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QUANTUM STATISTICAL PROPERTIES OF DEGENERATE PARAMETRIC AMPLIFICATION PROCESS

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Using the Heisenberg and Schrödinger pictures, the statistical properties of the degenerate parametric amplification process are investigated and the equivalence of both the pictures is shown. The statistics is described by the model of superposition of coherent and chaotic fields with correlated components in analogy to the statistics of radiation propagating through random media. The lossy mechanism is taken into account and the influence of damping and the contribution of chaotic energy from the reservoir is discussed. Particularly, the anticorrelation regime is studied on the basis of the time evolution of the photocounting distribution and its factorial moments up to the fifth order. Without the reservoir it is demonstrated that the decrease of uncertainty in the statistics is connected with the attenuation and vice versa. A field coherent at the beginning cannot be fully coherent again for any time, it is coherent only in the fourth order in a certain time, which is a typical property of fields having no classical analogs.

1. Introduction

Recently, considerable attention has been paid to the statistical properties of non-linear optical processes such as parametric amplification with classical pumping (e. g. [1, 2]) as well as with quantum pumping (e. g. [3–5] and references therein), higher harmonics and subharmonics generation (e. g. [5–7]), multiphoton absorption [8–13] and emission [9, 14–16] and Raman scattering [9, 17, 18]. Particularly, it has been shown that in some cases of the multiphoton absorption and stimulated emission, the photocounting statistics has less uncertainty than for one photon processes [12–14] and that in the multiphoton

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absorption processes and the degenerate parametric amplification process [19–21] regimes with antibunching (anticorrelation) effect occur [8, 10–13, 20, 21].

The purpose of this paper is to investigate systematically the statistical properties of the degenerate parametric amplification process with classical pumping adopting both the Heisenberg–Langevin approach (antinormal quantum characteristic function and related quasi-distribution) and Schrödinger approach (generalized Fokker-Planck equation). We also show the equivalence of these approaches. Particular attention is paid to the measurable photocounting statistics which is systematically studied with the help of the model of the superposition of coherent and chaotic fields which has been developed in studies of radiation propagating through random media [22–24]. We investigate the antibunching regime of this process following the time development of the photocounting distribution and its factorial moments up to the fifth order. All is done taking into account the lossy mechanism, i. e. the effect of damping and contribution of noise from the reservoir to the radiation field. It is directly demonstrated without the reservoir that the decrease of uncertainty in the statistics (narrowing of the photocounting distribution higher than for the corresponding coherent state) is connected with the attenuation of radiation and vice versa the amplification of the radiation is followed by an increase in chaotic noise.

In Section 2 the Heisenberg-Langevin and generalized Fokker-Planck equations are derived and solved and expressions for the photocounting distribution and its factorial moments are given. In Section 3 the numerical results are discussed.

2. *Theoretical description*

The optical process under consideration is described by the Hamiltonian

$$H_{\mathrm{rad}} = \hbar\omega a^{+}a-\tfrac{1}{2}\hbar g(a^{2}\exp(2i\omega t-i\varphi)+a^{+2}\exp(-2i\omega t+i\varphi)), \tag{2.1}$$

where 2ω is the pumping frequency, φ is the phase of pumping, a and a^{+} are annihilation and creation operators of the radiation mode with the frequency ω and g is a real coupling constant (phase matching is assumed).

In order to describe the lossy mechanism we introduce the usual additional Hamiltonian

$$H_{\mathrm{ad}} = \hbar\sum_{l}(\psi_{l}b_{l}^{+}b_{l}+\kappa_{l}a^{+}b_{l}+\kappa_{l}^{*}ab_{l}^{+}), \tag{2.2}$$

where ψ_l is the reservoir frequency of mode l described by the boson annihilation and creation operators b_l and b_l^{+}, respectively, κ_l are the coupling constants between the Gaussian reservoir and the radiative mode. Note that the process described by these Hamiltonians can be understood as prototype of an amplification process [15, 16].

Writing the Heisenberg equations for a and b_l and eliminating the reservoir operators b_l in the usual way [25, 23], we obtain the Heisenberg-Langevin equation

$$\dot{a} = -(i\omega+\gamma/2)\,a+ig\,a^{+}\exp(-2i\omega t+i\varphi)+L, \tag{2.3}$$

where $\gamma = 2\pi|\kappa(\omega)|^2 \varrho(\omega)$ is the damping constant ($\varrho(\omega)$ being the density function for damping oscillators) and

$$L(t) = -i\sum_l \kappa_l b_l \exp(-i\psi_l t) \tag{2.3a}$$

is the Langevin force (here and in the following the b_l's are initial values of reservoir operators) fulfilling the following conditions

$$\begin{aligned} \langle L(t)\rangle &= 0, \\ \langle L^+(t)\, L(t')\rangle &= \gamma\, \langle n_d\rangle\, \delta(t-t'), \\ \langle L(t)\, L^+(t')\rangle &= \gamma(\langle n_d\rangle + 1)\, \delta(t-t'); \end{aligned} \tag{2.4}$$

here the brackets mean the average over the reservoir variables and $\langle n_d\rangle$ denotes the mean number of reservoir oscillators which is assumed to be independent of l (flat reservoir spectrum). These conditions are consequences of the Gaussian properties of $\{b_l\}$.

An equivalent Schrödinger approach can be developed on the basis of the equation of motion for the density matrix ϱ,

$$i\hbar \frac{\partial \varrho}{\partial t} = [H, \varrho], \tag{2.5}$$

eliminating the Gaussian reservoir variables in the Markoff approximation [25, 26]. Performing the corresponding ordering and substituting $a \to \alpha$, $a^+ \to \alpha^*$ (α being the eigenvalue of a in the coherent state $|\alpha\rangle$) in the resulting equation, we obtain the corresponding generalized Fokker-Planck equation for the quasi-distribution. For instance, performing the antinormal ordering, we have the generalized Fokker-Planck equation for the Glauber-Sudarshan quasi-distribution $\phi_{\mathscr{N}}(\alpha) = \varrho^{(\mathscr{A})}(a \to \alpha, a^+ \to \alpha^*)/\pi$ [27].

