MODIFIED BLOCH EQUATIONS FOR AN ISOTROPIC FERROMAGNET

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The equations of motion for the nuclear magnetic moment in isotropic ferromagnetic materials in the presence of an oscillating external magnetic field are derived, by making use of the method of orthogonal operator expansion developed by Shimizu. It is argued that, if one assumes that the interaction between the nuclear and the electronic spin systems is weak then the relaxation times and energy shifts can be expressed through the correlation functions of the electronic spins.

1. *Introduction*

Nuclear magnetic resonance in crystals, whose localized electron spins form an antiferromagnetic or ferromagnetic configuration, exhibits some peculiarities connected with the strong local magnetic fields which act on the nuclei in these materials. One peculiarity is the difference in the value of the nuclear Larmor frequency which is about 2–3 orders greater than that in the paramagnetic state of the same materials. Another peculiarity is the enhancement of the NMR signal [1].

In a magnetic medium the nuclear spins are coupled by the Suhl-Nakamura indirect interaction. This interaction causes fluctuations of the local field which give rise to a broadening of the NMR line and a displacement of the NMR frequency in relation to the one corresponding to the average value of the local field.

Many attempts were made to express the features of the resonance, such as the origin of the line width, the line width and shape, through the correlation functions of the electron spin subsystem, but a complete description by means of a single method (*e. g.* the Green functions or kinetic equations method) has not yet been achieved.

By using the orthogonal operators expansion method [4] we derive in this paper the equations of motion for the nuclear magnetic moment in the presence of an oscillating magnetic field and by taking into account the indirect interaction between nuclear spins. With the assumption of weak interactions between the nuclear and the electronic spin

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systems the relaxation times and energy shifts are expressed through the correlation functions of the electronic spins.

In Section 2 the Hamiltonian of the system is expressed in a rotating reference frame. Section 3 outlines briefly Shimizu's method which in Section 4 is used in deriving the equations of motion for the nuclear magnetic moment.

2. *The Hamiltonian of the system in the revolving reference frame*

Let us consider a system of N sites, each of them having a nuclear spin I_j and an effective electronic spin S_j. The system is in an external, constant and homogeneous magnetic field H_0 which is applied along the z-axis, and there is a transverse, rotating magnetic field with frequency ω and amplitude H_1. The Hamiltonian of the system is supposed to have the form

$$\mathscr{H}'(t) = \mathscr{H}'_z + \mathscr{H}'_{ex} + \mathscr{H}'_{rf} + \mathscr{H}'_{SI}. \tag{1}$$

Here, $\mathscr{H}'_z$ is the Zeeman energy of the nuclear ($\boldsymbol{I} = \sum_j \boldsymbol{I}_j$) and electronic ($S = \sum_j S_j$) spins:

$$\mathscr{H}'_z = -\omega_n I_z - \omega_e S_z \tag{2}$$

where $\omega_n \cong \gamma_I(H_n + H_0)$ is the Larmor frequency of NMR, γ_I the nuclear magnetogyric value, $H_n = -\dfrac{A\,\langle S_{iz}\rangle_0}{\gamma_I}$ is the local field at the nucleus which is produced by its own "magnetic" electrons, A — constant of hyperfine interaction, $\langle\ldots\rangle_0 = \dfrac{\operatorname{Tr} e^{-\beta\mathscr{H}_e}(\ldots)}{\operatorname{Tr} e^{-\beta\mathscr{H}_e}}$, $\mathscr{H}_e$ — Hamiltonian of the electron spin system which includes the exchange interactions, $\omega_e = \gamma_s(H_0 + H_a)$ — frequency of ferromagnetic resonance and H_a — anisotropy field. Further, $\mathscr{H}'_{ex}$ is the isotropic exchange interaction energy between "magnetic" electrons, $\mathscr{H}'_{rf}$ is the energy of the system in the rotating field H_1:

$$\mathscr{H}'_{rf} = -\gamma_I \frac{H_1}{2}(I^+ e^{i\omega t} + I^- e^{-i\omega t}) - \gamma_s \frac{H_1}{2}(S^+ e^{i\omega t} + S^- e^{-i\omega t}),$$

