parallel}^2)^{-1}T_{p,\theta}]n\} \\ &+ \eta \frac{1}{r}\frac{\partial}{\partial \theta}\{[K_{r,\parallel}(K_{t,\parallel} - K_{c,\parallel}^2)^{-1}F_{p,\theta} - K_{c,\parallel}(K_{t,\parallel}K_{r,\parallel} - K_{c,\parallel}^2)^{-1}T_{p,r}]n\} \\ &+ \eta \frac{\partial}{\partial z}\left[K_{t,\perp}^{-1}\left(-kT\frac{\partial n}{\partial z} + F_{p,z}n\right)\right] = Q(r, \theta, z, n, t),\end{aligned} \tag{12}$$

where $F_{p,z}$, $F_{p,\theta}$, $F_{p,z}$ are the corresponding components of the force vector, and $T_{p,r}$, $T_{p,\theta}$, $T_{p,z}$ are the corresponding components of the torque vector. Under the steady-state conditions $\frac{\partial n}{\partial t} \equiv 0$, and assuming the term $Q(r, \theta, z, n, t)$ to be equal to zero, Eq. (12) is simplified considerably, and can be numerically solved, provided that the analytical expressions for $K_{t,\parallel}$, $K_{t,\perp}$, $K_{c,\parallel}$, $F_{p,r}$, $F_{p,\theta}$, $F_{p,z}$, $T_{p,r}$, $T_{p,\theta}$, $T_{p,z}$ are known. The tabulated values of $K_{t,\perp}$ as a function of H were given by Brenner [20] and by Goren [23]. Therefore

$$K_{t,\perp} = \frac{1}{6\pi a} F_1\left(\frac{z}{a} - 1\right),$$

where F_1 is the universal hydrodynamic function of the gap width $H = z/a - 1$ (cf. Fig. 1).

The tabulated values of $K_{r,\|}$, $K_{t,\|}$, $K_{c,\|}$ can be obtained by virtue of Goldman, Cox, and Brenner's [21] solution of the Stokes creeping motion equation for a sphere moving in a semi-infinite fluid parallel to a solid plane surface. In order to obtain the analytical expressions for $F_{p,r}$, $F_{p,\theta}$, $F_{p,z}$ we need to know the velocity vector field V of the liquid near the rotating disc. The problem of laminar flow due to the rotating disc was first solved by

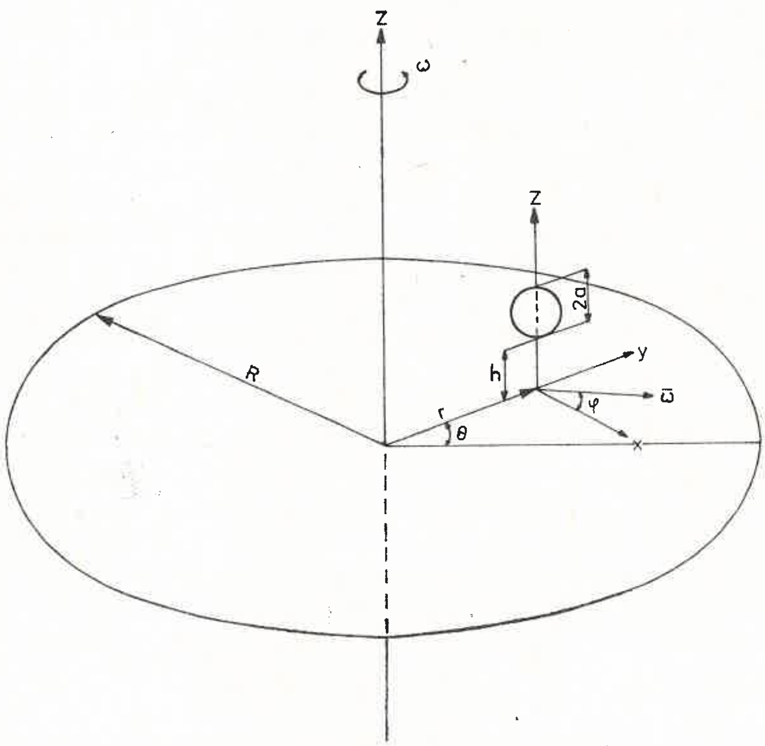

Fig. 1. Coordinate systems for the rotating disc

von Kármán [24] and later more exactly by Cochran [25]. At small distances from the disc surface, the components of the liquid velocity field V expressed in cylindrical coordinates relative to the disc surface are (through the first terms)

$$V_r = f(0)\,(\omega^3/\nu)^{\frac{1}{2}} rz \tag{14}$$

$$V_\theta = g(0)\,(\omega^3/\nu)^{\frac{1}{2}} rz \tag{15}$$

$$V_z = -f(0)\,(\omega^3/\nu)^{\frac{1}{2}} z^2 \tag{16}$$

where $f(0)$, $g(0)$ are the universal (dimensionless) constants for the rotating disc and ν is the kinematic viscosity of the liquid.

