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MAGNETIC FIELD INFLUENCE ON THE INDIRECT RUDERMAN-KITTEL-KASUYA-YOSIDA INTERACTION

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A new method for calculating the correction to the RKKY interaction due to the presence of magnetic field, is presented. This correction turns out to be small, but for some cases can be significant.

1. Introduction

The indirect exchange interaction among the electrons of unfilled shells (*d*- or *f*-) of the lattice ions follows *via* the conduction electrons (*s*-). This type of interaction has been introduced by Ruderman and Kittel [1], and since then a number of papers dealing with this problem appeared. Glasser [2] considered the influence of the magnetic field on the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction. He showed that the presence of the magnetic field influences markedly the magnitude of this interaction if instead of the rest mass m of the electron, the effective mass m^* is considered. If $m \gg m^*$, such effects have been observed (*e.g.* in Ga where $m^* \cong 0.051\, m$).

The RKKY interaction has been also considered by Izyumov and Vonsovskii [3]. They obtained the Hamiltonian for this interaction by eliminating the degrees of freedom of the conduction electrons. The general form on this Hamiltonian, although it contains quantities depending on the magnetic field, does not, however, give the explicit form of the dependence of the RKKY interaction on the magnetic field. It is the aim of the present paper to find this dependence.

2. Calculation

Izyumov and Vonsovskii obtained the following Hamiltonian describing the RKKY interaction:

$$H_{\text{ef}} = \frac{1}{N^2} \sum_{kk'} \sum_{nm} \frac{J^2(kk')}{E_k - E_{k'}} e^{i(k-k')R_{nm}} \{n_k^+(1-n_{k'}^-)S_n^- S_m^+ +$$

$$+ n_k^-(1-n_{k'}^+)S_n^+ S_m^- + [n_k^+(1-n_{k'}^+) + n_k^-(1-n_{k'}^-)]S_n^z S_m^z\} \tag{1}$$

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where:

$n_k^{\pm}$ — partition functions for the electrons with quasi-momentum $\boldsymbol{k}$ and spin $\sigma = \pm\frac{1}{2}$ (equal 0 or 1)

E_k — energy of the electron with a quasi-momentum $\boldsymbol{k}$

$J(kk')$ — s–d exchange integral

$\boldsymbol{R}_{nm}$ — radius vector between lattice sites n and m

N — number of electrons

$S_n^{\pm} = S_n^x \pm iS_n^y$

S_n^x, S_n^y, S_n^z — components of the total spin of the ion in the lattice site n.

The terms such that $E_k = E_{k'}$ should be omitted in (1), according to the usual perturbational procedure. Hence, in the limit $V \to \infty$, the principal values of singular integrals should be taken.

In temperatures near the absolute zero (*i. e.* $kT \ll E_F$)

$$n_k^+ = 1 \text{ for } E_F \geqslant E_k^+ \qquad n_{k'}^+ = 0 \text{ for } E_F < E_{k'}^+$$

$$n_k^- = 1 \text{ for } E_F \geqslant E_k^- \qquad n_{k'}^- = 0 \text{ for } E_F < E_{k'}^- \tag{2}$$

where E_F is the Fermi energy.

Energy $E_k^{\pm}$ of the electron with quasi-momentum $\boldsymbol{k}$ in the magnetic field H directed along the quantization axis is given by the formula [3]:

$$E_k^{\pm} = E_k - sJ(\boldsymbol{kk}) \pm \mu_0 H \pm J(\boldsymbol{kk})\frac{1}{N}\sum_n S_n^z \tag{3}$$

where:

$sJ(\boldsymbol{kk})$ — is the term leading to identical shift of the conduction electrons energy, regardless of their spins

μ_0 — magnetic moment of the electron

$J(\boldsymbol{kk})\frac{1}{N}\sum_n S_n^z$ — diagonal part of the s–d interaction Hamiltonian.