However, this quasi-distribution does not exist as an ordinary square integrable function (in general it is an ultradistribution) in most processes involving an interaction, at least for some time, including the process under discussion. Therefore for our analysis we adopt the quasi-distribution $\phi_{\mathscr{A}}(\alpha)$ $(= \varrho^{(\mathscr{N})}(a \to \alpha,\ a^+ \to \alpha^*)/\pi)$ related to the antinormal ordering which is bounded and non-negative [27]. In order to obtain the photocounting statistics we apply a special procedure [22, 27]. Thus the generalized Fokker-Planck equation for $\phi_{\mathscr{A}}(\alpha, t)$ is

$$\begin{aligned} \frac{\partial \phi_{\mathscr{A}}}{\partial t} = &\left\{\left(\frac{\gamma}{2} + i\omega\right)\frac{\partial \alpha}{\partial \alpha} + \text{c.c.} + \gamma(\langle n_d\rangle + 1)\, \frac{\partial^2}{\partial\alpha\partial\alpha^*}\right. \\ &\left. + \left[ig \exp(2i\omega t - i\varphi)\left(\alpha \frac{\partial}{\partial \alpha^*} + \tfrac{1}{2}\frac{\partial^2}{\partial \alpha^{*2}}\right)\right] + \text{c.c.}\right\} \phi_{\mathscr{A}}. \end{aligned} \tag{2.6}$$

The first and the second terms on the right-hand side correspond to the equation of motion for a damped harmonic oscillator, the first term together with the first conjugated one in the angular brackets correspond to the equation of motion (2.3). The third term represents the reservoir contribution (corresponding to the Langevin force L in (2.3a) with properties (2.4)); all terms containing g are the interaction ones. They arise from $[H', \varrho]/i\hbar$ performing the normal ordering and substituting $a \to \alpha$, $a^+ \to \alpha^*$, where H' is the interaction part of (2.1) ($a^k \varrho^{(\mathcal{N})} = \mathcal{N}\{(a+\partial/\partial a^+)^k \varrho\}$, $\varrho^{(\mathcal{N})} a^{+k} = \mathcal{N}\{(a^+ + \partial/\partial a)^k \varrho\}$, $\mathcal{N}$ being the operator of normal ordering, k is integer).

To solve equation (2.6) it is convenient to introduce the antinormal characteristic function

$$C^{(\mathscr{A})}(\beta, t) = \mathrm{Tr}\{\varrho(t) \exp(-\beta^* a) \exp(\beta a^+)\}$$

$$= \mathrm{Tr}\{\varrho(0) \exp(-\beta^* a(t)) \exp(\beta a^+(t))\}$$

$$= \int \phi_{\mathscr{A}}(\alpha, t) \exp(\alpha^* \beta - \alpha \beta^*) \, d^2\alpha, \tag{2.7a}$$

$$\phi_{\mathscr{A}}(\alpha, t) = \pi^{-2} \int C^{(\mathscr{A})}(\beta, t) \exp(\alpha \beta^* - \alpha^* \beta) \, d^2\beta, \tag{2.7b}$$

a and $a(t)$ is the radiation annihilation operator in the Schrödinger and Heisenberg picture, respectively, $\varrho(0) = \varrho_{\mathrm{rad}}(0)\, \varrho_{\mathrm{reserv}}(0)$ expressing the statistical independence of the radiation and the reservoir for $t = 0$.

Equation (2.6) provides performing substitutions $\alpha' = \alpha \exp(i\omega t)$, $\beta' = \beta \exp(i\omega t)$,

$$\frac{\partial C^{(\mathscr{A})}}{\partial t} = \left\{ -\frac{\gamma}{2}\left(\beta' \frac{\partial}{\partial \beta'} + \beta'^* \frac{\partial}{\partial \beta'^*}\right) - \gamma(\langle n_d \rangle + 1)\, |\beta'|^2 \right.$$
$$+ ig\left(\exp(-i\varphi)\beta' \frac{\partial}{\partial \beta'^*} - \exp(i\varphi)\beta'^* \frac{\partial}{\partial \beta'}\right)$$
$$\left. + \frac{ig}{2}\left(\exp(-i\varphi)\beta'^2 - \exp(i\varphi)\beta'^{*2}\right) \right\} C^{(\mathscr{A})}. \tag{2.8}$$

Equation (2.3) together with the Hermitian conjugate can be solved by direct elimination of a^+ (or using the Laplace transform method) to obtain [19]

$$a(t) = u(t) a + v(t) a^+ + \sum_l w_l(t) b_l + \sum_l z_l(t) b_l^+, \tag{2.9}$$

where $a = a(0)$, $a^+ = a^+(0)$,

$$u(t) = \exp(-i\omega t - \gamma t/2) \operatorname{ch} gt,$$

$$v(t) = i \exp(-i\omega t - \gamma t/2 + i\varphi) \operatorname{sh} gt,$$

$$w_l(t) = -\frac{i\kappa_l}{2}\left[\frac{1}{D_1}\left(\exp(-i\psi_l t) - \exp(g - i\omega - \gamma/2)t\right)\right.$$
$$\left. + \frac{1}{D_2}\left(\exp(-i\psi_l t) - \exp(-g - i\omega - \gamma/2)t\right)\right], \tag{2.10}$$

$$z_l(t) = \frac{\kappa_l^* \exp(i\varphi)}{2}\left[-\frac{1}{D_1^*}(\exp i(\psi_l - 2\omega)t\right.$$

$$\left. - \exp(g - i\omega - \gamma/2)t) + \frac{1}{D_2^*}(\exp i(\psi_l - 2\omega)t - \exp(-g - i\omega - \gamma/2)t)\right],$$

$$D_1 = i\omega - i\psi_l + \gamma/2 - g,\ D_2 = i\omega - i\psi_l + \gamma/2 + g.$$

Making use of the substitution $\sum_l \to \int \ldots \varrho(\psi)d\psi$ and the residuum theorem, we can verify that $[a(t), a^+(t)] = 1$ (i. e. $|u|^2 - |v|^2 + \sum_l |w_l|^2 - \sum_l |z_l|^2 = 1$), etc. regardless of the conditions $\gamma/2 \gtreqless g$.

