MEASUREMENT OF HALL MOBILITY IN ANISOTROPIC MATERIALS BY MEANS OF THE DISK METHOD

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The measurement of Hall mobility by means of the disk method has in case of low resistance samples many advantages owing to the lack of current contacts. The well known standard method elaborated for isotropic samples, however, cannot be applied directly in case of samples which exhibit an anisotropy of resistance. The present paper gives a function for the correction of the influence of anisotropy on the Hall voltage.

The method of measurement of Hall mobility in which the sample has an annular form or a form of a disk [1, 2] is in many cases more advantageous than the classical method. This concerns in particular samples with small resistivity. In order to obtain a measurable Hall voltage in the classical method in case of metallic sample where the mobility is *e. g.* of the order of 10 cm^2/Vs, the supplying current cannot be less than some tens or even hundreds of amperes. The heat produced by such a big current and the "$1/f$" noise due to contacts may decrease the precision of the measurement and even prohibit the latter. If this method is applied to the determination of the mobility of current carriers in low-resistance semiconducting samples, the measurement can be additionally disturbed by the introduction of carriers through current contacts. The measurement of the Hall mobility made with the sample in the form of a ring or disk located in an alternating magnetic field normal to the surface of the sample eliminates these spurious effects since it does not require external current supply. The electric field in the sample is produced by alternating magnetic field.

The method of measuring of the Hall mobility in sufficiently thin annular samples located in a sinusoidal magnetic field $\boldsymbol{B} = \boldsymbol{B}_0 \sin \omega t$ normal to the surface of the sample has been worked out by Pohl [1] in the case of isotropic samples. The magnetic field produces an electric field $\boldsymbol{E}$ which is normal to the radius at each point and thus the current carriers move along concentric circles. A Lorentz force which is perpendicular to

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both $\boldsymbol{B}$ and $\boldsymbol{E}$ acts on these carriers in the direction along the radius. Thus a radial Hall field E_H is produced in the sample which satisfied the following relationship

$$\mu B = \frac{E_H}{E}, \tag{1}$$

where μ is the mobility.

The Hall voltage thus produced between the edge and the centre of the disk is then [1] equal to

$$U_H = -\tfrac{1}{8}\,\mu\omega B_0^2 R^2 \sin 2\omega t, \tag{2}$$

where R is the radius of the disk.

In many cases single crystals both semiconducting and metallic exhibit an anisotropy of resistance. The value of anisotropy can be determined from the measurement of resistivity in the directions in which the latter is maximum and minimum. In anisotropic samples the distributions of the lines of current induced by the magnetic field are different than in case of isotropic samples. It is the purpose of the present paper to determine the influence of the anisotropy of resistance on the Hall voltage.

Let us consider the case of anisotropic samples in which the directions of maximum and minimum resistivities are perpendicular to each other. Let us also assume that the magnetic field is weak, *i. e.*, $(\mu B)^2 \ll 1$, to neglect the influence of $\boldsymbol{B}$ on anisotropy (weak influence of magnetoresistance). In this case the lines of current will be elliptic and not circular. The elongation of the ellipses is greatest near the centre of the sample while in the vicinity of the edges the ellipses gradually become circular owing to the circular shape of the disk. Such a circular anisotropic sample can be replaced by a fictitious elliptic, but isotropic sample with constant mobility. To do so one should construct an ellipse whose area would be equal to the area of the investigated circular sample and whose semi-axes ratio would be equal to the ratio of the corresponding mobilities of the anisotropic sample:

$$\eta = \frac{a}{b} = \frac{\mu_x}{\mu_y} = \frac{\sigma_x}{\sigma_y}, \tag{3}$$

where μ_x is the mobility in the direction of the x-axis and μ_y that in the direction of the y-axis of the circular sample. The (x, y) coordinate system is chosen so that the direction of maximum mobility is x and that of minimum mobility is y. In such a case:

$$\eta \geqslant 1. \tag{4}$$

We assume the effective mobility μ to be the geometrical mean of these two extreme values:

$$\mu = (\mu_x \mu_y)^{\frac{1}{2}}. \tag{5}$$

To calculate the field strength $\boldsymbol{E}(x, y)$ at an arbitrary point of the sample one should find an orthogonal coordinate system (u, v) composed of lines to which the vector $\boldsymbol{E}$ is tangent at any point and of such lines to which the vector $\boldsymbol{E}$ is perpendicular.