$$I^{\pm} = I_x \pm i I_y, \qquad S^{\pm} = S_x \pm i S_y,$$

and $\mathscr{H}'_{SI}$ is the hyperfine interaction:

$$\mathscr{H}'_{SI} = A \sum_{j=1}^{N} I_{zj}\delta S_{zj} + A \sum_{j=1}^{N} (I_{xj}S_{xj} + I_{yj}S_{yj}),$$

where

$$\delta S_{zj} = S_{zj} - \langle S_{zj}\rangle_0.$$

Let us go over to a reference frame rotating around the z-axis with angular frequency ω. This can be done with the aid of the transformation

$$B = e^{-i\omega(I_z+S_z)t} B' e^{i\omega(I_z+S_z)t}. \tag{3}$$

In the new reference frame the Hamiltonian becomes

$$\mathscr{H}(t) = -(\omega_n-\omega)I_z-(\omega_e-\omega)S_z-\gamma_s H_1 S_x-\gamma_I H_1 I_x+\mathscr{H}_{ex}+\mathscr{H}_{SI}. \tag{4}$$

To obtain the effect of the enhancement of the NMR signal it is necessary to further change the reference frame for the electronic spins to an "effective" one. This frame has an axis of quantization which is tilted at an angle α to the z-axis and is arranged to coincide with the effective static field for the electronic spins in the rotating reference frame. This change can be done with the aid of the transformation given in [5].

In this frame the Hamiltonian has the form

$$\mathscr{H}^*(t) = -(\omega_n-\omega)I_z-(\eta+1)\gamma_I H_1 I_x-\omega_e S^z+\mathscr{H}_{ex}+A\sum_{j=1}^{N} I_{zj}(\delta S_j^z-\alpha S_j^x)+$$
$$+A\sum_{j=1}^{N} I_{xj}(S_j^x+\alpha\delta S_j^z)+A\sum_{j=1}^{N} I_{yj}S_j^y \tag{5}$$

where the enhancement factor is

$$\eta = \frac{A\sigma\alpha}{\gamma_I H_1} = \frac{H_n}{H+H_a}, \tag{6}$$

$$\sigma = |\langle S_{zj}\rangle_0|, \tag{7}$$

$$\sin\alpha = \frac{\gamma_s H_1}{(\omega_e-\omega)^2+(\gamma_s H_1)^2}. \tag{8}$$

3. *Outline of Shimizu's method*

Shimizu [4] obtained equations for macroscopic variables by making use of the method of orthogonal operator expansion. Here orthogonal relations for operators are defined by

$$\mathrm{Tr}\,\{O_i^+ O_j\} = C\delta_{ij}$$

where C represents a normalization factor and O_i^+ denotes the Hermitian conjugate operator of O_i. This can be written in the notation used in that paper as

$$\mathrm{Tr}\,\{AB^+\} = \langle A|B\rangle.$$

Here $\langle A|$, $\langle B|$ are vector representations of the operators A, B and $|B\rangle$ is the Hermitian conjugate to $\langle B|$.

The average value of an operator A is represented as

$$\langle A\rangle \equiv \mathrm{Tr}\,\{A\varrho\} = \langle A|\varrho\rangle,$$

$|\varrho\rangle$ being the vector representation of the density matrix ϱ.

The von Neumann equation in this notation can be written as

$$\frac{d}{dt}|\varrho(t)\rangle = -i\hat{\mathscr{H}}(t)|\varrho(t)\rangle$$

where $\hat{\mathscr{H}}(t)$ stands for a matrix representation of the time-dependent Hamiltonian $\mathscr{H}(t)$. Next, Shimizu expanded $|\varrho(t)\rangle$ into a linear combination of orthogonal operators O_k $(k = 1, 2, \dots m)$ using the projection operator P in the form

$$P \equiv \sum_{k=1}^{m} \frac{|O_k\rangle\langle O_k|}{\langle O_k|O_k\rangle}$$

