ON THE g-FACTOR IN THE MULTICOMPONENT FERROMAGNETIC FERMI LIQUID

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The equations for the g-factor for the two-component ferromagnetic Fermi liquid are discussed and solved.

1. *Introduction*

One of the most important parameters in the theory of the interaction of an electron with magnetic field is the g-factor. It is connected with the Pauli term and has the form

$$\frac{1}{2}\mu_B g_{ab}(p) H_a \sigma_b, \tag{1.1}$$

where σ_a is the Pauli spin operator and $\mu_B = e\hbar/2mc$ is the Bohr magneton. In formula (1,1) the g-factor has the tensoral form of the p-dependence.

In the free electron theory the g-factor of a free electron is the ratio of the spin magnetic moment in Bohr units to the spin angular momentum in units of $\hbar$ and it has the value $g_0 = 2$.

There are two important corrections to the magnetic moment. One has relativistic origin and is of the order $1/c^2$, the other arises from the interaction with the zero-point vibrations of the radiation field. For a review of the g-factor (theoretical and experimental results) in a model using a free electron theory see Yafet [1].

However, since the interaction between electrons is not weak, the free electron picture is not in full adequate and must be modified. The Fermi liquid theory first proposed by Landau [2] and Silin [3] describes a system of interacting electrons even the Coulomb force between electrons.

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The purpose of this paper is to discuss the g-factor in the two-component ferromagnetic Fermi liquid as defined in [4]. We restrict ourselves to the isotropic case and spherical Fermi surface and we shall discuss the scalar g-factor only.

The equation for the electron g-factor in the spirit of the Landau-Silin theory could be derived very simply if one recognizes that the change in the electron energy in a magnetic field has two components. The first arises from the fact that the magnetic field interacts with the magnetic moment of the electron, the second is due to the fact that the change in distribution function changes the energy of the quasiparticles.

Using the idea mentioned, one can find the integral equation for the electron g-factor [5]

$$g(\mathbf{p}) = 2 - \mathrm{Tr}\,\sigma\, \mathrm{Tr}\,\sigma'(\boldsymbol{\sigma\sigma}') \int d\Omega' F(\mathbf{p}, \sigma, \mathbf{p}', \sigma') g(\mathbf{p}'), \tag{1.2}$$

where $F(\mathbf{p}, \sigma, \mathbf{p}', \sigma')$ is the effective interaction of the quasiparticles.

Equation (1,1) forms the basis for the discussion which we shall give in the next sections.

2. One-component liquid with spin-orbit coupling

If we restrict ourselves to the isotropic case the effective interaction needs [6, 7]

$$F(\mathbf{p}, \sigma, \mathbf{p}', \sigma') = F^S(\mathbf{p}, \mathbf{p}') + F^A(\mathbf{p}, \mathbf{p}')(\boldsymbol{\sigma\sigma}') + A(\mathbf{p}, \mathbf{p}')(\boldsymbol{\sigma p})(\boldsymbol{\sigma}'\,\mathbf{p}')$$

$$+ B(\mathbf{p}, \mathbf{p}')(\boldsymbol{\sigma}\mathbf{p}')(\boldsymbol{\sigma}'\mathbf{p}) + C(\mathbf{p}, \mathbf{p}')(\boldsymbol{\sigma} - \boldsymbol{\sigma}')(\mathbf{p} \times \mathbf{p}'), \tag{2.1}$$

where $F^S(\mathbf{p}, \mathbf{p}')$ and $F^A(\mathbf{p}, \mathbf{p}')$ are the usual direct and exchange parts of the effective interaction, and the new quantities A, B and C describe the spin-orbit coupling. In our isotropic case those five functions depend on the $(\mathbf{p} \cdot \mathbf{p}')$ only and can be expanded into a series of the Legendre polynomials with the expansion coefficients f_l^S, f_l^A, a_l, b_l and c_l, respectively.