Assuming $J(\boldsymbol{kk}) = J_0$, after substituting $E_k = k^2/2m$, $E_{k'} = k'^2/2m$, $E_F = k_0^2/2m$, $2msJ_0 = \xi$, $2m\left(\mu_0 H + \frac{J_0}{N}\sum_n S_n^z\right) = \eta$, and taking into account the conditions for k and k' following from (2), and denoting $k_0^{\pm} = \sqrt{k_0^2 + \xi \pm \eta}$, we get instead of (1):

$$H_{\text{ef}} = \frac{2mJ_0^2}{N^2}\sum_{nm}\left\{\sum_{k\leqslant k_0^-}\sum_{k'>k_0^+}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^-S_m^+ + \sum_{k\leqslant k_0^+}\sum_{k'>k_0^-}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^+S_m^- + \right.$$

$$\left. + \left[\sum_{k\leqslant k_0^-}\sum_{k'>k_0^-}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2} + \sum_{k\leqslant k_0^+}\sum_{k'>k_0^+}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}\right]S_n^zS_m^z\right\}. \tag{4}$$

The restrictions on sums over k' can be withdrawn, as the sums appearing in square bracket in (4) can be rewritten in the following way:

$$\left\{\sum_{k\leqslant k_0^-}\sum_{k'}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^-S_m^+ - \sum_{k\leqslant k_0^-}\sum_{k'\leqslant k_0^+}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^-S_m^+ +\right.$$
$$+\sum_{k\leqslant k_0^+}\sum_{k'}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^+S_m^- - \sum_{k\leqslant k_0^+}\sum_{k'\leqslant k_0^-}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^+S_m^- +$$
$$+\sum_{k\leqslant k_0^-}\sum_{k'}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^zS_m^z - \sum_{k\leqslant k_0^-}\sum_{k'\leqslant k_0^-}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^zS_m^z +$$
$$\left.+\sum_{k\leqslant k_0^+}\sum_{k'}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^zS_m^z - \sum_{k\leqslant k_0^+}\sum_{k'\leqslant k_0^+}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}S_n^zS_m^z\right\} \tag{5}$$

whereas

$$\sum_{k\leqslant k_0^-}\sum_{k'\leqslant k_0^-}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2} = 0,\quad \sum_{k\leqslant k_0^+}\sum_{k'\leqslant k_0^+}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2} = 0,$$
$$\sum_{k\leqslant k_0^-}\sum_{k'\leqslant k_0^+}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2} = -\sum_{k\leqslant k_0^+}\sum_{k'\leqslant k_0^-}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}. \tag{6}$$

In deriving the above formulae we have exchanged variables under sums. It should be also noted that the expression in the nominator is, after integrating over angular variables, invariant with respect to the change of variables.

Upon expressing $S_n^{\pm}$ and $S_m^{\pm}$ by means of the x and y components, and taking into account Eqs (6) we get instead of (4):

$$H_{ef} = \frac{2mJ_0^2}{N^2}\sum_{nm}\left\{\sum_{k\leqslant k_0^-}\sum_{k'}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}(S_nS_m) + \sum_{k\leqslant k_0^+}\sum_{k'}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}(S_nS_m)\right\} +$$
$$+\frac{2mJ_0^2}{N^2}\sum_{nm}\left\{\sum_{k\leqslant k_0^+}\sum_{k'\leqslant k_0^-}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}[S_n^-S_m^+ - S_n^+S_m^-] +\right.$$
$$+\sum_{k\leqslant k_0^-}\sum_{k'}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}i[S_n^yS_m^x - S_n^xS_m^y] +$$
$$\left.+\sum_{k\leqslant k_0^+}\sum_{k'}\frac{e^{i(k-k')R_{nm}}}{k^2-k'^2}i[S_n^xS_m^y - S_n^yS_m^x]\right\}. \tag{7}$$

After summing over n and m the second term in (7) vanishes (invariance with respect to the change of summation variables), and as a result H_{ef} is defined by the first part of the expression (7).