and then derived exact equations for macroscopic variables from the von Neumann equation. These exact equations have the form

$$\frac{d}{dt}\langle O_k(t)\rangle = -i\sum_{j=1}^{m}\frac{\langle O_k|\hat{\mathscr{H}}(t)|O_j\rangle}{\langle O_j|O_j\rangle}\langle O_j(t)\rangle -$$

$$-i\langle O_k|\hat{\mathscr{H}}(t)S(t,0)(1-P)|\varrho\rangle -$$

$$-\sum_{j=1}^{m}\int_0^t dt'\langle O_k|\hat{\mathscr{H}}(t)S(t,t')(1-P)\hat{\mathscr{H}}(t')|O_j\rangle\frac{\langle O_j(t')\rangle}{\langle O_j|O_j\rangle}$$

$$(k = 1, 2, \dots m) \tag{9}$$

where

$$S(t,t') = Te^{-i\int_{t'}^{t}(1-P)\hat{\mathscr{H}}(\tau)d\tau}.$$

4. *Equations of motion for the nuclear magnetic moment*

In order to obtain linear equations for $\langle I_x(t)\rangle$, $\langle I_y(t)\rangle$ and $\langle I_z(t)\rangle$ Shimizu [4] chose I_x, I_y, I_z as O_1, O_2 and O_3 in Eq. (9). Then the equations for these variables, up to the second order in $\hat{V}(t)$ (to ensure this, it is necessary to use the first or second term of the expansion of $S(t,t')$ in powers of $\hat{\mathscr{H}}(t)$, become

$$\frac{\partial}{\partial t}\langle I_a(t)\rangle = -i\langle I_a|\hat{\mathscr{H}}_0|\varrho(t)\rangle - \sum_{b=x,y,z}\int_0^t dt'\frac{\langle I_a|\hat{V}e^{-i\hat{\mathscr{H}}_0(t-t')}\hat{V}_1 I_b\rangle}{\langle I_b|I_b\rangle}\langle I_b(t')\rangle -$$

$$-i\langle I_a|\hat{V}(1-P)|\varrho\rangle - \int_0^t dt'\langle I_a|\hat{V}e^{-i\hat{\mathscr{H}}_0(t-t')}\hat{V}(1-P)|\varrho\rangle$$

$$(a = x, y, z) \tag{10}$$

where

$$P = \sum_{b=x,y,z}\frac{|I_b\rangle\langle I_b|}{\langle I_b|I_b\rangle} \tag{11}$$

and

$$\mathscr{H}_0 = \mathscr{H}_z^{ef} + \mathscr{H}_e.$$

Here, $\mathscr{H}_z^{ef}$ is the Zeeman energy of the system[1] with respect to the effective field in the rotating frame:

$$\mathscr{H}_z^{ef} = -\Delta I_z - (\eta+1)\omega_{1l}I_x, \quad (\Delta = \omega_n - \omega)$$

$$\mathscr{H}_e = -\omega_e S^z + \mathscr{H}_{ex} \tag{12}$$

and V is

$$V = \sum_\gamma V^\gamma, \quad V^\gamma = \sum_{j=1}^{N} I_j^\gamma Q_j^{(-\gamma)} \quad (\gamma = -1, 0, 1) \tag{13}$$

where the following abbreviations are used:

$$Q_j^{-1} = \frac{A}{2}(S_j^- + \alpha\delta S_j^z), \quad Q_j^0 = A(\delta S_j^z - \alpha S_j^z), \quad Q_j^{+1} = \frac{A}{2}(S_j^+ + \alpha\delta S_j^z). \tag{14}$$

The initial condition is chosen in the same manner as in [4]:

$$\varrho = \varrho(0) = \frac{e^{-\beta\mathscr{H}_0}}{\mathrm{Tr}\, e^{-\beta\mathscr{H}_0}} \quad \left(\beta = \frac{1}{kT}\right). \tag{15}$$

Let us consider the kernels:

$$K_{ab}(t-t') \equiv \frac{\langle I_a | \hat{V} e^{-i\hat{\mathscr{H}}_0(t-t')} \hat{V} | I_b \rangle}{\langle I_b | I_b \rangle} \quad (a, b = x, y, z). \tag{16}$$

It can easily be checked that all $K_{ab}(t-t')$ are real.