Substitution of (2.1) into (1.2) gives

$$g(\mathbf{p}) = 2 - \int d\Omega' W(\mathbf{p} \cdot \mathbf{p}') g(\mathbf{p}'), \tag{2.2}$$

where

$$W(\mathbf{p}\mathbf{p}') = [A(\mathbf{p}\mathbf{p}') + B(\mathbf{p}\mathbf{p}')]\,\mathbf{p} \cdot \mathbf{p}' + F^A(\mathbf{p} \cdot \mathbf{p}'). \tag{2.3}$$

The solution of (1.2) can be easily found in terms of the inverse operators as

$$g(\mathbf{p}) = 2[1 - \int d\Omega' \tilde{W}(\mathbf{p}\mathbf{p}')], \tag{2.4}$$

while the quantity $W(\mathbf{p}\mathbf{p}')$ must fulfil the integral equation

$$W(\mathbf{p}\mathbf{p}') = \tilde{W}(\mathbf{p}\mathbf{p}') + \int d\Omega'' W(\mathbf{p}\mathbf{p}'')\, \tilde{W}(\mathbf{p}'\mathbf{p}''). \tag{2.5}$$

With rotation symmetry, the interaction function at the Fermi surface depends only on the angle between $\mathbf{p}$ and $\mathbf{p}'$. Hence, it can be expanded in a series of Legendre polynomials

$$W(\mathbf{pp}') = \sum_l (2l+1) W_l P_l(\hat{\mathbf{p}}\hat{\mathbf{p}}'), \quad \tilde{W}(\mathbf{pp}') = \sum_l (2l+1)\, \tilde{W}_l P_l(\hat{\mathbf{p}}\hat{\mathbf{p}}'), \tag{2.6}$$

where $\hat{p}$ is a unit vector, $\hat{p} = p/|p|$. We obtain the formula

$$P_l(\hat{p}\hat{p}') = \sum_{m=-l}^{m=l} \frac{4\pi}{2l+1} Y_{lm}^*(\hat{p}) Y_{lm}(\hat{p}').$$

Now, we can get the relation between W_l and $\tilde{W}_l$

$$W_l = \frac{\tilde{W}_l}{1-W_l}. \tag{2.7}$$

From the first formula in (2.6) we have

$$W_l = \int_{-1}^{1} W(\cos \vartheta) P_l(\cos \vartheta) d(\cos \vartheta), \tag{2.8}$$

where $\cos \vartheta = \hat{p} \cdot \hat{p}'$. Using (2, 3) we can write

$$W_l = \int_{-1}^{1} F^A(\cos \vartheta) P_l(\cos \vartheta) d(\cos \vartheta) + \int_{-1}^{1} [A(\cos \vartheta) + B(\cos \vartheta)] P_l(\cos \vartheta) \cos \vartheta d(\cos \vartheta).$$

If we expand the functions $F^A(\cos \vartheta)$, $A(\cos \vartheta)$, $B(\cos \vartheta)$ in a series of Legendre polynomials we obtain

$$W_l = F_l^A + \sum_n (A_n + B_n) \int_{-1}^{1} P_n(\cos \vartheta) P_l(\cos \vartheta) \cos \vartheta d(\cos \vartheta).$$

We use the formula

$$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x).$$

After simple calculations we obtain

$$W_l = F_l^A + (A_{l-1} + B_{l-1}) \frac{l}{2l-1} + (A_{l+1} + B_{l+1}) \frac{l}{2l+1} \quad \text{for } l \neq 0$$

$$W_0 = F_0^A. \tag{2.9}$$

3. *Two-component ferromagnetic Fermi liquid*

In this section we intend to discuss the influence of the Fermi liquid effects on the g-factor in the isotropic model. Our basic assumptions follow from the proceeding work [4]. Here we give another discussion of the g-factor than that of Section 2. The equations for the g-factor of the s and d electrons are [4]

$$\mu_s^\alpha(p) = \alpha - \frac{1}{(2\pi)^3} \sum_\beta \int d^3p' G_s^{\alpha\beta}(p, p') \mu_s^\beta(p') - \sum_{\beta r'} G_{sd}^{\alpha\beta}(p, r') \mu_d^\beta(r'), \tag{3.1}$$

