Vol. A52 (1977) ACTA PHYSICA POLONICA No 4

# AN APPROXIMATE DETERMINATION OF THE ANISOTROPY CONSTANT IN THIN FILMS OF FERROMAGNETS WITH THE UNIAXIAL ANISOTROPY

By A. Szymanowska

Institute of Mathematics, Polish Academy of Sciences, Łódź*

*(Received August 13, 1976; revised version received December 23, 1976)*

This paper contains an application of the approximate method of Ławrynowicz and Wojtczak (*Acta Phys. Pol.* **A41**, 11 (1972)) to analyze domain structures in thin films of ferromagnets with uniaxial anisotropy. Effective calculations of the anisotropy constant for multi-domain structures of the Landau and Lifshitz type are given.

## *1. Introduction*

At the present time there exists no general theory of domains which would give a complete distribution of magnetization inside a sample, i. e., the domain structure and its parameters. It is possible to calculate parameters of domain structure assuming their form based on experiments [2, 10, 11]. Brown presented the domain structure problem with variational equations [1]. Nevertheless, their effective solution is difficult.

This paper gives applications of the approximate method, obtained by Ławrynowicz and Wojtczak [3], to the basic domain structures in thin films of ferromagnets with the uniaxial anisotropy.

The method is concerned with the problem of determining the domain structure in ferromagnets on the basis of physical parameters of a sample: the exchange integral, the uniaxial and cubic anisotropy constans, and the geometrical dimensions of the sample. The authors determine approximately the possible directions of magnetization, by minimizing the free energy within the class of eigenstates in which the Hamiltonian is diagonal. In the first step this approximation assumes that the domain walls have negligible thickness, and that the magnetization changes by jumps over the boundaries of regions in question. In this step the magnetization vectors are found and the possible boundaries of domains are determined. The second step of the approximation gives the elimination of the error connected with the assumption about the domain wall thickness.

* Address: Instytut Matematyczny, PAN, Kilińskiego 86, 90-012 Łódź, Poland.

In the present paper the equations having solutions which determine the configurations of boundaries of magnetic films are established. These equations are dependent only on the demagnetizing factor, which may be obtained on the basis of the geometrical dimensions of the sample. The paper presents a numerical verification of these equations and gives calculations of the uniaxial anisotropy constant for multi-domain structure of the Landau and Lifshitz type.

## 2. *The Hamiltonian*

Let us consider a ferromagnetic film with uniaxial anisotropy parallel to the sample. We confine ourselves to films so thin that their magnetic structure is homogeneous across the film. In this case the sample can be considered as a film which is a superposition of $m$ monoatomic layers, each of them with thickness $a$. If $x_1$, $x_2$, $x_3$ denote the rectangular coordinates of a point $x$, then we define by $\nu = x_3/a$ the layer which contains this point. The position of an atom in the plane of a layer $x_3 = \nu a$ is given by $z = x_1 + ix_2$, where $i$ denotes the imaginary unit. We assume that the easy axis of magnetization is directed along $\mathrm{Re}\, z = 0$ and the cosines of the angles between the axis $(x_\alpha)$ and the vector of magnetization at the point $z_j$ resp. $z_{j_\nu}$ are equal to $\gamma_{j,\alpha}$ resp. $\gamma_{j_\nu,\alpha}$, $\alpha = 1, 2, 3$.

Suppose that the properties of the sample in question are described by the Hamiltonian

$$H = H_e + H_a + H_d, \tag{2.1}$$

where $H_e$, $H_a$, $H_d$ denote the isotropic Heisenberg exchange term, the anisotropic term, and the demagnetizing factor, respectively. The terms appearing in (2.1) are given by the formula (cf. e. g. [3]):

$$H_e = -mI \sum_{\langle j,j_\nu \rangle} \sum_{\alpha} S_{j,\alpha} S_{j_\nu,\alpha}, \quad H_a = -m \sum_j K_{\|} S_{j,2}^2, \quad H_d = -m \sum_j \sum_\alpha M_{j,\alpha} S_{j,\alpha}^2.$$

Here $I$ is the exchange integral and $K_{\|}$, denotes the uniaxial anisotropy constant parallel to the sample. Further $M_{j,\alpha}$ is the demagnetizing factor corresponding to the $\alpha$-component of the atom $z_j$; $S$ is the value of spin, and $S_{j,\alpha}$, $S_{j_\nu \alpha}$ denote the $\alpha$-components of the spin operator at $z_j$ resp. $z_{j_\nu}$. Here $z_j$ is situated in the plane of any fixed layer, while $z_{j_\nu}$ is situated in the plane of the layer $x_3 = \nu a$.