In the usual notation $K_{ab}(t-t')$ have the form

$$K_{ab}(t-t') = \frac{1}{\langle I_b | I_b \rangle} \sum_{\gamma,\gamma'} \mathrm{Tr}\, \{e^{i\mathscr{H}_0(t-t')}[I_a, V^\gamma] e^{-i\mathscr{H}_0(t-t')}[V^{\gamma'}, I_b]\} \tag{17}$$

with the help of Eq. (13). It is convenient to use the notation

$$K_{\mu\nu}(t) = \frac{1}{\langle I_\nu | I_\nu \rangle} \sum_{\gamma,\gamma'} \mathrm{Tr}\, \{e^{i\mathscr{H}_0 t}[I^\mu, V^\gamma] e^{-i\mathscr{H}_0 t}[V^{\gamma'}, I^\nu]\} \tag{18}$$

$$(\mu, \nu = \pm 1, 0).$$

The connections between $K_{\mu\nu}(\mu, \nu = \pm 1, 0)$ and $K_{ab}(a, b = x, y, z)$ are given by Eqs (2.32) in [4].

The following relations are useful in calculating Eqs (18):

$$[I_k^\mu, I_j^\gamma] = A_j^\mu I_j^{\mu+\gamma} \tag{19}$$

where the values of A_γ^μ are as follows:

$$A_0^\mu = -\mu; \quad A_{\gamma=\pm1}^{\mu=\pm1} = \mu - \gamma; \quad A_{\gamma=\pm1}^0 = \gamma.$$

It can be proved [6] that

$$e^{i\mathscr{H}_z^{ef}t}I^{\mu+\gamma}e^{-i\mathscr{H}_z^{ef}t} = \sum_{\lambda,\kappa} p_\lambda^{\mu+\gamma} q_\kappa^\lambda e^{-i\lambda\omega' t} I^\kappa$$

$$(\omega' = \sqrt{\Delta^2+(\eta+1)^2\omega_{1I}^2}) \tag{20}$$

where the coefficients $p_\lambda^{\mu+\gamma}$ and q_ν^λ are given in Table I.

TABLE I

$p_\lambda^{\mu+\gamma}$ | q_ν^λ

λ \ $\mu+\gamma$	1	0	−1	ν \ λ	1	0	−1
1	$\frac{1}{2}(\cos\theta+1)$	$-\frac{1}{2}\sin\theta$	$\frac{1}{2}(\cos\theta-1)$	1	$\frac{1}{2}(\cos\theta+1)$	$\frac{1}{2}\sin\theta$	$\frac{1}{2}(\cos\theta-1)$
0	$\sin\theta$	$\cos\theta$	$\sin\theta$	0	$-\sin\theta$	$\cos\theta$	$-\sin\theta$
−1	$\frac{1}{2}(\cos\theta-1)$	$-\frac{1}{2}\sin\theta$	$\frac{1}{2}(\cos\theta+1)$	−1	$\frac{1}{2}(\cos\theta-1)$	$\frac{1}{2}\sin\theta$	$\frac{1}{2}(\cos\theta+1)$

where

$$\tan\theta = \frac{(\eta+1)\omega_{1I}}{\Delta}.$$

By making use of these relations the kernels (16) become

$$K_{\mu\nu}(t) = \frac{-N}{\langle I^\nu|I^\nu\rangle}\sum_{\gamma,\gamma'}\sum_{\lambda,\kappa} A_\gamma^\mu A_{\gamma'}^\nu p_\lambda^{\mu+\gamma} q_\kappa^\lambda e^{-i\lambda\omega' t}\times$$

$$\times \mathrm{Tr}_I\,\{I_j^\kappa I_j^{\nu+\gamma'}\}\,\mathrm{Tr}_S\,\{Q_j^{(-\gamma)}(t)Q_j^{(-\gamma')}\} \tag{21}$$

where

$$Q_j^{(-\gamma)}(t) = e^{i\mathscr{H}_{et}}Q^{(-\gamma)}e^{-i\mathscr{H}_{et}}.$$