$$\mu_d^\alpha(r) = \alpha - \frac{1}{(2\pi)^3} \sum_\beta \int d^3p' G_{ds}^{\alpha\beta}(r, p') \mu_s^\beta(p') - \sum_{\beta r'} G_d^{\alpha\beta}(r, r') \mu_d^\beta(r'), \tag{3.2}$$

where $G_{sd}^{\alpha\beta}(\boldsymbol{p}, \boldsymbol{r})$ is an interaction function, which describes the interaction between the s electron with momentum $\boldsymbol{p}$ and spin α and the d electron with spin β at the point $\boldsymbol{r}$; $G_{ds}^{\alpha\beta}(\boldsymbol{r}, \boldsymbol{p})$, respectively, an interaction function between the d electron with spin α at the point $\boldsymbol{r}$ and the s electron with momentum $\boldsymbol{p}$ and spin β; $G_s^{\alpha\beta} = G_{ss}^{\alpha\beta}$:, $G_d^{\alpha\beta} = G_{dd}^{\alpha\beta}$. There is, of course, $G_{sd}^{\alpha\beta} = G_{ds}^{\beta\alpha}$.

Now using the rotation symmetry of interaction functions, we expand in a series of the Legendre polynomials

$$\mu_s^{\alpha}(\boldsymbol{p}) = \sum_{lm} \mu_{slm}^{\alpha} Y_{lm}(\hat{\boldsymbol{p}}),$$

$$G_s^{\alpha\beta}(\boldsymbol{p}, \boldsymbol{p}') = C \sum_{l} G_{sl}^{\alpha\beta}(2l+1) P_l(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}'),$$

$$G_{sd}^{\alpha\beta}(\boldsymbol{p}, \boldsymbol{r}) = C \sum_{lm} G_{sdlm}^{\alpha\beta} Y_{lm}(\hat{\boldsymbol{p}}), \tag{3.3}$$

where

$$C = (2\pi)^3 \frac{v}{p_0^2}.$$

We can assume that the interaction function $G_d^{\alpha\beta}\boldsymbol{r}, \boldsymbol{r}')$ depends only on the difference $\boldsymbol{r}-\boldsymbol{r}'$. Hence, we have

$$G_d^{\alpha\beta}(\boldsymbol{r}-\boldsymbol{r}') = \int d^3k G_d^{\alpha\beta}(\boldsymbol{k}) e^{i\boldsymbol{k}(\boldsymbol{r}-\boldsymbol{r}')}. \tag{3.4}$$

We now assume that μ_d^{α} has a space dependence of the form $\exp(i\boldsymbol{q}\boldsymbol{r})$. It is possible when the surface effects are not considered. Thus, we have

$$\sum_{\beta \boldsymbol{r}'} G_d^{\alpha\beta}(\boldsymbol{r}-\boldsymbol{r}')\mu_d^{\beta}(\boldsymbol{r}') = \sum_{\beta} G_d^{\alpha\beta}(\boldsymbol{q}) e^{i\boldsymbol{q}\boldsymbol{r}}. \tag{3.5}$$

Using (3.3) and (3.5) we can rewrite (3.1) and (3.2) in the following form

$$\sum_{\beta} (\delta\alpha\beta + G_{sl}^{\alpha\beta})\mu_{slm}^{\beta} + \sum_{\beta} G_{sdlm}^{\alpha\beta}\mu_d^{\beta} = \alpha\delta_{l0}\delta_{m0},$$

$$\sum_{\beta lm} G_{dslm}^{\alpha\beta}\mu_{slm}^{\beta} + \sum_{\beta} (\delta\alpha\beta + G_d^{\alpha\beta})\mu_d^{\beta} = \alpha. \tag{3.6}$$

As we see equations (3), (6) can be solved.

In Sect. 2 we have discussed the problem of the g-factor for the one-component liquid only. Here we have shown the relationship between the suitable g-factors for s and d electrons.

Afterwards, we shall need solutions of equations (3), (6) for further discussion of the problems of the two-component ferromagnetic Fermi liquid.

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