According to [3], the energy $E$ of the system of spins, given by (2.1), becomes

$$E = -mS^2 \sum_j \left[K_{\|} \gamma_{j,2}^2 + \sum_\alpha M_{j,\alpha} \gamma_{j,\alpha}^2 + I \sum_{j_\nu} \gamma_{j,\alpha} \gamma_{j_\nu,\alpha}\right)]. \tag{2.2}$$

## 3. *An approximate determination of the domain structure*

In order to determine the possible directions of magnetization within the sample we have to minimize the energy $E$ with respect to the direction cosines. In the presented approximation we assume that the domain walls have a negligible thickness and that the magnetization changes by jumps over the boundaries of the regions in question. This assumption allows us to consider the neighbouring atoms as having the same directions of spins. An error appears only for atoms lying in the nearest neighbourhood of the boundaries, and is connected with the ratio of the area of walls and domains. The elimination of this error is realized in [4].

Owing to the above assumptions formula (2.2) becomes

$$E = -mS^2 \iint_{\Sigma} [K_{\|}\gamma_2^2 + \sum_{\alpha} (M_\alpha \gamma_\alpha^2 + \zeta I \gamma_\alpha^2)] d\sigma, \tag{3.1}$$

where $\sum$ denotes the section of the sample by the $z$-plane, $d\sigma$ — the area element, $\zeta$ — the number of nearest neighbours of an atom in the film, while $M_\alpha$ and $\gamma_\alpha$, $\alpha = 1, 2, 3$, are step functions of the variable $z$, constant within a fixed domain and corresponding to $M_{j,\alpha}$ and $\gamma_{j,\alpha}$ respectively, $z_j$ ranging over all atoms in the $z$-plane. Since $\gamma_1^2+\gamma_2^2+\gamma_3^2 = 1$ and in our case $\gamma_3 = 0$, the formula (3.1) yields

$$\bar{E} = -mS^2 \iint_{\Sigma} E d\sigma, \tag{3.2}$$

where

$$\bar{E} = s_0 + \mathrm{Re}\, s - (\mathrm{Re}\, s - \mathrm{Im}\, s)\gamma_2^2 = s_0 + \mathrm{Im}\, s + (\mathrm{Re}\, s - \mathrm{Im}\, s)\gamma_1^2,$$

$$s_0 = \zeta I, \qquad s = M_1 + i(M_2 + K_{\|}).$$

The minimum of $E$ with respect to the direction cosines is realized in the following cases:

(a) $\gamma_2 = 0, \quad \gamma_1 = \pm 1, \quad \mathrm{Re}\, s > \mathrm{Im}\, s$,

(b) $\gamma_1 = 0, \quad \gamma_2 = \pm 1, \quad \mathrm{Re}\, s < \mathrm{Im}\, s$,

(c) $\gamma_1$ and $\gamma_2$ are arbitrary such that $\gamma_1^2+\gamma_2^2 = 1$ and $\mathrm{Re}\, s = \mathrm{Im}\, s$.

Physically the conditions (a) and (b) denote the possibility of four directions of magnetization in the sample. They are: parallel or antiparallel to the easy axis of magnetization and perpendicular to this axis in the direction parallel or antiparallel with the section of the sample by $z$-plane. Condition (c) corresponds to the domain wall in which the magnetization changes from the direction specified in (a) to the direction specified in (b) or vice versa.

## *4. The equations of the boundaries*

The solutions of the equation $\mathrm{Re}\, s = \mathrm{Im}\, s$, in condition (c), are the boundaries separating the domains specified in (a) and (b). In accordance with (3.2) this equation becomes

$$M_1(z, \tilde{z}) - M_2(z, \tilde{z}) = K_{\|}. \tag{4.1}$$

Curves of the form $\tilde{z} = f(z)$, determiend as solutions of the equation (4.1), represent, boundaries of magnetic domains.

Now we derive the exact form of this equation for the multi-domain structure of the Landau and Lifshitz type, within the thin film of a ferromagnet with the dimensions

$D$, $D$, $ma$. The corresponding domain structure is shown on Fig. 1. It contains $2n$ domains with the magnetization as specified in (a) and $2(2n-1)$ domains with the magnetization as specified in (b).

