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ON DOMAIN STRUCTURES IN FERROMAGNETS

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The domain structure of ferromagnets is approximately determined by the authors on the basis of physical and geometrical parameters of a sample, *i. e.* by assuming a suitable Hamiltonian. The possible directions of magnetization appearing in the sample are calculated under the assumption that the domain walls have no thickness. The boundaries of domains are determined by some equations depending on the demagnetizing factors. Next it is stated that if for the domains calculated in such a way the domain walls have some thickness, then our method leads to the well-known, standard variational procedure for calculating domain parameters. The general method is illustrated by effective calculations given in some particular cases.

Introduction

This paper is concerned with the problem of determination of the domain structure in ferromagnets on the basis of physical parameters of a sample: the exchange integral, the uniaxial and cubic anisotropy constants, and the geometrical dimensions of this sample which, in particular, imply the demagnetizing factors. The domain structure is determined by the direction of magnetization in each site of the crystallographic lattice. This direction can be derived by minimizing the free energy calculated within the class of eigenstates in which the Hamiltonian is diagonal. Given physical parameters of the sample, which have been mentioned above, we do not assume the geometric form of domains, but obtain it as a result of our considerations. For the sake of simplicity we confine ourselves to the case where the absolute temperature is zero, and the sample may be regarded as a thin film. The first assumption implies that the free energy reduces to that part of the internal energy

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which corresponds to the ground energy of ferromagnets without spin deviations. The second assumption enables us to treat the problem in two dimensions since the distribution of magnetization across the film may be regarded as homogeneous.

1. The Hamiltonian of a sample

Let x_1, x_2, x_3 denote the rectangular coordinates of a point x. Consider a sample as a film being a superposition of n monoatomic layers labelled by $\nu = x_3/a$, where a can be expressed effectively by the lattice constant. Throughout the paper we confine ourselves to films so thin that the domain structure across the thickness is homogeneous. The position of an atom in the plane of a layer $x_3 = \nu a$ is given by $z = x_1 + ix_2$, where i is the imaginary unit; $z^* = x_1 - ix_2$. The easy axis of magnetization is supposed to be directed along $\mathrm{Re}\, z = 0$.

Suppose that properties of the sample in question are described by the Hamiltonian

$$H = H_e + H_a + H_d, \tag{1}$$

where H_e, H_a, and H_d denote the isotropic Heisenberg exchange term, the anisotropic uniaxial, unidirectional and cubic terms, and the demagnetizing term, respectively. The Hamiltonian is composed from terms used in various approaches to the problem (*cf. e.g.* [4, 6, 7]). More exactly,

$$H_e = -nI \sum_{\langle j, j_\nu \rangle} \sum_\alpha S_{j,\alpha} S_{j_\nu,\alpha},$$

$$H_a = -n \sum_j (K_\perp S_{j,3}^2 + K_{||} S_{j,2}^2 + K_\dagger S S_{j,2} + K_0 S^{-2} \sum_\alpha S_{j,\alpha}^4),$$

$$H_d = -n \sum_j \sum_\alpha M_{j,\alpha} S_{j,\alpha}^2.$$

Here I is the exchange integral, $K_\perp$, $K_{||}$, $K_\dagger$, and K_0 denote the anisotropy constants: uniaxial perpendicular to the sample, uniaxial parallel with the sample, unidirectional, and cubic, respectively. Further, $M_{j,\alpha}$ is the classical demagnetizing factor corresponding to the α-component at the atom z_j, S is the value of spin, and $S_{j,\alpha}$ resp. $S_{j_\nu,\alpha}$ denote the α-components of the spin operator at z_j resp. z_{j_ν}. Here z_j is situated in the plane of any fixed layer, while z_{j_ν} is situated in the plane of the layer $x_3 = \nu a$.

From the quantum mechanical point of view the demagnetizing field is connected with the magnetic dipolar interactions of long range. However, this leads to repeated summation over all the sites of the sample in the term H_d. Thus, since the remaining terms of H are given in the nearest neighbours approximation, we replace the long range interactions by the effective demagnetizing field at the site considered. This demagnetizing field is supposed to be proportional to magnetization with a factor of proportionality which can be interpreted as the demagnetizing factor. The energy of demagnetizing field, introduced in such a way, was discussed in [4]. This approximation is not essential for the method presented by the authors, its aim being to simplify further calculations.

The Hamiltonian (1) is written in a coordinate system, in which the easy axis of magnetization is supposed to be directed along $\mathrm{Re}\, z = 0$. In order to find the eigenvalues of (1), for each z_j resp. z_{j_ν} we transform the spin operators into some local rectangular coordinate system (x'_α) such that the magnetization of z'_j resp. z'_{j_ν} is directed along $Re\, z' = 0$. This transformation, usually applied in the theory of screw structures (*cf. e.g.* [5]), gives at the absolute temperature zero the following results:

$$S_{j,\alpha} = S\gamma_{j,\alpha} \text{ resp. } S_{j_\nu,\alpha} = S\gamma_{j_\nu,\alpha}, \tag{2}$$

where $\gamma_{j,\alpha}$ resp. $\gamma_{j_\nu,\alpha}$ denote the direction cosines of x'_2 corresponding to z_j resp. x'_2 corresponding to z_{j_ν} with respect to the coordinate system (x_α). Formulae (2) may be checked by the standard procedure for small number of excited magnons after letting the number of magnons to decrease to zero since this corresponds to the state of the sample at the absolute temperature zero.