Now, let us consider the inhomogeneous terms of Eqs (10):

$$D_a(t) \equiv -i\langle I_a|\hat{V}(1-P)|\varrho\rangle - \int_0^t dt'\langle I_a|\hat{V}e^{-i\mathscr{H}_0(t-t')}\hat{V}(1-P)|\varrho\rangle \tag{22}$$

$$(a = x, y, z).$$

With the help of Eqs (15) and (16) this term becomes

$$D_a(t) = -i\langle I_a|\hat{V}|\varrho\rangle + i\sum_{b=x,y,z}\frac{\langle I_a|\hat{V}|I_b\rangle}{\langle I_b|I_b\rangle}\langle I_b\rangle_0 -$$

$$-\int_0^t dt'\langle I_a|\hat{V}e^{-i\mathscr{H}_0(t-t')}\hat{V}|\varrho\rangle + \sum_{b=x,y,z}\int_0^t dt' K_{ab}(t')\langle I_b\rangle_0 \tag{23}$$

where

$$\langle I_b\rangle_0 = \langle I_b|\varrho\rangle = \mathrm{Tr}\,\{\varrho I_b\}.$$

The first and second terms of the r. h. s. of Eqs (23) are equal to zero, and the third term can be written as

$$D_a^1(t) = -\int_0^t dt'\langle[e^{i\mathscr{H}_0 t'}[I_a, V]e^{-i\mathscr{H}_0 t'}, V]\rangle_0 =$$

$$= -\sum_{\gamma,\gamma'}\int_0^t dt'\langle[e^{i\mathscr{H}_0 t'}[I_a, I^\gamma]Q^{(-\gamma)}e^{-i\mathscr{H}_0 t'}, I^{\gamma'}Q^{(-\gamma')}]\rangle_0. \tag{24}$$

It is convenient to use the notation

$$I_z = I^0;\quad I_x = \tfrac{1}{2}(I^1+I^{-1});\quad I_y = \frac{1}{2i}(I^1-I^{-1});$$

$$D_z^1(t) = D_0^1(t);\quad D_x^1(t) = \tfrac{1}{2}(D_1^1(t)+D_{-1}^1(t));\quad D_y^1(t) = \frac{1}{2i}(D_1^1(t)-D_{-1}^1(t)); \tag{25}$$

$$D_\mu^1(t) = -\sum_{\gamma,\gamma'}\int_0^t dt'\langle[e^{i\mathscr{H}_0 t'}[I^\mu, I^\gamma]Q^{(-\gamma)}e^{-i\mathscr{H}_0 t'}, I^{\gamma'}Q^{(-\gamma')}]\rangle_0. \tag{26}$$

By using Eqs (19), (20) Eqs (26) assume the form

$$D_\mu^1(t) = -\sum_{\substack{\gamma,\gamma'\\ \lambda,\nu\\ j,k}}\int_0^t dt' e^{-i\lambda\omega' t'} A_{\gamma}^{\mu} p_{\lambda}^{\mu+\gamma} q_{\nu}^{\lambda}\langle[I_j^{\nu}Q_j^{(-\gamma)}(t'), I_k^{\gamma'}Q_k^{(-\gamma')}]\rangle_0 =$$

$$= -\sum_{\substack{\gamma,\gamma'\\ \lambda,\nu\\ j,k}}\int_0^t dt' e^{-i\lambda\omega' t'} A_{\gamma}^{\mu} p_{\lambda}^{\mu+\gamma} q_{\nu}^{\lambda}(\langle I_j^{\nu} I_k^{\gamma'}\rangle_0\langle Q_j^{(-\gamma)}(t')Q_k^{(-\gamma')}\rangle_0 -$$

$$-\langle I_k^{(\gamma')} I_j^{\nu}\rangle_0\langle Q_k^{(-\gamma')}Q_j^{(-\gamma)}(t')\rangle_0). \tag{27}$$