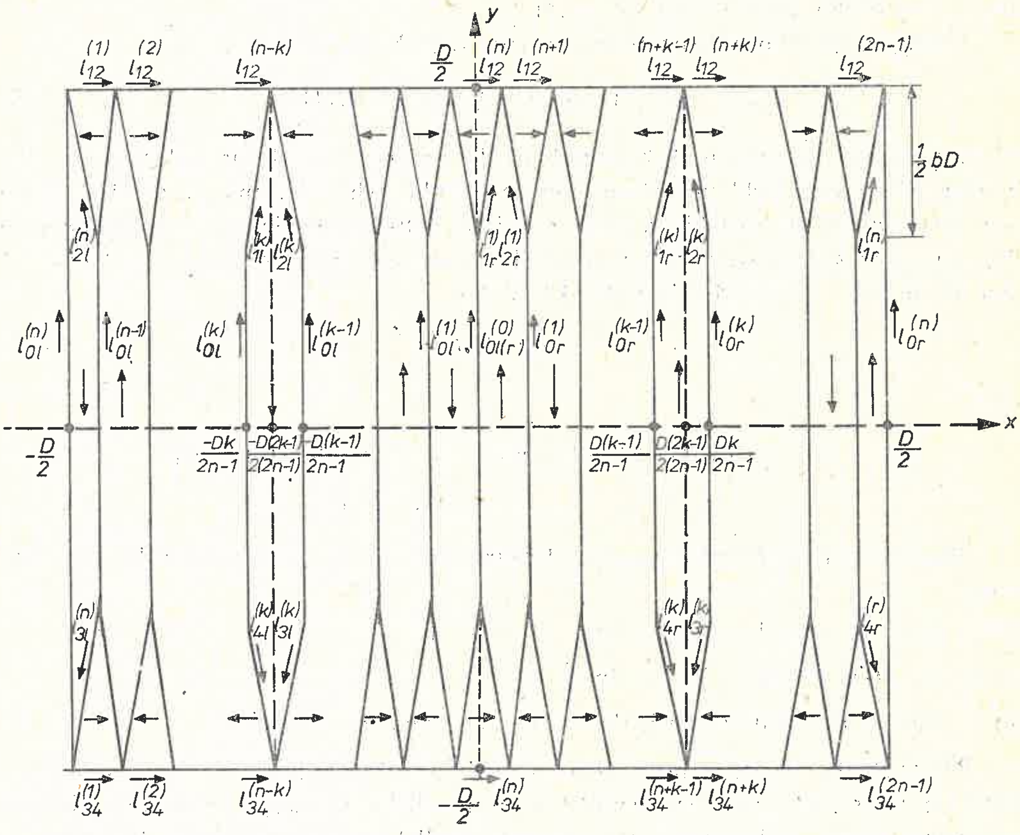

Fig. 1. Configuration of the domain structure in a thin ferromagnetic film

Let us consider the $k$-th domain with the magnetization directed parallel to the easy axis of magnetization, and calculate the demagnetizing factor at a point $z^* = (x_1^*, x_2^*)$ of the layer $x_3 = va$, lying in the boundary separating this domain and the neighbouring domain with the magnetization antiparallel to the axis $\operatorname{Im} z = 0$ and lying above the axis $\operatorname{Re} z = 0$. According to [3], [7] and [9] the components of the demagnetizing factor at the point $z^*$ of the layer $x_3 = va$ are given by the formulae

$$M_{1k}^{(h)} = c_k^{(h)} \sum_j \int_{t_j}^{t'_j} \overline{M}_j^{(h)} \operatorname{Re}(z_j^{(h)} - z^*) \operatorname{Im} \frac{d}{dt} z_j^{(h)} dt,$$

$$M_{2k}^{(h)} = c_k^{(h)} \sum_j \int_{t_j}^{t'_j} \overline{M}_j^{(h)} \operatorname{Im}(z_j^{(h)} - z^*) \operatorname{Re} \frac{d}{dt} z_j^{(h)} dt, \tag{4.2}$$

where

$$\overline{M}_j^{(h)} \approx ma|z_j^{(h)}-z^*|^{-2}(|z_j^{(h)}-z^*|^2+m^2a^2)^{-1/2}.$$

Here the summation is carried out over all domains bounded by the curves $z = z_j^{(h)}(t)$, $t_j \leqslant t \leqslant t_j'$, with the direction of the magnetization vector parallel to the easy axis for $h = 1$ and antiparallel to the axis $\operatorname{Im} z = 0$ for $h = 2$. The constants $c_k^{(h)}$ are determined by the condition

$$M_{1k}^{(h)}+M_{2k}^{(h)} = g\mu^2/v_0, \tag{4.3}$$

where $g$ is the gyromagnetic factor, $\mu_B$ is the Bohr magneton, and $v_0$ is the volume of an elementary cell.