Spielman and Fitzpatrick [8] decomposed the undisturbed velocity field V near the disc, by the introduction of local Cartesian coordinates (x, y, z) which are tangential to the directions (r, θ, z) and have their origin at the point $(r, \theta, 0)$ see Fig. 1, and local cylindrical coordinates (ϖ, ϕ, z) with the same origin as the (x, y, z) coordinates.

According to [8] $V = V_{st} + V_{sh,y} + V_{sh,x} + V_{sp}$, where

$$V_{st} = f(0)\,(\omega^3/\nu)^{\frac{1}{2}}(\varpi z \boldsymbol{i}_{\varpi} - z^2 \boldsymbol{i}_z), \tag{17}$$

$$V_{sh,y} = f(0)\,(\omega^3/\nu)^{\frac{1}{2}} r z_{iy}, \tag{18}$$

$$V_{sh,x} = g(0)\,(\omega^3/\nu)^{\frac{1}{2}} r z \boldsymbol{i}_x, \tag{19}$$

$$V_{sp} = g(0)\,(\omega^3/\nu)^{\frac{1}{2}} \varpi z \boldsymbol{i}_{\phi}, \tag{20}$$

here $\boldsymbol{i}_x, \boldsymbol{i}_y, \boldsymbol{i}_z, \boldsymbol{i}_{\varpi}, \boldsymbol{i}_{\phi}$ are the unit vectors, and V_{st} is the axisymmetrical stagnation field which induces a purely z-directed force on the particle, $V_{sh,y}$ is the uniform shear field which induces a hydrodynamic force parallel to the disc surface in the y-direction (which for the particle, coincides with the r-direction), and a hydrodynamic torque in the x, (θ) direction, $V_{sh,x}$ induces a force parallel to the disc in the x-direction (which for the particle, coincides with the θ-direction) and a hydrodynamic torque in the y, (r) direction, and finally V_{sp} is a spinning flow which tends to spin the particle in place about the z-axis. The particle motion induced by the fields $V_{sh,y}$ and V_{sp} cause no net contribution to the collection of particles by the disc.

By virtue of the above considerations Spielman and Fitzpatrick have determined the hydrodynamic force $F_{h,z}$ on particles immersed in an axisymmetrical stagnation field, giving the expression.

$$F_{h,z} = -6\pi\eta a f(0)\,(\omega^3/\nu)^{\frac{1}{2}} z^2 F_2(H) \tag{21}$$

where $F_2(z/a-1)$ is the universal hydrodynamic function, known for all H from the exact solution of Stokes equation, given by Goren [26] and Goren and O'Neill [27].

The expressions for $F_{h,r}$ and $T_{h,\theta}$ are given by Goldman, Cox and Brenner [28], thus

$$F_{h,r} = 6\pi\eta a f(0)\,(\omega^3/\nu)^{\frac{1}{2}} F_3(H) r z, \tag{22}$$

$$T_{h,\theta} = 4\pi\eta a^3 f(0)\,(\omega^3/\nu)^{\frac{1}{2}} F_4(H) r, \tag{23}$$

where $F_3(H)$ and $F_4(H)$ are universal hydrodynamic functions of the gap width H.

Putting (13), (21), (22) and (23) into equation (12) and considering that $F_{p,\theta}$ and $T_{p,r}$ do not depend on θ, and that the external force is purely z-directed, we get (under the steady-state conditions)

$$\frac{\partial}{\partial z}\left\{F_1(H)\left\{\frac{\partial n}{\partial z}\frac{kT}{6\pi\eta a} + \left[f(0)\,(\omega^3/\nu)^{\frac{1}{2}} z^2 F_2(H) + \frac{F_{ex}}{6\pi\eta a}\right] n\right\}\right\}$$
$$= 2f(0)\,(\omega^3/\nu)^{\frac{1}{2}} z F_5(H) n, \tag{24}$$

where

$$F_5(H) = K_{r,\parallel}(K_{t,\parallel} - K_{c,\parallel}^2)^{-1} F_3(H) - K_{c,\parallel}(K_{t,\parallel}K_{r,\parallel} - K_{c,\parallel}^2)^{-1} F_4(H)\,\tfrac{2}{3}\,\frac{a}{h}, \tag{25}$$

is another universal hydrodynamic function given by Goldman, Cox and Brenner [28].