Substituting (2.9) into (2.7a), making use of the Baker-Hausdorff identity for obtaining the normal order in the initial operators and the Glauber-Sudarshan representation of the density matrix for radiation and reservoir and averaging over the Gaussian reservoir variables [22], we arrive at

$$C^{(\mathscr{A})}(\beta, t) = \left\langle \exp\left(-B(t)|\beta|^2 + \frac{C^*(t)}{2}\beta^2 + \frac{C(t)}{2}\beta^{*2} + \beta\alpha^*(t) - \beta^*\alpha(t)\right)\right\rangle, \quad (2.11)$$

where the brackets mean the average over the initial complex amplitude α with the probability distribution $\phi_{\mathscr{N}}(\alpha)$ and

$$B(t) = |u(t)|^2 + (1 + \langle n_d\rangle)\sum_l |w_l(t)|^2 + \langle n_d\rangle \sum_l |z_l(t)|^2,$$

$$C(t) = u(t)v(t) + (1 + 2\langle n_d\rangle)\sum_l w_l(t)z_l(t),$$

$$\alpha(t) = u(t)\alpha + v(t)\alpha^*. \quad (2.12)$$

Using (2.9) and (2.10) in (2.12) and applying the residuum theorem again (substituting the integral for the sum), we obtain for $\gamma/2 \gtrless g$

$$B(t) = c_1 - c_2\exp(-\gamma t)\operatorname{ch} 2gt - c_3\exp(-\gamma t)\operatorname{sh} 2gt,$$

$$C(t) = i\exp(-2i\omega t + i\varphi)\,[-c_2\exp(-\gamma t)\operatorname{sh} 2gt + c_3(1 - \exp(-\gamma t)\operatorname{ch} 2gt)], \quad (2.13a)$$

$$c_1 = \frac{\gamma^2(1 + \langle n_d\rangle) - 2g^2}{\gamma^2 - 4g^2}, \quad c_2 = \frac{\gamma^2\langle n_d\rangle + 2g^2}{\gamma^2 - 4g^2}, \quad c_3 = \frac{\gamma g(1 + 2\langle n_d\rangle)}{\gamma^2 - 4g^2},$$

and for $\gamma/2 = g$

$$B(t) = \frac{7 + 2\langle n_d\rangle}{8} + \frac{1 + 2\langle n_d\rangle}{4}\gamma t + \frac{1 - 2\langle n_d\rangle}{8}\exp(-2\gamma t),$$

$$C(t) = i\exp(-2i\omega t + i\varphi)\left[\frac{1 + 2\langle n_d\rangle}{4}\gamma t + \frac{1 - 2\langle n_d\rangle}{8}(1 - \exp(-2\gamma t))\right]. \quad (2.13b)$$

Substituting (2.11) with unknown $B'(t)$, $C'(t)$, $\alpha'(t)$ and β' into (2.8) and comparing coefficients at $|\beta'|^2$, β'^2, β'^{*2}, β' and β'^*, we obtain the following equations

$$\dot{B}' = -\gamma B' + ig(C'^* \exp(-i\varphi) - C' \exp(i\varphi)) + \gamma(\langle n_d\rangle + 1),$$

$$\tfrac{1}{2}\dot{C}' = -\frac{\gamma}{2} C' + ig \exp(-i\varphi)(B' - \tfrac{1}{2}),$$

$$\tfrac{1}{2}\dot{C}'^* = -\frac{\gamma}{2} C'^* - ig \exp(i\varphi)(B' - \tfrac{1}{2}),$$

$$\dot{\alpha}' = -\frac{\gamma}{2}\alpha' + ig\alpha'^* \exp(i\varphi),$$

$$\dot{\alpha}'^* = -\frac{\gamma}{2}\alpha'^* - ig\alpha' \exp(-i\varphi) \tag{2.14}$$

with the initial conditions $B'(0) = 1$, $C'(0) = 0$, $\alpha'(0) = \alpha$, i. e. $C^{(\mathscr{A})}(\beta, 0) = \exp(-|\beta|^2 + \beta\alpha^* - \beta^*\alpha)$. Successive elimination of the variables (or the use of the Laplace transform method) provides solutions $\alpha'(t) = \alpha(t)\exp(i\omega t)$, $B'(t) = B(t)$, $C'(t) = C(t)\exp(2i\omega t)$, where $\alpha(t)$ is given in (2.12) with u and v obtained in (2.10) and B and C are determined by (2.13a, b). This is an explicit demonstration of the equivalence of the Heisenberg and Schrödinger pictures.

Performing the Fourier transformation (2.7b), we obtain from (2.11) [22]

$$\phi_{\mathscr{A}}(\alpha, t) = \left\langle (\pi K^{1/2}(t))^{-1} \exp\left[-\frac{B(t)}{K(t)}|\alpha - \alpha(t)|^2 + \frac{C^*(t)(\alpha - \alpha(t))^2 + \text{c.c.}}{2K(t)}\right]\right\rangle, \tag{2.15}$$

where $K(t) = B^2(t) - |C(t)|^2$. Thus if the radiation in a coherent state is incident, then the statistics of this optical process is described by the superposition of a signal $\alpha(t)$ and noise $K(t)$ with correlated real and imaginary parts of the complex amplitude α. This correlation disappears if $C(t) = 0$ when $K(t) = B^2(t)$.