It is seen from Fig. 1 that all bounding curves consist of segments given by the equation

$$z_L(t) = (A_1+B_1t, A_2+B_2t), \tag{4.4}$$

where $0 \leqslant t \leqslant 1$ and $A_1, A_2, B_1, B_2$ are constants which may be obtained from Fig. 1. Therefore, the calculation of $M_{1k}^{(h)}$ and $M_{2k}^{(h)}$ reduces to calculations analogous to the integrals for the curves (4.4) occurring in (4.2).

Now let us set for the curve (4.4)

$$N_{1L} = \int_0^1 \frac{ma \operatorname{Re}(z_L-z^*) \operatorname{Im} \frac{d}{dt} z_L}{|z_L-z^*|^2(|z_L-z^*|^2+\frac{1}{4} m^2a^2)^{1/2}} dt,$$

$$N_{2L} = \int_0^1 \frac{ma \operatorname{Im}(z_L-z^*) \operatorname{Re} \frac{d}{dt} z_L}{|z_L-z^*|^2(|z_L-z^*|^2+\frac{1}{4} m^2a^2)^{1/2}} dt. \tag{4.5}$$

After elementary calculations we obtain the following results:

(*i*) if $(x_1^*, x_2^*) \notin z_L$, then

$$N_{1L} = \frac{B_2}{B_1^2+B_2^2}\{B_2[J(u_1, v_1)-J(u_2, v_2)]+B_1[I(u_2^*, v_2^*)-I(u_1^*, v_1^*)]\},$$

$$N_{2L} = \frac{B_1}{B_1^2+B_2^2}\{B_1[J(u_2, v_2)-J(u_1, v_1)]+B_2[I(u_2^*, v_2^*)-I(u_1^*, v_1^*)]\}, \tag{3.6}$$

where

$$u_1 = u_2 = 2[B_1(A_2-x_2^*)-B_2(A_1-x_1^*)]/(B_1^2+B_2^2)^{1/2},$$

$$v_1 = 2[B_1(A_1-x_1^*)+B_2(A_2-x_2^*)]/(B_1^2+B_2^2)^{1/2},$$

$$v_2 = 2[B_1^2+B_2^2+B_1(A_1-x_1^*)+B_2(A_2-x_2^*)]\,(B_1^2+B_2^2)^{1/2},$$

$$u_1^* = 2(A_1-x_1^*),\quad v_1^* = 2(A_2-x_2^*),\quad u_2^* = 2(A_1+B_1-x_1^*),\quad v_2^* = 2(A_2+B_2-x_2^*),$$

and

$$I(u, v) = \log \left| \frac{(u^2+v^2+m^2a^2)^{1/2}-ma}{(u^2+v^2+m^2a^2)^{1/2}+ma} \right|,$$

$$J(u, v) = 2 \arctan \frac{ma}{u} \frac{v}{(u^2+v^2+m^2a^2)^{1/2}}, \tag{3.7}$$

(*ii*) if $(x_1^*, x_2^*) \in z_L$, then

$$N_{1L} = N_{2L} = \frac{B_1 B_2}{B_1^2+B_2^2} [I(u_2^*, v_2^*) - I(u_1^*, v_1^*)],$$

where $u_1^*, v_1^*, u_2^*, v_2^*$ have the same meaning as in (*i*).

Now let $N_{1jh}^{(1)}$ and $N_{2jh}^{(1)}$ be defined by (4.5), where $z_L$ is the curve bounding the $j$-th domain with the magnetization parallel to the easy axis of magnetization (Fig. 1). The coefficient $h = l$ denotes that the domain in question lies to the left of the imaginary axis and $h = r$ denotes that this domain lies to the right of the imaginary axis. Further, suppose that $N_{1jh}^{(s)}$ and $N_{2jh}^{(s)}$, $s = 2, 3$, are defined by (4.5), where $z_L$ is the curve bounding the $j$-th domain with the magnetization antiparallel to the imaginary axis. The coefficient $s = 2$ resp. $s = 3$ corresponds to the situation, where the domain in question lies above resp. below the real axis.