By (2), the energy E of the system of spins, being equal to the energy of its ground eigenstate, is given by the formula

$$E = -nS^2 \sum_j [K_\perp \gamma_{j,3}^2 + K_{||}\gamma_{j,2}^2 + K_\dagger \gamma_{j,2} + \\ + \sum_\alpha (K_0 \gamma_{j,\alpha}^4 + M_{j,\alpha}\gamma_{j,\alpha}^2 + I \sum_{\langle j_\nu \rangle} \gamma_{j,\alpha}\gamma_{j_\nu,\alpha})]. \tag{3}$$

Now, in order to find the distribution of magnetization within the sample, we have to minimize E with respect to direction cosines.

2. *Main steps of the method*

Let us suppose that the sample in question satisfies the hypotheses of Section 1. Our method consists of three steps:

(*i*) We determine approximatively the possible directions of magnetization within the sample.

(*ii*) We determine the possible boundaries of domains. The domains are defined as sets of neighbouring sites, where the magnetization is nearly constant. The magnetization vectors are found in Step (*i*). Therefore we discuss the possible domain structures within the sample.

(*iii*) We determine more exactly the possible directions of magnetization in the neighbourhood of the domain boundaries, *i.e.* we determine the domain walls.

3. *The first approximation*

In Step (*i*) we consider regions with homogeneous magnetization, which may be interpreted as magnetic domains. In the first approximation we assume that the domain walls have no thickness, the magnetization changing by jumps over the boundaries of regions in question. This assumption allows us to consider the neighbouring atoms as having the same directions of spin. An error appears only for atoms lying in a nearest neighbourhood of the boundaries, and is connected with the ratio of area of walls and domains. This error is rather small and will be eliminated in the second approximation.

Owing to the first approximation the formula (3) becomes

$$E = -nS^2 \iint_D [K_\perp \gamma_3^2 + K_\parallel \gamma_2^2 + K_\dagger \gamma_2 + \sum_\alpha (K_0 \gamma_\alpha^4 + M_\alpha \gamma_\alpha^2 + \mathfrak{z} I \gamma_\alpha^2)] d\sigma, \tag{4}$$

where D denotes the section of the sample by the z-plane, $d\sigma$ — the area element, $\mathfrak{z}$ — the number of nearest neighbours of an atom in the film, while M_α and γ_α — step functions of z and z^* corresponding to $M_{j,\alpha}$ and $\gamma_{j,\alpha}$, respectively, z_j ranging over all atoms in the z-plane. Since $\gamma_3^2 = 1 - \gamma_1^2 - \gamma_2^2$, formula (4) yields

$$E = -nS^2 \iint_D \overline{E} d\sigma, \tag{5}$$

where

$$\overline{E} = s_0 + K_\dagger \gamma_2 + (\mathrm{Re}\, s - 2K_0)\gamma_1^2 + (\mathrm{Im}\, s - 2K_0)\gamma_2^2 + 2K_0(\gamma_1^4 + \gamma_1^2\gamma_2^2 + \gamma_2^4),$$

$$s_0 = \mathfrak{z} I + M_3 + K_0 - K_\perp, \quad s = M_1 - M_3 - K_\perp + i(M_2 - M_3 + K_\parallel - K_\perp).$$

For better illustration of our method we confine ourselves in this section to the case where $K_0 = 0$ and $K_\dagger = 0$. A more general discussion will appear in the two forthcoming sections.

Under the above hypotheses

$$\overline{E} = s_0 + (\mathrm{Re}\, s)\gamma_1^2 + (\mathrm{Im}\, s)\gamma_2^2.$$

Now we have the following possibilities:

(A) γ_1 is arbitrary such that $\gamma_1^2 + \gamma_2^2 \neq 1$ for $\mathrm{Re}\, s = 0$, while $\gamma_1 = 0$ for $\mathrm{Re}\, s \neq 0$; γ_2 is arbitrary (such that $\gamma_1^2 + \gamma_2^2 \neq 1$) for $\mathrm{Im}\, s = 0$, while $\gamma_2 = 0$ for $\mathrm{Im}\, s \neq 0$; $\overline{E}_{\max} = s_0$.

(B) $\gamma_1^2 + \gamma_2^2$; γ_2 is arbitrary such that $\gamma_2 \neq 1, -1$ for $\mathrm{Re}\, s = \mathrm{Im}\, s$, while $\gamma_2 = 0$ for $\mathrm{Re}\, s \neq \neq \mathrm{Im}\, s$; $\overline{E}_{\max} = s_0 + \mathrm{Re}\, s$.

(C) $\gamma_1 = 0$; $\gamma_2 = 1$ and -1; $\overline{E}_{\max} = s_0 + \mathrm{Im}\, s$.