Note that upon introducing the real variables x and y ($\alpha = x + iy$), it can be shown that the existence of the above Fourier transform is ensured by the conditions $B \pm (C + C^*)/2 = \text{ch}\, gt\,(\text{ch}\, gt \pm \text{sh}\, gt \sin(2\omega t - \varphi)) > 0$, $K(t) = B^2 - |C|^2 = \text{ch}^2\, gt > 0$ (without the reservoir). Such conditions are not fulfilled for the normal characteristic function $C^{(\mathscr{N})}(\beta, t) = \text{Tr}\{\varrho(0)\exp(\beta a^+(t))\exp(-\beta^* a(t))\}$ when $B \pm (C + C^*)/2 = \text{sh}\, gt \times (\text{sh}\, gt \pm \text{ch}\, gt \sin(2\omega t - \varphi))$ and $K(t) = -\text{sh}^2\, gt$.

The photocounting statistics are determined by the photocounting generating function [22]

$$C^{(W)}_{\mathscr{A}}(is, t) = \int \phi_{\mathscr{A}}(\alpha, t)\exp(isW)d^2\alpha = \langle \exp(isW)\rangle_{\mathscr{A}}$$

$$= \left\langle (1 - is/F)^{-1/2}(1 - is/E)^{-1/2} \exp\left[\frac{isA_1}{1 - is/F} + \frac{isA_2}{1 - is/E}\right]\right\rangle, \tag{2.16}$$

where $W = |\alpha|^2$ and

$$E = (B + |C|)^{-1}, \quad F = (B - |C|)^{-1},$$

$$A_{1,2} = \tfrac{1}{2}\left[|\alpha(t)|^2 \mp \frac{1}{|C|}(\alpha^2(t)C^* + \text{c.c.})\right]. \tag{2.17}$$

The photocounting distribution and its factorial moments are obtained by differentiation from the normally ordered photocounting generating function [27]

$$C_{\mathcal{N}}^{(W)}(is, t) = (1+is)^{-1}\left\langle \exp\frac{isW}{1+is}\right\rangle_{\mathcal{A}}$$

$$= \left\langle\left[1-is\left(\frac{1}{F}-1\right)\right]^{-1/2}\left[1-is\left(\frac{1}{E}-1\right)\right]^{-1/2}\right.$$

$$\left.\times\exp\left[\frac{isA_1}{1-is\left(\frac{1}{F}-1\right)}+\frac{isA_2}{1-is\left(\frac{1}{E}-1\right)}\right]\right\rangle. \quad (2.18)$$

The term -1 represents the subtraction of the physical vacuum noise.

Consequently the photocounting distribution and its factorial moments are expressed in terms of the Laguerre polynomials [22][1]

$$p(n, t) = \left\langle (EF)^{1/2}(1-E)^n \exp(-FA_1-EA_2)\sum_{k=0}^{n}\frac{1}{\Gamma(k+1/2)\Gamma(n-k+1/2)}\right.$$

$$\left.\left(\frac{1-F}{1-E}\right)^k L_k^{-1/2}\left(-\frac{A_1F^2}{1-F}\right)L_{n-k}^{-1/2}\left(-\frac{A_2E^2}{1-E}\right)\right\rangle, \quad (2.19a)$$

$$\langle W^k\rangle_{\mathcal{N}} = k!\left\langle\left(\frac{1}{E}-1\right)^k\sum_{l=0}^{k}\frac{1}{\Gamma(l+1/2)\Gamma(k-l+1/2)}\right.$$

$$\left.\times\left[\frac{E(1-F)}{F(1-E)}\right]^l L_l^{-1/2}\left(-\frac{A_1F}{1-F}\right)L_{k-l}^{-1/2}\left(-\frac{A_2E}{1-E}\right)\right\rangle. \quad (2.19b)$$

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[1] In the case without lossy mechanism this photocounting distribution can be rewritten into the form [15] ($u = \mu^*, v = -\nu$)

$$p(n, t) = \frac{1}{2^n n!}\frac{1}{|u|}\left(\frac{|v|}{|u|}\right)^n\left\langle \exp\left(-|\alpha|^2-\frac{v^*}{2u^*}\alpha^2-\frac{v}{2u}\alpha^{*2}\right)H_n\left(\frac{\alpha}{i\sqrt{2u^*v}}\right)H_n\left(\frac{\alpha^*}{i\sqrt{2uv^*}}\right)\right\rangle,$$

where the identity

$$\sum_{k=0}^{n}\frac{(-1)^k}{\Gamma(k+1/2)\,\Gamma(n-k+1/2)}L_k^{-1/2}(x^2)L_{n-k}^{-1/2}(y^2) = \frac{1}{2^n n!}H_n\left(\frac{x+y}{\sqrt{2}}\right)H_n\left(\frac{x-y}{\sqrt{2}}\right)$$

has been used (it can be proved making use of $H_{2n}(x) = (-1)^n 2^{2n} n!\, L_n^{-1/2}(x^2)/\Gamma(n+1/2)$, definition of the Hermite polynomials $H_n(x) = (-1)^n \exp(x^2)\, d^n\exp(-x^2)/dx^n$ and simple operator algebra). Here $\alpha(t)$ has been expressed in the form given in (2.12) ($B = |u|^2$, $C = uv$). Similarly, the generating function (2.18) can be written in the simplified form [15]

$$C_{\mathcal{N}}^{(W)}(is, t) = \tau^{-1/2}\left\langle \exp\left\{\left(\frac{1+is}{\tau}-1\right)|\alpha|^2-\left[1-\frac{(1+is)^2}{\tau}\right]\left(\frac{v^*}{2u^*}\alpha^2+\frac{v}{2u}\alpha^{*2}\right)\right\}\right\rangle,$$

where $\tau = |u|^2-(1+is)^2|v|^2$.