If a point $z'$ tends to $z^*$ in the $k$-th domain and the magnetization is parallel to the easy axis, according to (4.6) and (4.7), we get

$$\lim_{z' \to z^*} N_{1kr}^{(1)} = N_{1kr}^{(1)} + \frac{2\pi(2n-1)^2 b^2}{(2n-1)^2 b^2+1},$$

and

$$\lim_{z' \to z^*} N_{2kr}^{(1)} = N_{2kr}^{(1)} - \frac{2\pi}{(2n-1)^2 b^2+1}.$$

Similary, if a point $z'$ tends to $z^*$ in the neighbouring domain and the magnetization is antiparpllel to the imaginary axis, and this domain lies above the real axis, we obtain

$$\lim_{z' \to z^*} N_{1kr}^{(2)} = N_{1kr}^{(2)} + \frac{2\pi(2n-1)^2 b^2}{(2n-1)^2 b^2+1},$$

and

$$\lim_{z' - z^*} N_{2kr}^{(2)} = N_{2kr}^{(2)} - \frac{2\pi}{(2n-1)^2 b^2+1}.$$

Here we recall that, since $z^*$ belongs to $L_{1r}^{(k)}$ (Fig. 1), we have

$$x_2^* = (2n-1) b x_1^* + \tfrac{1}{2} [1-(2k-1)b] D.$$

Next utilizing the above calculation procedure we have derive formulae for the components of the demagnetizing factor at a point $z^*$ of $L_{1r}^{(k)}$. According to (4.2)–(4.7) we have

$$M_{1k}^{(j)} = c_k^{(j)}(N_{1kr}^{(j)} + R_{ik}^{(j)}), \quad i, j = 1, 2, \tag{4.8}$$

Here $j = 1$ denotes the domain with the magnetization parallel to the easy axis of magnetization and $j = 2$ — the domain with the magnetization antiparallel to the imaginary axis. Further, if $n$ is an odd number, we have

$$R_{ik}^{(1)} = \sum_{\substack{j=1(2)\\ j\neq k}}^{n} N_{ijr}^{(1)} + \sum_{j=2(2)}^{n-1} N_{ijl}^{(1)}, \quad i = 1, 2,$$

and

$$R_{ik}^{(2)} = \sum_{\substack{j=1(2)\\ j\neq k}}^{n} N_{ijr}^{(2)} + \sum_{j=2(2)}^{n-1} (N_{ijl}^{(2)} + N_{ijr}^{(3)}) + \sum_{j=1(2)}^{n-2} N_{ijl}^{(3)}, \quad i = 1, 2.$$

Similary, if $n$ is an even number, we have

$$R_{ik}^{(1)} = \sum_{\substack{j=2(2)\\ j\neq k}}^{n} N_{ijr}^{(1)} + \sum_{j=1(2)}^{n-1} N_{ijl}^{(1)}, \quad i = 1, 2,$$

and

$$R_{ik}^{(2)} = \sum_{\substack{j=2(2)\\ j\neq k}}^{n} N_{ijr}^{(2)} + \sum_{j=1(2)}^{n-1} (N_{ijl}^{(2)} + N_{ijr}^{(3)}) + \sum_{j=2(2)}^{n-2} N_{ijl}^{(3)}, \quad i = 1, 2.$$

In turn, since we need to evaluate $K_{\|}$ for $z^*$ of $L_{1r}^{(k)}$ which, in fact, is a domain wall, we calculate in analogy to [3] the mean demagnetizing factor given by the formula

$$\tilde{M}_{ik}^{(j)} = \lim_{z'\to z^*} \left\{ \left[ \frac{M_{1k}^{(j)}(z^*)}{c_k^{(j)}(z^*)} + \frac{M_{1k}^{(j)}(z')}{c_k^{(j)}(z')} \right] \Big/ \left[ \frac{1}{c_k^{(j)}(z^*)} + \frac{1}{c_k^{(j)}(z')} \right] \right\}.$$

Here:

(*i*) $i = 1, 2$.