Eq. (24) is an ordinary linear differential equation of the second order and is subject to the following boundary conditions:

$$n = 0 \quad \text{at} \quad z = \delta$$

$$n = n_{\infty} \quad \text{at} \quad z \to \infty, \tag{26}$$

where δ denotes the closest distance between the particle and the disc surfaces, and n_{∞} is the particle number concentration in the bulk.

4. *Method of solution*

Equation (25) and the corresponding boundary conditions (26) were transformed into dimensionless form by the substitutions

$$\bar{n} = n/n_{\infty}, \quad H = z/a-1, \quad D_{\infty} = kT/6\pi\eta a, \tag{27}$$

where D_{∞} is the diffusion coefficient of a spherical particle in an unbounded liquid. Thus, we obtain

$$\frac{d}{dH}\left\{F_1(H)\left[\frac{d\bar{n}}{dH} + \mathrm{Pe}(H+1)^2F_2(H)\bar{n} + \frac{aF_{\mathrm{ex}}}{kT}\bar{n}\right]\right\} = \mathrm{Pe}(H+1)F_5(H)\bar{n}, \tag{28}$$

where

$$\mathrm{Pe} = \frac{2aV_H}{D_{\infty}} = 2f(0)\,(\omega^3/\nu)^{\frac{1}{2}}\,\frac{a^3}{D_{\infty}}$$

is the dimensionless Péclet number calculated for the liquid H-velocity component at the point $H = 1$, $(z = a)$. The appropriate boundary conditions are now

$$n = 0 \quad \text{at} \quad H = \bar{\delta}, \tag{29}$$

$$n = 1 \quad \text{at} \quad H \to \infty,$$

where $\bar{\delta} = \delta/a$ is the dimensionless closest distance between the collector and particle surfaces.

Let us assume that the external force consists of:

1. Net gravity and buoyancy force given by the expression

$$F_g - F_b = (m_p - m_l)g = 4/3\pi a^3(\varrho_p - \varrho_l)g, \tag{30}$$

where m_p, ϱ_p are the particle mass and density respectively, and m_l, ϱ_l are the liquid mass and density respectively, and g is the acceleration due to gravity.

2. Retarded London–van der Waals force, having evidently an H-component only

$$F_l = \frac{A}{6a^2}\,\frac{\lambda(\lambda/a+22.232H)}{H^2(\lambda/a+11.116H)^2} \tag{31}$$

where A is the Hamaker constant, and λ is the characteristic wavelength in the dispersion energy theory. Here, the formula given by Suzuki, Ho and Higuchi [29] was adopted. The electrical double-layer forces, and the body force due to the radial acceleration of the particle have been neglected.

Using Eqs (30) and (31), Eq. (28) can be rearranged to the form

$$\frac{d}{dH}\left\{F_1(H)\left\{\frac{d\bar{n}}{dH}+\tfrac{1}{2}\,\mathrm{Pe}\,[(H+1)^2F_2(H)+\mathrm{Gr}]\bar{n}+\mathrm{Ad}\,\frac{(\lambda/a)\,[(\lambda/a)+22.232H]}{H^2[(\lambda/a)+11.116H]^2}\,\bar{n}\right\}\right\}$$

$$= \mathrm{Pe}\,(H+1)F_5(H)\bar{n}, \tag{32}$$

where

$$\mathrm{Gr} = \frac{4\Delta\varrho g}{9f(0)\omega^{\frac{3}{2}}\nu^{\frac{1}{2}}\varrho_l^{\frac{1}{2}}}$$

is the dimensionless gravity number, and $\mathrm{Ad} = \dfrac{A}{6kT}$ is the dimensionless adhesion number. The boundary conditions are the same as expressed by Eq. (29).

The particle flux at the disc surface can be expressed as

$$j_{z=\delta} = D(\delta)\left(\frac{dn}{dz}\right)_\delta = \frac{D(\bar{\delta})}{a}\left(\frac{d\bar{n}}{dH}\right)_{\bar{\delta}} n_\infty = \frac{D_\infty n_\infty}{a}\,F_1(\bar{\delta})\left(\frac{d\bar{n}}{dH}\right)_{\bar{\delta}}, \tag{33}$$

where $D(\delta)$, $D(\bar{\delta})$ are the diffusion coefficients at the point δ and $\bar{\delta}$ respectively.