By making use of Eqs (16) and (23) Eq. (10) becomes

$$\frac{\partial}{\partial t}\langle I_a(t)\rangle = \sum_{b=x,y,z}\Omega_{ab}\langle I_b(t)\rangle - \sum_{b=x,y,z}\int_0^t dt' K_{ab}(t')\,\langle I_b(t-t')\rangle + D_a(t), \tag{28}$$

$$(a = x, y, z)$$

$$\Omega_{xy} = -\Omega_{yx} = \Delta,\quad \Omega_{yz} = -\Omega_{zy} = (\eta+1)\omega_{1I},\quad \Omega_{xx} = \Omega_{xz} = \Omega_{yy} = \Omega_{zx} = \Omega_{zz} = 0.$$

If we assume that the "memory" of the kernels $K_{ab}(t)$ extends only over the finite time interval $0 < t < T_c$,

$$K_{ab}(t) \cong 0 \text{ if } t > T_c, \tag{29}$$

then Eqs (28) reduce to

$$\frac{d}{dt}\langle I_x(t)\rangle = \left(\Delta - \frac{1}{T_{xy}}\right)\langle I_y(t)\rangle - \frac{1}{T_x}(\langle I_x(t)\rangle - \langle I_x\rangle_0) - \frac{1}{T_{xz}}(\langle I_z(t)\rangle - \langle I_z\rangle_0) + D_x^1(t),$$

$$\frac{d}{dt}\langle I_y(t)\rangle = -\left(\Delta - \frac{1}{T_{xy}}\right)\langle I_x(t)\rangle + \left((\eta+1)\omega_{1I} - \frac{1}{T_{yz}}\right)\langle I_z(t)\rangle - \frac{1}{T_y}\langle I_y(t)\rangle -$$

$$-\frac{1}{T_{xy}}\langle I_x\rangle_0 + \frac{1}{T_{yz}}\langle I_z\rangle_0 + D_y^1(t),$$

$$\frac{d}{dt}\langle I_z(t)\rangle = -\left((\eta+1)\omega_{1I} - \frac{1}{T_{yz}}\right)\langle I_y(t)\rangle - \frac{1}{T_z}(\langle I_z(t)\rangle - \langle I_z\rangle_0) -$$

$$-\frac{1}{T_{xz}}(\langle I_x(t)\rangle - \langle I_x\rangle_0) + D_z^1(t) \tag{30}$$

where the relaxation times are given by

$$\frac{1}{T_{ab}} \equiv \int_0^\infty dt' K_{ab}(t'), \qquad \frac{1}{T_a} = \frac{1}{T_{aa}} (a = x, y, z) \tag{31}$$

and we have utilized the relations

$$K_{xy}(t') = -K_{yx}(t'), \quad K_{zx}(t') = K_{xz}(t'), \quad K_{zy}(t') = -K_{yz}(t').$$

Let us turn to the relaxation times T_{ab}. By using the notation

$$C_{\xi\eta}(t) = \mathrm{Tr}_S\,\{Q^\xi(t)Q^\eta\} \quad (\xi, \eta = \pm 1, 0) \tag{32}$$

for the correlation functions and

$$j_{\xi\eta}(\omega) = \int_0^\infty dt e^{i\omega t} c_{\xi\eta}(t) \tag{33}$$

for their one-sided Fourier transforms, we obtain

$$\frac{1}{T_{\mu\nu}} = \frac{-N}{\langle I^\nu | I^\nu\rangle} \sum_{\substack{\gamma,\gamma' \\ \lambda,\kappa}} A_\gamma^\mu A_{\gamma'}^\nu p_\lambda^{\mu+\gamma} q_\kappa^\lambda \,\mathrm{Tr}_I\,\{I_j^\kappa I_j^{\nu+\gamma'}\} j_{-\gamma,-\gamma'}(-\lambda\omega'). \tag{34}$$