(*ii*) We suppose that $z'$ tends to $z^*$ within the domain bounded by the curve

$$L_{4r}^{(k)} - L_{3r}^{(k)} + L_{0r}^{(k)} + L_{2r}^{(k)} - L_{1r}^{(k)} - L_{0r}^{(k-1)}, \quad \text{for } j = 1$$

and

$$L_{1r}^{(k)} - L_{12}^{(n+k-1)} - L_{2r}^{(k-1)} \quad \text{for } j = 2.$$

The contribution of the remaining domains to $K_{\parallel}$ is very small ([3], pp. 23). Putting $[M_1, M_2] = [\tilde{M}_{1k}^{(2)}, \tilde{M}_{2k}^{(1)}]$ we further rearange equation (4.1) and finally arrive at the relation

$$K_{\parallel} \approx \frac{g\mu_B^2}{v_0}\left[\frac{N_{1kr}^{(2)}+R_{1k}^{(2)}+\dfrac{\pi(2n-1)^2b^2}{(2n-1)^2b^2+1}}{N_{1kr}^{(2)}+N_{2kr}^{(2)}+R_{1k}^{(2)}+R_{2k}^{(2)}+\dfrac{\pi(2n-1)^2b^2-\pi}{(2n-1)^2b^2+1}}\right.$$

$$\left.-\frac{N_{2kr}^{(1)}+R_{2k}^{(1)}-\dfrac{\pi}{(2n-1)^2b^2+1}}{N_{1kr}^{(1)}+N_{2kr}^{(1)}+R_{1k}^{(1)}+R_{2k}^{(1)}+\dfrac{\pi(2n-1)^2b^2-\pi}{(2n-1)^2b^2+1}}\right]. \tag{4.9}$$

## 5. *Influence of the nature of domain walls on the change of sign of the uniaxial anisotropy constant*

As a particular case we consider equation (4.9) with $n = 1$, first given in [3]:

$$K_{\parallel} \approx \frac{g\mu_B^2}{v_0}\left[\frac{A_2b-B_2b^2}{B_2-C_2+2A_2b+(B_2+C_2)b^2}-\frac{B_1+A_1b}{B_1-C_1+2A_2b-(B_1+C_1)b^2}\right], \tag{5.1}$$

where

$$A_1 = I(D-2x_1^*, D-2x_2^*)+I(D-2x_1^*, D+2x_2^*)$$
$$-I(-2x_1^*, D-bD-2x_2^*)-I(-2x_1^*, D-bD+2x_2^*),$$
$$A_2 = I(D-2x_1^*, D-2x_2^*)+I(D+2x_1^*, D-2x_2^*)$$
$$-I(-2x_1^*, D-bD-2x_2^*)-I(2x_1^*, D-bD-2x_2^*),$$
$$B_1 = \pi+J\left(\frac{D+2x_2^*-b(D-2x_1^*)}{(1+b^2)^{1/2}}, \frac{D-2x_1^*+b(D+2x_2^*)}{(1+b^2)^{1/2}}\right)$$
$$-J\left(\frac{D-bD+2x_2^*+2bx_1^*}{(1+b^2)^{1/2}}, \frac{2x_1^*+b(D-bD+2x_2^*)}{(1+b^2)^{1/2}}\right),$$
$$B_2 = -\pi+J\left(\frac{D-2x_2^*-b(D+2x_1^*)}{(1+b^2)^{1/2}}, \frac{D+2x_1^*+b(D-2x_2^*)}{(1+b^2)^{1/2}}\right)$$
$$-J\left(\frac{D-bD-2x_2^*-2bx_1^*}{(1+b^2)^{1/2}}, \frac{2x_1^*+b(D-bD-2x_2^*)}{(1+b^2)^{1/2}}\right),$$
$$C_1 = J(D-2x_1^*, D-2x_2^*)+J(D-2x_1^*, D+2x_2^*)$$
$$-J(-2x_1^*, D-bD-2x_2^*)-J(-2x_1^*, D-bD+2x_2^*),$$
$$C_2 = J(D-2x_2^*, D-2x_1^*)+J(D-2x_2^*, D+2x_1^*),$$

and

$$x_2^* = bx_1^* + \tfrac{1}{2}(1-b)D.$$

The present author has evaluated $K_{\|}$ for $0 < b < 1$, $x_1^* = 0.0045$ cm, $D = 1.5 \cdot 10^{-2}$ cm, $ma \approx 10^{-5}$ cm. Since $g\mu_B^2/v_0 \approx 10^3$ erg/cm$^3$, we are led to the results given in Fig. 2.