For $k = 1$

$$\langle W \rangle_{\mathcal{N}} = \langle a^{+}(t)\, a(t) \rangle = |\alpha(t)|^2 + B(t) - 1 \tag{2.20}$$
$$= (\operatorname{ch} 2gt + \operatorname{sh} 2gt \sin(2\vartheta - \varphi))|\alpha|^2 + B(t) - 1,$$

assuming here and in the following that the incident radiation is in the coherent state $|\alpha\rangle$ (the brackets in (2.19a, b) are to be omitted in this case); if $\gamma = \langle n_d \rangle = 0$, then $B(t) - 1 = |v(t)|^2 = \operatorname{sh}^2 gt$, $\alpha = |\alpha| \exp i\vartheta$.

It should be noted that in (2.18) and (2.19a, b) the quantities E, F (B, C) are related to the chaotic (noise) component while the quantities $A_{1,2}$ (determined by the complex amplitude $\alpha(t)$) represent the coherent (signal) component of the superposition.

3. *Discussion of results*

We have investigated numerically, on the basis of the above expressions, the time developing photocounting distribution and its factorial moments including the mean intensity $\langle W \rangle_{\mathcal{N}}$ and the chaotic component $K(t)$ of the superposition in the antibunching

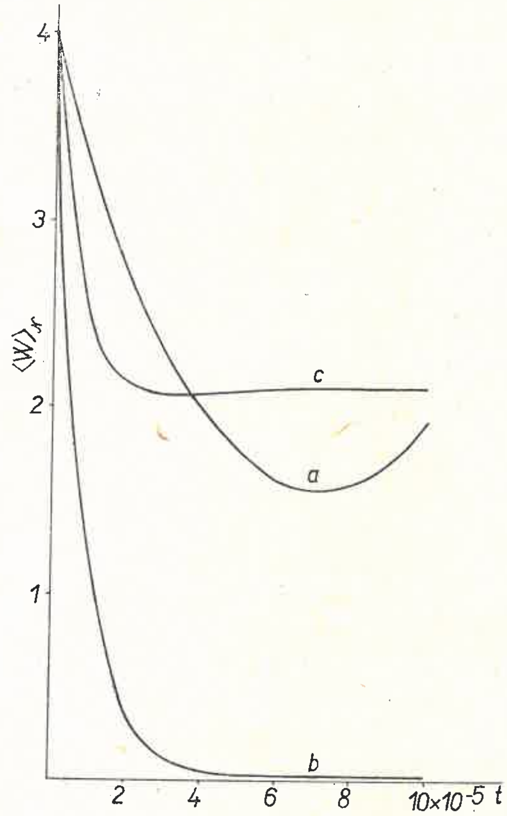


Fig. 1. The mean intensity $\langle W \rangle_{\mathcal{N}}$ for $|\alpha(0)| = 2$, $2\vartheta - \varphi = -\pi/2$, $g = 10^4\ \mathrm{s}^{-1}$ and $a - \gamma = \langle n_d \rangle = 0$, $b - \gamma = 10^5\ \mathrm{s}^{-1}$, $\langle n_d \rangle = 0$, $c - \gamma = 10^5\ \mathrm{s}^{-1}$, $\langle n_d \rangle = 2$

regime, i. e. for $2\vartheta-\varphi = -\pi/2$ (ϑ being the phase of α) [20], for the incident photon number $|\alpha|^2 = 4$, $g = 10^4\ s^{-1}$, t from 0 to 10^{-4} s and $a - \gamma = \langle n_d \rangle = 0$, $b - \gamma = 10^5\ s^{-1}$, $\langle n_d \rangle = 0$, $c - \gamma = 10^5\ s^{-1}$, $\langle n_d \rangle = 2$.

In Fig. 1 the time behaviour of the mean intensity $\langle W \rangle_{\mathcal{N}}$ is shown for the above cases, in Fig. 2 we see the reduced factorial moments $\langle W^k \rangle_{\mathcal{N}} / \langle W \rangle^k_{\mathcal{N}} - 1$ for $k = 2, 3, 4, 5$ for the above case a). Calculations are performed according to (2.19b) and (2.20). With respect