After analysing the coordinate systems given in literature the authors accepted that defined by the following transformation rule [3]:

$$\hat{Z} = \frac{2a}{\pi} \ln \operatorname{tg} \frac{\hat{W}}{2} - ia, \tag{6}$$

where

$$\hat{Z} = x + iy$$
$$\hat{W} = u + iv.$$

The dependences between the Cartesian coordinates and the present ones are the following:

$$x = \frac{a}{\pi} \ln \frac{\sin^2 \frac{u}{2} + \operatorname{sh}^2 \frac{v}{2}}{\cos^2 \frac{u}{2} + \operatorname{sh}^2 \frac{v}{2}}$$

$$y = \frac{2a}{\pi} \operatorname{arc\,ctg} \frac{\operatorname{sh} v}{\sin u}\,. \tag{7}$$

The v = const lines (Fig. 1) are lines of current while the u = const-lines are quasi-potential lines. The shape of the lines of current is nearly elliptic. This approximation is

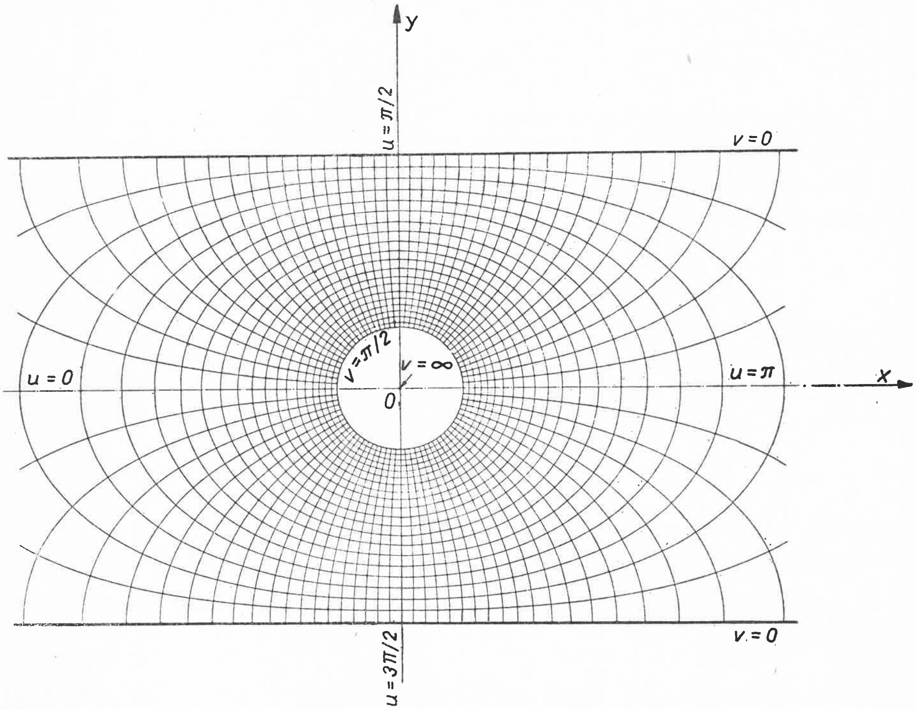

Fig. 1

very good for small values of anisotropy. For large values of anisotropy the area bounded by the $v = \text{const}$ curve is greater than the area of an ellipse with the same semi-axes ratio η, whereas the length of the curve does not differ practically from the perimeter of the ellipse (the maximum error of such an assumption, *i. e.*, that the perimeters of the $v = \text{const}$ line and of the ellipse with the same semi-axes ratio η are equal, is 3.5%).