Eqs (27) can also be written in the form

$$D_\mu^1(\infty) = -N\sum_\lambda \int_0^t dt' e^{-i\lambda\omega' t'}(A_1^\mu p_\lambda^{\mu+1} q_1^\lambda((I(I+1) - \langle I_{jz}^2\rangle_0)\langle[Q_j^-(t')Q_j^+]\rangle_0 +$$

$$+\langle I_{jz}\rangle_0\langle\{Q_j^-(t'), Q_j^+\}\rangle_0) +$$

$$+A_0^\mu p_\lambda^\mu q_0^\lambda \langle I_{jz}^2\rangle_0\langle[Q_j^z(t'), Q_j^z]\rangle_0 +$$

$$+A_{-1}^\mu p_\lambda^{\mu-1} q_{-1}^\lambda((I(I+1) - \langle I_{jz}^2\rangle_0)\langle[Q_j^+(t'), Q_j^-]\rangle_0 -$$

$$-\langle I_{jz}\rangle_0\langle\{Q_j^+(t'), Q_j^-\}\rangle_0)).$$

If we introduce the retarded double-time Green functions

$$G_r^{\pm}(t') = \langle\langle A(t')B(0)\rangle\rangle_r^{\pm} = -i\theta(t)\,\langle[A(t'), B]_{\pm}\rangle_0,$$

$$G_r^{\pm}(t') = \frac{1}{2\pi}\int_{-\infty}^{\infty} G_r^{\pm}(\omega)e^{-i\omega t'}d\omega,$$

$$G_r^{\pm}(\omega) = {}_r\langle\langle A|B\rangle\rangle_{\omega}^{\pm} = \int_{-\infty}^{\infty} G_r^{\pm}(t')e^{i\omega t'}dt'$$

then $D_{\mu}^{1}(\infty)$ becomes

$$D_{\mu}^{1}(\infty) = -iN\sum_{\lambda} A_1^{\mu}p_{\lambda}^{\mu+1}q_1^{\lambda}((I(I+1)-\langle I_{jz}^2\rangle_0)\,{}_r\langle\langle Q_j^-|Q_j^+\rangle\rangle_{-\lambda\omega'}^{-}+$$
$$+\langle I_{jz}\rangle_0\,{}_r\langle\langle Q_j^-|Q_j^+\rangle\rangle_{-\lambda\omega'}^{+})+$$
$$+A_0^{\mu}p_{\lambda}^{\mu}q_0^{\lambda}\langle I_{jz}^2\rangle_0\,{}_r\langle\langle Q_j^z|Q_j^z\rangle\rangle_{-\lambda\omega'}^{-}+$$
$$+A_{-1}^{\mu}p_{\lambda}^{\mu-1}q_{-1}^{\lambda}((I(I+1)-\langle I_{jz}^2\rangle_0)\,{}_r\langle\langle Q_j^+|Q_j^-\rangle\rangle_{-\lambda\omega'}^{-}-$$
$$-\langle I_{jz}\rangle_0\,{}_r\langle\langle Q_j^+|Q_j^-\rangle\rangle_{-\lambda\omega'}^{+}).$$

5. *Conclusions*

Eqs. (30) represent modified Bloch equations for an isotropic ferromagnet, referred to a coordinate system rotating with the driving field. It should be noted that the behaviour of the nuclear spin system is described by four independent relaxation times T_x, T_y, T_z, T_{xz} and two energy shifts $\frac{1}{T_{xy}}$, $\frac{1}{T_{yz}}$, and all these values depend on three frequency parameters of the system: ω, $(\eta+1)\,\omega_{1I}$, and ω_n. The system here under consideration has been examined by Bariachtar *et al.* [3]. With the assumption of weak interaction between the nuclear and electronic spin systems the equations obtained in that paper are less accurate, because they describe the changes in time of the transverse and longitudinal components of the nuclear spin moment independently from each other. There are only two relaxation times and one energy shift in the equations obtained by Bariachtar *et al.*, and these quantities are expressed through the correlation functions of the electron spin system, in a manner quite similar to that in the present paper.

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