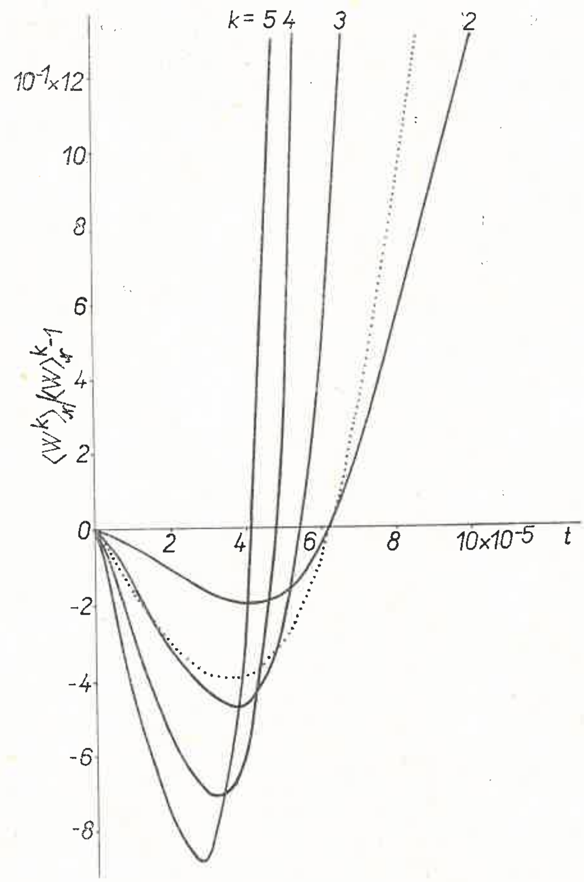


Fig. 2. The reduced factorial moments $\langle W^k \rangle_{\mathcal{N}} / \langle W \rangle^k_{\mathcal{N}} - 1$, $k = 2-5$ (full curves) for $|\alpha(0)| = 2$, $2\vartheta - \varphi = -\pi/2$, $g = 10^4\ s^{-1}$ and $\gamma = \langle n_d \rangle = 0$. The dotted curve represents the quantity $(\langle W^2 \rangle_{\mathcal{N}} - \langle W \rangle^2_{\mathcal{N}}) / \langle W \rangle_{\mathcal{N}}$

to the definition of the antibunching (anticorrelation) effect [28, 29, 8, 10–13, 20, 21] it holds that $\langle (\Delta W)^2 \rangle_{\mathcal{N}} = \langle W^2 \rangle_{\mathcal{N}} - \langle W \rangle^2_{\mathcal{N}} < 0$ (or $\langle (\Delta W)^2 \rangle_{\mathcal{N}} / \langle W \rangle^2_{\mathcal{N}} = \langle W^2 \rangle_{\mathcal{N}} / \langle W \rangle^2_{\mathcal{N}} - 1 < 0$), which can be observed in Fig. 2.[2] For the comparison with the Fock state it is more convenient to use the quantity $\langle (\Delta W)^2 \rangle_{\mathcal{N}} / \langle W \rangle_{\mathcal{N}}$ (or $\langle (\Delta W)^2 \rangle_{\mathcal{N}} / \langle W \rangle_{\mathcal{N}} + 1 = \langle (\Delta n)^2 \rangle / \langle n \rangle$)

[2] In the antibunching regime the "chaotic photon mean number" $B - |C| - 1$ is negative in (2.18) and in the following equations, which causes the photocounting distribution to be narrower than the Poisson distribution corresponding to the coherent state (cf. Figs 4–6).

which is equal to -1 for the Fock state. This quantity is shown in Fig. 2 by the dotted curve. Its minimum compared to -1 for the Fock state demonstrates the value of deviation of the state under discussion from the Fock state. From Fig. 2 we see that the antibunching disappears for t slightly higher than 6×10^{-5} s. The antibunching also reflects itself in the higher moments shown. The different intersection points with the t-axis of the single moments mean that at no time a coherent state can be reached again (except for $t = 0$), since for coherent states $\langle W^k\rangle_{\mathcal{N}} = \langle W\rangle_{\mathcal{N}}^k$ has to be true for all k. Only coherent radiation in the fourth order ($k = 2$) is generated for $t \approx 6\times 10^{-5}$ s when $\langle W^2\rangle_{\mathcal{N}}/\langle W\rangle_{\mathcal{N}}^2 - 1 = 0$. This is a typical property of quantum fields having no classical analogs (for fields having classical analogs all order coherence follows from the second and fourth order coherence [30]). Comparing curve for $k = 2$ in Fig. 2 and curve a in Fig. 1 we observe that the decrease of uncertainty in the statistics (cf. also the photocounting

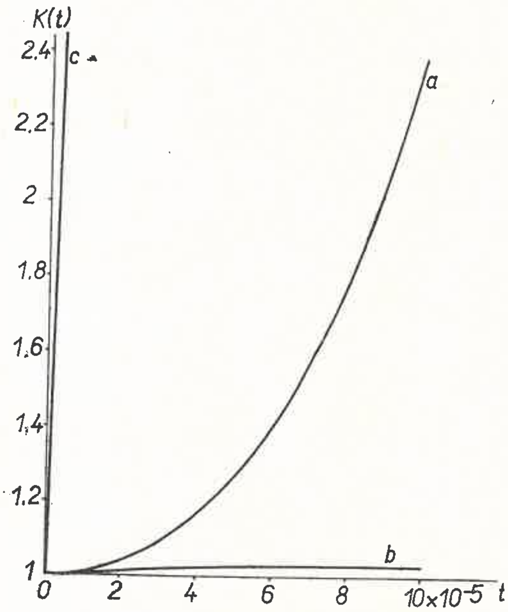


Fig. 3. The noise component $K(t)$ of the quasi-distribution for $|\alpha(0)| = 2$, $2\vartheta - \varphi = -\pi/2$, $g = 10^4\ \mathrm{s}^{-1}$ and a — $\gamma = \langle n_d\rangle = 0$, b — $\gamma = 10^5\ \mathrm{s}^{-1}$, $\langle n_d\rangle = 0$, c — $\gamma = 10^5\ \mathrm{s}^{-1}$, $\langle n_d\rangle = 2$

distributions in Figs 4–6) is connected with the attenuation of radiation and vice versa when amplification occurs, it is connected with the increase in noise (cf. also Fig. 1 in a paper by Mollow and Glauber [1]). This is also pointed out by the time behaviour of the noise component $K(t)$ of the quasi-distribution demonstrated in Fig. 3 for cases a — c. However, here one can observe only the increase in the noise component having the quasi-distribution related to the antinormal ordering where the physical vacuum noise is included. From Figs 1 and 3 the effect of the reservoir (damping of radiation caused by the flux of coherent energy from radiation to the reservoir and noise contribution from the reservoir to the radiation) can be seen, too. For instance, comparing curves a and b in Fig. 1

we observe the effect of damping in the mean intensity $\langle W\rangle_{\mathcal{N}}$, in curve c the reservoir noise with $\langle n_d\rangle = 2$ is added. Similarly curve b in Fig. 3 compared to curve a demonstrates the effect of damping in the noise component ($\langle n_d\rangle = 0$), in curve c the reservoir noise is involved again ($\langle n_d\rangle = 2$).