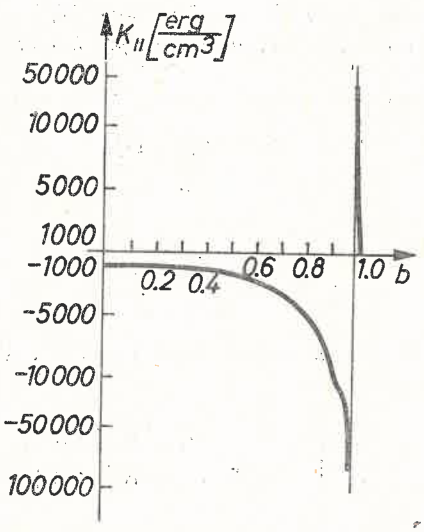


Fig. 2. Dependence of the anisotropy constant $K_{\|}$ on the dimension of domains in the case where the vector of magnetization is parallel to the real axis at the point $x_1^*$ — 0.0045 cm

The calculations have been performed on a computer ODRA 1304. For other points $(x_1^*, x_2^*)$ the dependence of $K_{\|}$ on $b$ is analogous to that shown in Fig. 2. The cases where $x_1^*$ is either close to 0 or to $\frac{1}{2}D$ are exceptions. For these points, i. e. if $x_1^* \to 0^+$ or $\frac{1}{2}D^-$,

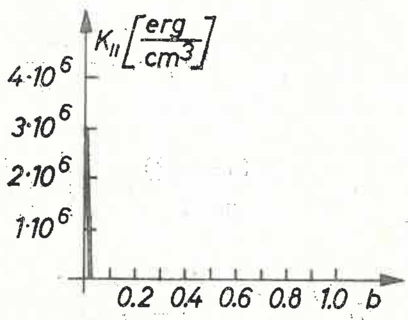


Fig. 3. Dependence of the anisotropy constant $K_{\|}$ on the dimension of domains in the case where the vector of magnetization is parallel to the real axis at the points $x_1^*$, close to 0 cm $\frac{1}{2}$ $D$ — 0.0075 cm

by (5.1), we obtain $A_1, A_2 \to -\infty$ for $0 < b < 1$. Hence $K_{\|} \to 0^-$. On the other hand, it is known [3] that $K_{\|} \to 0$ for $b \to 1^-$, and $K_{\|} \to 3.51 \cdot 10^6$ erg/cm$^3$ for $b \to 0^+$. These results are visualized in Fig. 3.

In this situation domains with the magnetization parallel and antiparallel to the real axis vanish and we have only the 180° domain walls. The results obtained show that for 90° domain walls ($0 < b < 1$ and $x_1 \neq 0, \frac{1}{2}D$) the sign changes into 1. This is consistent with at may be expected in view of experiments described in [5, 6].

### *6. Dependence of the uniaxial anisotropy constant on the number of domains for stripe domain structure*

Using the results of Section 4 we consider now a stripe domain structure with $b \to 0^+$,

$$x_1^* \to \frac{D(k-1)}{2n-1}$$

or

$$x_1^* \to \frac{D(2k-1)}{2(2n-1)}.$$

Now, for example, we take $k = n$, $n$ is an odd number and $x_1^* \to \frac{1}{2} D$. In this case (4.9) takes the form

$$K_{\parallel} \approx \frac{g\mu_B^2}{v_0}\left[-\frac{B_{1r}^{(n)}-R_{2n}^{(1)}+\pi}{B_{1r}^{(n)}-C_{1r}^{(n)}-R_{1n}^{(1)}-R_{2n}^{(1)}+\pi}\right],$$

where

$$B_{1r}^{(n)} = J\left(2D, \frac{D}{2n-1}\right), \quad C_{1r}^{(n)} = \pi+J\left(\frac{D}{2n-1}, 2D\right),$$

$$R_{1n}^{(1)} = \sum_{j=1(2)}^{n-2}\left[J\left(\frac{2D(n-j-\frac{1}{2})}{2n-1}, 2D\right)-J\left(\frac{2D(n-j+\frac{1}{2})}{2n-1}, 2D\right)\right]$$