The flux of particles was determined after numerical integration of Eq. (32) using Hamming's predictor-corrector method with initial values calculated according to the 4-th order Runge-Kutta's method [30] on the CYBER-72 computer. Further details of the calculation can be found elsewhere [31].

5. *Results and discussion*

The numerical solution to Eq. (32) gives the flux of the particles at the disc surface as a function of the dimensionless parameters Pe, Gr, Ad, λ/a or as an explicit function of the physico-chemical parameters describing the state of the system such as temperature, viscosity and density of the liquid, density and radius of the particle, disc angular velocity etc. Most of the computation results and the discussion were presented elsewhere [31].

Fig. 1 shows the dependence of the normalized flux $\bar{j} = j/n_\infty$ on the particle radius a, at fixed selected values of the apparent densities of the particle $\Delta\varrho$. The temperature was assumed to be 293°K, the liquid viscosity 10^{-2} g/cm sec, the disc angular velocity $\omega = 25$ rad/sec, the Hamaker constant $A = 10^{-13}$ erg. The straight line *3* drawn as a reference denotes the flux calculated according to Levich's formula

$$\bar{j} = 0.62\omega^{\frac{1}{2}}D^{\frac{2}{3}}\nu^{-\frac{1}{6}}$$

As it can be seen in Fig. 2 our predictions approach asymptotically Levich's values as the radius of the particle becomes small (for $a < 0.2$ µm independently of the particle density and the Hamaker constant. On the other hand, for larger particles, it is clear, that the Levich equation cannot be applied even as a crude approximation (especially

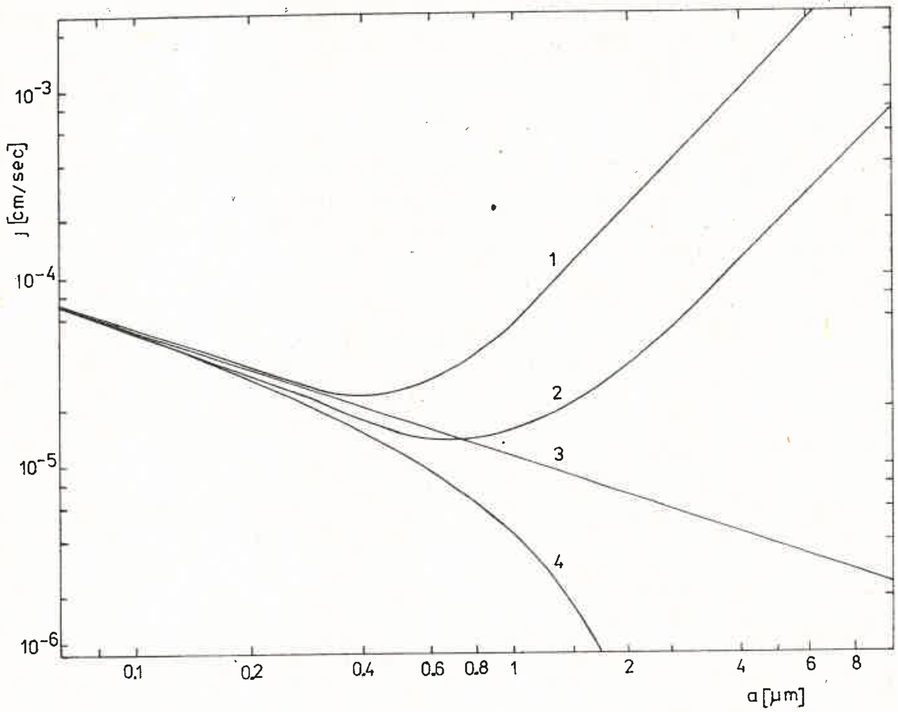

Fig. 2. The normalized flux $\bar{j} = j/n_\infty$ at the disc surface as a function of the particle radius a. The temperature was assumed to be $T = 293$ K, the liquid viscosity $\eta = 10^{-2}$ g/cm sec, the Hamaker constant $A = 10^{-13}$ erg; the disc angular velocity $\omega = 25$ rad/sec. Curve *1*: for the apparent density of the particle $\Delta\varrho = 0.3$ g/cm³. Curve *2*: for the apparent density equal to zero. Curve *3*: for diffusion alone Levich's formula $\bar{j} = 0.62\,\omega^{\frac{1}{2}} D^{\frac{2}{3}} \nu^{-\frac{1}{6}}$. Curve *4*: for apparent density of the particle -0.3 g/cm³

for dense particles). Therefore, for a particle with a radius of 3 µm and an apparent density of 0.3 g/cm³ Levich's formula gives a flux value of about two orders of magnitude smaller than our predictions, and for a particle with apparent density -0.3 g/cm³ a flux value of about two orders of magnitude greater (the minus sign of the apparent density value denotes the net gravity and buoyancy force acting away from the disc surface).