In Figs 4–6 full curves represent the photocounting distributions calculated after (2.19a). They can be compared with the Poisson distribution

$$p(n, t) = \frac{\langle W\rangle_{\mathcal{N}}^n}{n!} \exp\left(-\langle W\rangle_{\mathcal{N}}\right), \tag{3.1}$$

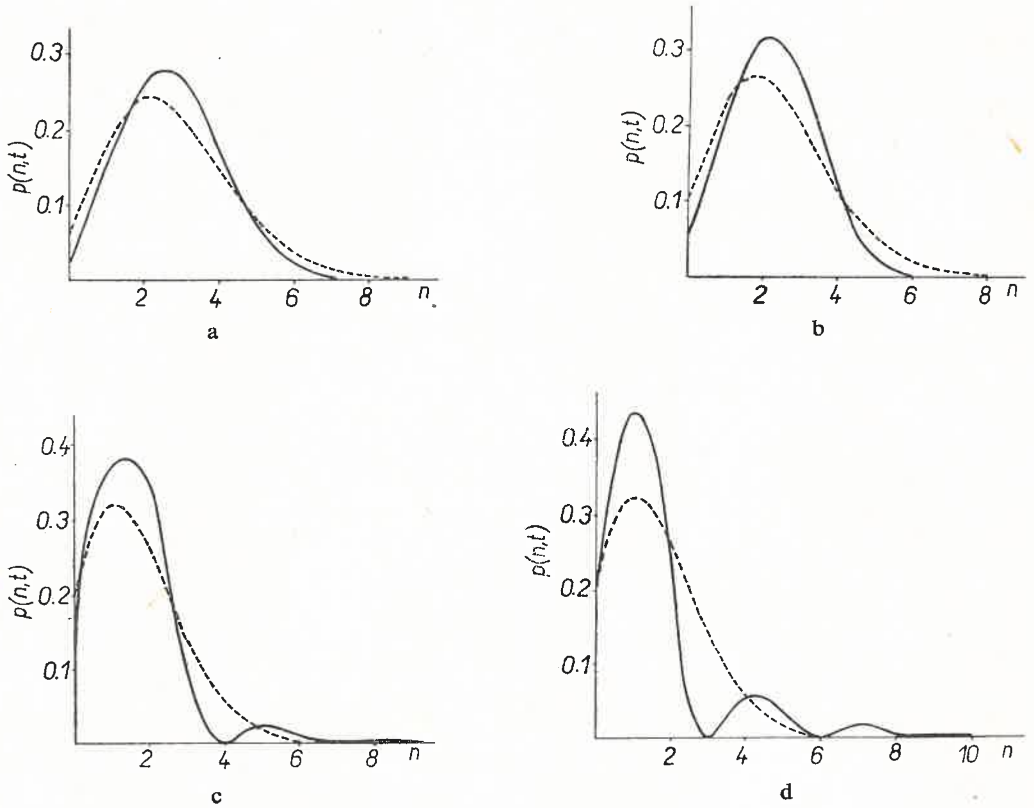
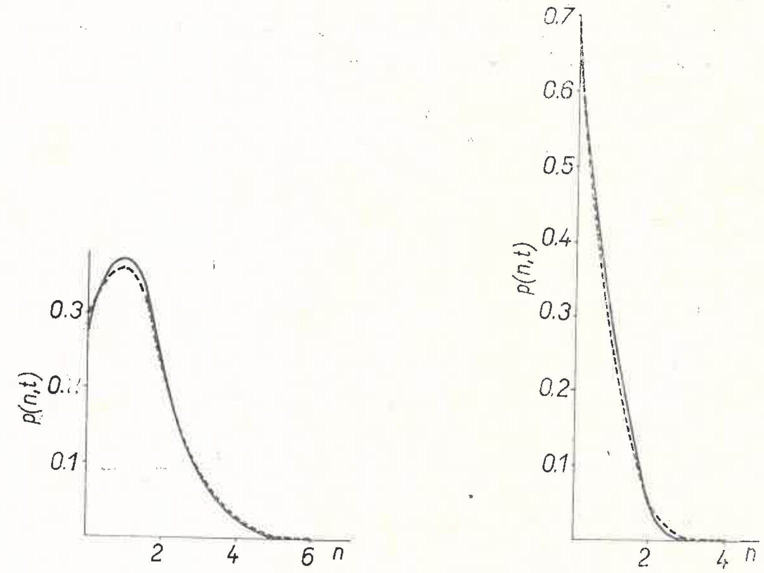


Fig. 4. The photocounting distribution $p(n, t)$ for $|\alpha(0)| = 2$, $2\vartheta - \varphi = -\pi/2$, $g = 10^4\,\mathrm{s}^{-1}$, $\gamma = \langle n_d\rangle = 0$ and a) $t = 2.10^{-5}$ s, b) $t = 3.10^{-5}$ s, c) $t = 6.10^{-5}$ s, d) $t = 8.10^{-5}$ s. Full curves are calculated after (2.19a) and broken ones are calculated after (3.1)

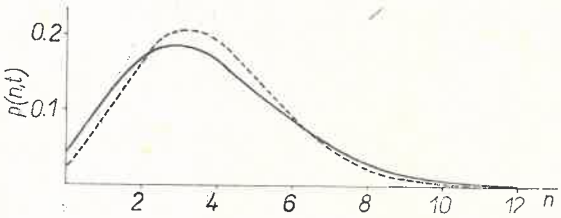
where $\langle W\rangle_{\mathcal{N}}$ is given in (2.20), which corresponds to a fully coherent field with the same $\langle a^+(t)a(t)\rangle$. These photocounting distributions are shown by broken curves. For all numerical calculations $\sum_{n=0}^{\infty} p(n, t) = 1$ with good accuracy. In Figs 4a-d one can follow the time evolution of $p(n, t)$, in Figs 4a and 4b antibunching occurs and it disappears in Fig. 4c for $t \approx 6\times 10^{-5}$ s in rough agreement with curve $k = 2$ in Fig. 2 and curve a in Fig. 1.