$$+\sum_{j=2(2)}^{n-1}\left[J\left(\frac{2D(n+j-\frac{1}{2})}{2n-1}, 2D\right)-J\left(\frac{2D(n+j-1\frac{1}{2})}{2n-1}, 2D\right)\right],$$

$$R_{2n}^{(1)} = \sum_{j=1(2)}^{n-2}\left[J\left(2D, \frac{2D(n-j+\frac{1}{2})}{2n-1}\right)-J\left(2D, \frac{2D(n-j-\frac{1}{2})}{2n-1}\right)\right]$$

$$-\sum_{j=2(2)}^{n-1}\left[J\left(2D, \frac{2D(n+j-\frac{1}{2})}{2n-1}\right)-J\left(2D, \frac{2D(n+j-1\frac{1}{2})}{2n-1}\right)\right],$$

and $J(u, v)$ is defined in (4.7).

It can easily be checked that the order of $K_{\parallel}$ is $10^6$ erg/cm$^3$ and that $K_{\parallel}$ decreases with an increase in the number of domains, which is consistent with the physical interpretation of $K_{\parallel}$.

The last result can easily be obtained when noticing that $R_{2n}^{(1)} \ll \pi$ and $B_{1r}^{(n)} \ll \pi$. Then we have

$$K_{\|} \approx \frac{g\mu_B\pi}{v_0} \Big/ \left\{ -f(n, n+\tfrac{1}{2}) + \sum_{j=1(2)}^{n-2} [f(n, -j-\tfrac{1}{2}) - f(n, j+\tfrac{1}{2}) - f(n, j-\tfrac{1}{2}) + f(n, -j+\tfrac{1}{2})] \right\},$$

where

$$f(n, \alpha) = J\left(2D, \frac{2D(n+\alpha)}{2n-1}\right) - J\left(\frac{2D(n+\alpha)}{2n-1}, 2D\right) \approx \frac{ma}{2D} \frac{(n+\alpha)^2-(2n-1)^2}{[(n+\alpha)^2+(2n-1)^2]^{1/2}}.$$

The function $f$ with values

$$-f(n, -n+\tfrac{1}{2}) \approx \frac{ma}{D} \frac{4(2n-1)^2-1}{4(2n-1)^2+1}$$

increases as $n$ grows, All values

$$[f(n, -\alpha) - f(n, \alpha)] - [f(n, -\alpha+1) - f(n, \alpha+1)],$$

$\alpha > 0$, are much smaller than $-f(n, -n+\frac{1}{2})$ and they grow, when $n$ grows, since

$$\frac{d}{dn}(f(n, -\alpha) - f(n, \alpha)) > 0$$

and

$$\frac{d}{d\alpha}\left(\frac{d}{dn}(f(n, -\alpha) - f(n, \alpha))\right) > 0 \quad \text{for } \alpha > 0.$$

## 7. *Conclusions*

The considerations and calculations of the above lead to the following conclusions.

The sign of the uniaxial anisotropy constant changes from $-1$ into 1 when the four-domain structure of the Landau-Lifschitz type changes into the stripe structure.

For the stripe structure the order of the uniaxial anisotropy constant is $10^6$ erg/cm$^3$ and it decreases with an increase in the number of domains in the sample.

## REFERENCES

[1] W. E. Brown, *Micromagnetics*, New York 1963.
[2] L. Landau, E. Lifshitz, *Phys. Z. Sowjetunion* **8**, 153 (1935).
[3] J. Ławrynowicz, L. Wojtczak, *Acta Phys. Pol.* **A41**, 11 (1972).
[4] J. Ławrynowicz, L. Wojtczak, *Acta Phys. Pol.* **A46**, 19 (1974).
[5] I. B. Puchalska, A. Sukiennicki, *Phys. Status Solidi* **17**, K1 (1966).
[6] I. B. Puchalska, A. Sukiennicki, *Phys. Status Solidi* **21**, K57 (1967).
[7] E. Schlömann, *J. Appl. Phys.* **33**, 9 (1962).
[8] J. Silcox. *Phil. Mag.* **8**, 7, 1395 (1963).
[9] R. Soohoo, *Magnetic Thin Films*, Harper and Row Publishers, New York 1965.
[10] W. Ziętek, *Acta Phys. Pol.* **21**, 175, (1962).
[11] W. Ziętek, *Acta Phys. Pol.* **25**, 117 (1964).