6. Conclusions

1. The transport equation presented above describes the convective diffusion of spherical particles in an arbitrary force field to a solid surface of an arbitrary geometrical shape.

2. This equation formulated explicitly for the rotating disc system becomes an ordinary linear differential equation of the second order.

3. The numerical integration of this equation gives the flux at the disc surface as a function of dimensionless parameters Pe, Gr, Ad, λ/a, describing the influence of hydrodynamic, gravity and London-van der Waals forces, respectively.

4. For very small particles (with radius less than about 0.2 μm) our predictions become identical with those calculated according to Levich's formula. For greater particles the flux is mainly due to interception and sedimentation.

REFERENCES

[1] G. E. Clint, J. H. Clint, J. M. Corkill, T. J. Walker, *J. Colloid. Interface Sci.* **44**,121 (1973).
[2] L. A. Spielman, S. K. Friedlander, *J. Colloid Interface Sci.* **46**, 22 (1974).
[3] E. Ruckenstein, D. C. Prieve, *J. Chem. Soc. Faraday Trans. II*, **69**, 1522 (1974).
[4] E. Ruckenstein, D. C. Prieve, *A. I. Ch. E. J.* **22**, 276 (1976).
[5] B. Dahneke, *J. Colloid Interface Sci.* **48**, 520 (1974).
[6] B. D. Bowen, S. Levine, N. Epstein, *J. Colloid Interface Sci.* **54**, 375 (1976).
[7] L. A. Spielman, S. L. Goren, *Environ. Sci. Technol.* **4**, 135 (1970).
[8] L. A. Spielman, J. A. Fitzpatrick, *J. Colloid Interface Sci.* **42**, 607 (1973).
[9] L. A. Spielman, P. M. Cukor, *J. Colloid Interface Sci.* **43**, 51 (1973).
[10] E. Ruckenstein, D. C. Prieve, *A. I. Ch. E. J.* **20**, 1178 (1974).
[11] J. K. Marshall, J. A. Kitchener, *J. Colloid Interface Sci.* **22**, 342 (1966).
[12] M. Hull, J. A. Kitchener, *Trans. Faraday Soc.* **65**, 3093 (1969).
[13] V. G. Levich, *Physicochemical Hydrodynamics*, Prentice Hall, Englewood Cliffs, New York 1962.
[14] H. Brenner, *J. Colloid Interface Sci.* **20**, 104 (1965).
[15] H. Brenner, *Chem. Eng. Sci.* **19**, 599 (1964).
[16] H. Brenner, *J. Colloid Interface Sci.* **23**, 407 (1967).
[17] H. Brenner, *J. Fluid Mech.* **18**, 144 (1964).
[18] R. G. Cox, H. Brenner, *Chem. Eng. Sci.* **22**, 1753 (1967).
[19] J. Happel, H. Brenner, *Low Reynolds Number Hydrodynamics*, Prentice Hall, Englewood Cliffs, New York 1965.
[20] H. Brenner, *Chem. Eng. Sci.* **16**, 242 (1961).
[21] A. J. Goldman, R. G. Cox, H. Brenner, *Chem. Eng. Sci.* **22**, 637 (1967).
[22] S. L. Goren, M. E. O'Neill, *Chem. Eng. Sci.* **26**, 325 (1971).
[23] S. L. Goren, *J. Colloid Interface Sci.* **44**, 356 (1973).
[24] Th. von Kármán, *Z. Angew. Math. Mech.* **1**, 235 (1923).
[25] W. G. Cochran, *Proc. Cambr. Philos.* **30**, 365 (1934).
[26] S. L. Goren, *J. Fluid Mech.* **41**, 619 (1970).
[27] S. L. Goren, M. E. O'Neill, *Chem. Eng. Sci.* **26**, 235 (1971).
[28] A. J. Goldman, R. G. Cox, H. Brenner, *Chem. Eng. Sci.* **22**, 653 (1967).
[29] A. Suzuki, N. F. H. Ho, W. I. Higuchi, *J. Colloid Interface Sci.* **29**, 552 (1969).
[30] A. Ralston, *A First Course in Numerical Analysis*, Mc Grow-Hill, New York (1965).
[31] T. Dąbroś, Z. Adamczyk, J. Czarnecki, *J. Colloid Interface Sci.*, **62**, 529 (1977).