The electric field $\boldsymbol{E}$ with the components $\boldsymbol{E}(E_u, E_v)$ produced in an isotropic elliptic sample by the magnetic field $\boldsymbol{B} = \boldsymbol{B}_0 \sin \omega t$ must fulfil the following conditions:

a) the field may not contain any sources since no electric charge may accumulate at any point of the sample,

b) the line integral

$$\oint \boldsymbol{E}(u, v) \cdot d\boldsymbol{l} = -S \frac{\partial B}{\partial t}, \tag{8}$$

where S is the area closed by the integration contour, since the amplitude of the magnetic field $\boldsymbol{B}_0$ is constant in the whole sample

c) the component $\boldsymbol{E}_v$ which is normal to the boundary of the sample, *i. e.*, $v = v_0$, must vanish at this boundary.

In the coordinate system under consideration the vector $\boldsymbol{E}(u, v)$ with the components:

$$E_u = -\frac{2a}{\pi} B_0 \omega \cos(\omega t) (\sin^2 u + \text{sh}^2 v)^{\frac{1}{2}} \ln \text{ctgh}\, v$$

$$E_v = 0 \tag{9}$$

satisfies all the above-mentioned conditions. The electric field with these components is thus a curl field induced in the sample under the influence of the magnetic field.

The knowledge of $\boldsymbol{E}(u, v)$ permitted the calculation of $\boldsymbol{E}_H(u, v)$ by means of Eq. (1) and then the voltage U_H between the edge and the centre of the sample:

$$U_H = \int E_H(u, v) dl. \tag{10}$$

Consequently:

$$U_H = -\frac{4a^2}{\pi^2} \mu B \frac{\partial B}{\partial t} \int_{\infty}^{v_0} \ln \text{ctgh}\, v dv. \tag{11}$$

One can asign to the closed oval line $v_0 = \text{const}$ (which corresponds to the edge of the sample) an ellipse such that the shorter semi-axes of the former and the latter as well as their areas would be equal. In such a case the ratio of the semi-axes of the ellipse is equal to anisotropy

$$\eta = \frac{2 \ln \text{ctgh}\, v_0}{[\text{arc ctg}\,(\text{sh}\, v_0)]^2}. \tag{12}$$

The constant a has been determined by comparing the area of the surface to a circular sample with the radius R:

$$\frac{8a^2}{\pi} \ln \operatorname{ctgh} v_0 = \pi R^2, \tag{13}$$

and thus

$$U_H = -\tfrac{1}{8}\mu B_0^2 R^2 \omega \sin(2\omega t) f(v_0), \tag{14}$$

where

$$f(v_0) = \frac{2\int\limits_{\infty}^{v_0} \ln \operatorname{ctgh} v dv}{\ln \operatorname{ctgh} v_0}. \tag{15}$$

The function $f(v_0)$ has been calculated numerically. Making use of the connection between v_0 and η (Eq. (12)) we have calculated the function $f(\eta)$ whose graph is shown

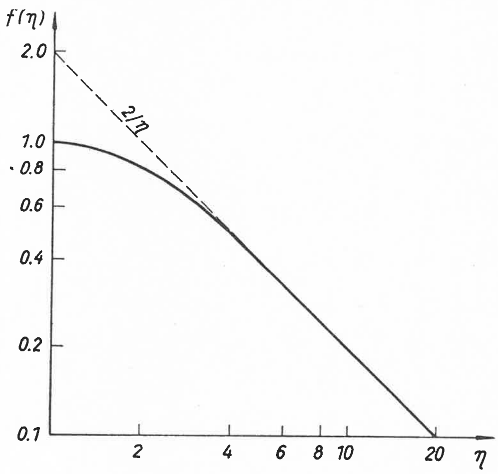

Fig. 2

in Fig. 2. For large values of the anisotropy parameter η, *i. e.* small v_0 (Fig. 1) $f(\eta) = 2/\eta$. In this case one can write:

$$U_H = -\frac{R^2\omega}{4\eta}\mu B_0^2 \sin 2\omega t. \tag{16}$$

The above dependence permits μ to be calculated for samples with large anisotropy.

In case of small anisotropy, of the order of several or a dozen per cent, one can make use of Eq. (2) derived for isotropic samples, since for $\eta \to 1$, $f(\eta) \to 1$ and Eq. (14) goes over into (2).

For anisotropic samples the Hall voltage drop is considerable compared to isotropic samples and the correction for anisotropy is necessary.

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