Passing from sums to integrals we obtain:

$$H_{ef} = \frac{2mJ_0^2}{N^2} \sum_{nm} \left\{ \frac{V^2}{(2\pi)^6} \int_0^{k_0^-} d^3k \int_0^{\infty} d^3k' \frac{e^{i(k-k')R_{nm}}}{k^2-k'^2} + \right.$$

$$\left. + \frac{V^2}{(2\pi)^6} \int_0^{k_0^+} d^3k \int_0^{\infty} d^3k' \frac{e^{i(k-k')R_{nm}}}{k^2-k'^2} \right\} (S_n S_m). \tag{8}$$

Calculating the first of the integrals in (8) we get:

$$\int_0^{k_0^-} d^3k \int_0^{\infty} d^3k' \frac{e^{i(k-k')R_{nm}}}{k^2-k'^2} = 4\pi^2 \int_0^{k_0^-} k^2 dk \int_{-1}^{1} e^{ikR_{nm}\mu} d\mu \int_0^{\infty} \frac{k'^2}{k^2-k'^2} dk' \int_{-1}^{1} e^{ik'R_{nm}\mu'} d\mu' =$$

$$= -\frac{16\pi^2}{R_{nm}^2} \int_0^{k_0^-} k \sin kR_{nm} dk \int_0^{\infty} \frac{k'}{k^2-k'^2} \sin k'R_{nm} dk' =$$

$$= -\frac{16\pi^2}{R_{nm}^2} \int_0^{k_0^-} k \sin kR_{nm} dk \left[-\frac{\pi}{2} \cos kR_{nm} \right] =$$

$$= \frac{4\pi^3}{R_{nm}^2} \int_0^{k_0^-} k \sin 2kR_{nm} dk = -16\pi^3 \frac{2k_0^- R_{nm} \cos 2k_0^- R_{nm} - \sin 2k_0^- R_{nm}}{(2R_{nm})^4}. \tag{9}$$

We have taken the principial value of the integral of the function $k'(k^2-k'^2)^{-1} \cdot \sin k'R_{nm}$ over k', as in the first-order perturbation theory we neglect terms in which the energies appearing in denominators have the same value. The second integral is calcuted in the same way.

After inserting the results obtained into (8) and substituting $V = 3\pi^2 N_s \cdot k_0^{-3}$, we get finally:

$$H_{ef} = -\frac{9}{4}\pi \frac{J_0^2}{E_F} \left(\frac{N_s}{N}\right)^2 \sum_{nm} \left\{ \frac{2k_0^- R_{nm} \cos 2k_0^- R_{nm} - \sin 2k_0^- R_{nm}}{(2k_0 R_{nm})^4} + \right.$$

$$\left. + \frac{2k_0^+ R_{nm} \cos 2k_0^+ R_{nm} - \sin 2k_0^+ R_{nm}}{(2k_0 R_{nm})^4} \right\} (S_n S_m) \tag{10}$$

where

$$k_0^{\pm} = \sqrt{k_0^2+2msJ_0 \pm 2m\mu_0 H \pm \frac{J_0}{N}\sum_n S_n^z}.$$

In the field-free case ($H = 0$), Eq. (10) reduces to the formula derived by Ruderman and Kittel.

3. *Discussion*

Our result indicates a very important fact that the presence of the magnetic field does not induce anisotropy in the RKKY interaction.

To evaluate the magnitude of the correction resulting from the presence of the magnetic field, $k_0^{\pm}$ and functions of $k_0^{\pm}$ appearing in Eq. (10) should be expanded in a power series. The correction calculated in this manner, as compared with the magnitude of the RKKY interaction with out the magnetic field is of the order 3×10^{-8} and therefore negligibly small.

It does not, however, mean that considering the influence of the magnetic field on the RKKY interaction is pointless. This influence, as shown by Glasser, when taking into account certain effects, may be eularged. This problem will be further discussed.

REFERENCES

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