Figs 5a, b demonstrate similar results with damping, slight antibunching can be still observed. In Figs 6a, b chaotic reservoir contribution is included with $\langle n_d \rangle = 2$ leading to the disappearence of antibunching. In the antibuching regime $\langle W \rangle_{\mathcal{N}}$ is decreasing, which

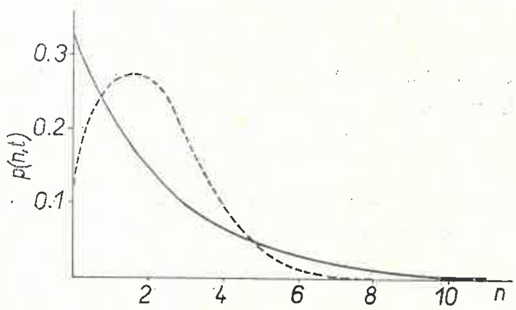


a b

Fig. 5. As in Fig. 4 for $\gamma = 10^5\,s^{-1}$, $\langle n_d \rangle = 0$ and a) $t = 10^{-5}$ s, b) $t = 2.10^{-5}$ s



a



b

Fig. 6. As in Fig. 4 for $\gamma = 10^5\,s^{-1}$, $\langle n_d \rangle = 2$ and a) $t = 10^{-6}$ s, b) $t = 4.10^{-5}$ s

shifts the peaks of the photocounting distributions to the left with increasing time. In Fig. 6b the chaotic reservoir statistics is already dominant.

Thus the antibunching effect is also clearly demonstrated in the photocounting distribution when results obtained for photocounting statistics of radiation propagating through random media are applied. We have found a similar behaviour of the photocounting distribution for the degenerate parametric amplification process in the antibunching regime as has been found for the two-photon absorption process by Bandilla and Ritze [11].

REFERENCES

[1] B. R. Mollow, R. J. Glauber, *Phys. Rev.* **160**, 1076, 1097 (1967).
[2] L. Mišta, *Czech. J. Phys.* **B19**, 443 (1969).
[3] D. F. Walls, R. Barakat, *Phys. Rev.* **A1**, 446 (1970).
[4] R. Graham, in *Springer Tracts in Modern Physics*, Vol. 66, ed. by G. Höhler, Springer, Berlin 1973, p. 1.
[5] J. Peřina, *Czech. J. Phys.* **B26**, 140 (1976).
[6] P. Dewael, *J. Phys.* **A8**, 1614 (1975).
[7] V. Peřinová, J. Peřina, L. Knesel, *Czech. J. Phys.* **B27**, 487 (1977).
[8] N. Tornau, A. Bach, *Opt. Commun.* **11**, 46 (1974).
[9] K. J. McNeil, D. F. Walls, *J. Phys.* **A7**, 617 (1974).
[10] H. D. Simaan, R. Loudon, *J. Phys.* **A8**, 539, 1140 (1975).
[11] A. Bandilla, H. H. Ritze, *Ann. Phys.* (Germany) **33**, 207 (1976).
[12] M. Schubert, B. Wilhelmi, in *Proc. ICO-10*, ed. by J. Blabla, Publ. Techn. Lit., Prague 1977.
[13] H. Paul, U. Mohr, W. Brunner, *Opt. Commun.* **17**, 145 (1976).
[14] K. J. McNeil, D. F. Walls, *J. Phys.* **A8**, 104, 111 (1975).
[15] H. P. Yuen, *Phys. Lett.* **51A**, 1 (1975); *Phys. Rev.* **A13**, 2226 (1976).
[16] H. P. Yuen, *Appl. Phys. Lett.* **26**, 505 (1975).
[17] D. F. Walls, *J. Phys.* **A6**, 496 (1973).
[18] H. D. Simaan, *J. Phys.* **A8**, 1620 (1975).
[19] M. T. Raiford, *Phys. Rev.* **A2**, 1541 (1970); **A9**, 2060 (1974).
[20] D. Stoler, *Phys. Rev. Lett.* **33**, 1397 (1974).
[21] A. Bandilla, *Veränderung der Photonenstatistik mit Hilfe des entarteten parametrischen Verstärkers*, report in *Diskussionstreffen Nichtlineare Optik*, Juliusruh 1976.
[22] J. Peřina, V. Peřinová, L. Mišta, *Czech. J. Phys.* **B24**, 482 (1974).
[23] J. Peřina, V. Peřinová, L. Mišta, R. Horák, *Czech. J. Phys.* **B24**, 374 (1974).
[24] J. Peřina, V. Peřinová, P. Diament, M. C. Teich, Z. Braunerová, *Czech. J. Phys.* **B25**, 483 (1975).
[25] W. H. Louisell, *Quantum Statistical Properties of Radiation*, J. Wiley, New York 1973, Chapts 6 and 7.
[26] G. S. Agarwal, in *Progress in Optics*, Vol. XI, ed. by E. Wolf, North-Holland, Amsterdam 1973, p. 1.
[27] J. Peřina, *Coherence of Light*, Van Nostrand, London 1972 (Russian transl., Mir, Moscow 1974), Ch. 13.
[28] M. M. Miller, E. A. Mishkin, *Phys. Lett.* **24A**, 188 (1967).
[29] P. P. Bertrand, E. A. Mishkin, *Phys. Lett.* **25A**, 204 (1967).
[30] U. M. Titulaer, R. J. Glauber, *Phys. Rev.* **B140**, 676 (1965).