An easy calculation gives the result that the regions in the s-plane, corresponding to the cases (A), (B), and (C), are determined by the inequalities $-\pi \leqslant -\arg(-s) \leqslant -\frac{1}{2}\pi$, $-\frac{1}{2}\pi \leqslant \arg s \leqslant \frac{1}{4}\pi$, and $\frac{1}{4}\pi \leqslant \arg s \leqslant \pi$, respectively, where we include the point $s = 0$ to each of the above regions.

Our discussion implies that the only possible directions of magnetization within a magnetic domain in the case in question are:

(a) perpendicular to the easy axis of magnetization in the direction parallel or antiparallel with the normal to the section of the sample by the z-plane,

(b) perpendicular to the same axis in the direction parallel or antiparallel with the section of the sample by the z-plane,

(c) parallel or antiparallel with the same axis.

The other directions of magnetization obtained above correspond to boundaries of the magnetic domains and this shows the self-consistency of our assumption that the magnetization changes by jumps over the boundaries of regions in question.

4. *Films without cubic anisotropy*

Now we consider the more general case where the cubic anisotropy constant K_0 vanishes and no restrictions for the unidirectional anisotropy constant $K_\dagger$ are made. Then the energy E of the system of spins is given by formula (5), where

$$\overline{E} = s_0 + K_\dagger \gamma_2 + (\mathrm{Re}\, s)\gamma_1^2 + (\mathrm{Im}\, s)\gamma_2^2,$$

$$s_0 = \mathfrak{z}I + M_3 + K_\perp, \quad s = M_1 - M_3 - K_\perp + i(M_2 - M_3 + K_{||} - K_\perp).$$

We shall consider, separately, two cases: $K_\dagger \geqslant 0$ and $K_\dagger < 0$.

In the case where $K_\dagger \geqslant 0$ we have the following possibilities:

1.1. γ_1 is arbitrary such that $\gamma_1^2+\gamma_2^2 \neq 1$ for $\mathrm{Re}\, s = 0$, while $\gamma_1 = 0$ for $\mathrm{Re}\, s \neq 0$; γ_2 is arbitrary (such that $\gamma_1^2+\gamma_2^2 \neq 1$) for $\mathrm{Im}\, s = 0$ and $K_\dagger = 0$, while $\gamma_2 = -\frac{1}{2}K_\dagger/\mathrm{Im}\, s \neq 1$, -1 for $\mathrm{Im}\, s \neq 0$ or $K_\dagger \neq 0$; $\overline{E}_{\max} = E_{11} \equiv s_0 - \frac{1}{4}K_\dagger^2/\mathrm{Im}\, s$. This possibility may hold in the case where $|\mathrm{Im}\, s| \geqslant \frac{1}{2}K_\dagger$ only. If $K_\dagger \neq 0$ this restriction may be replaced by $|\mathrm{Im}\, s| > \frac{1}{2}K_\dagger$.

1.2. $\gamma_1^2+\gamma_2^2 = 1$; γ_2 is arbitrary such that $\gamma_2 \neq 1, -1$ for $\mathrm{Re}\, s = \mathrm{Im}\, s$ and $K_\dagger = 0$, while $\gamma_2 = \frac{1}{2}K_\dagger/(\mathrm{Re}\, s - \mathrm{Im}\, s) \neq 1, -1$ for $\mathrm{Re}\, s \neq \mathrm{Im}\, s$ or $K_\dagger \neq 0$; $\overline{E}_{\max} = E_{12} \equiv s_0 + \mathrm{Re}\, s + \frac{1}{4}K_\dagger^2/(\mathrm{Re}\, s - \mathrm{Im}\, s)$. This possibility may hold in the case where $|\mathrm{Re}\, s - \mathrm{Im}\, s| \geqslant \frac{1}{2}K_\dagger$ only. If $K_\dagger \neq 0$ this restriction may be replaced by $|\mathrm{Re}\, s - \mathrm{Im}\, s| > \frac{1}{2}K_\dagger$.

1.3. $\gamma_1 = 0$; $\gamma_2 = 1$ and -1 for $K_\dagger = 0$, while $\gamma_2 = 1$ for $K_\dagger \neq 0$; $\overline{E}_{\max} = E_{13} \equiv s_0 + \mathrm{Im}\, s + K_\dagger$.

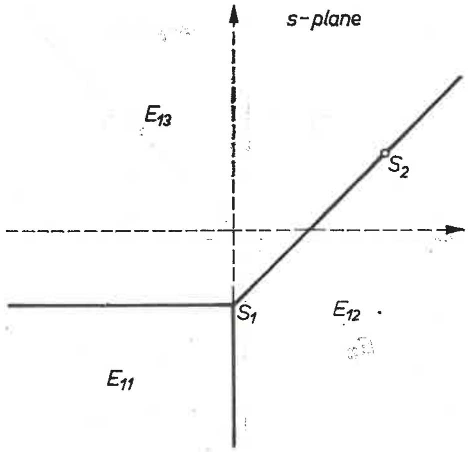

Fig. 1

An easy calculation gives the result that the regions in the s-plane, corresponding to the cases $\overline{E}_{\max} = E_{1\beta}$, $\beta = 1, 2, 3$, are such as shown in Fig. 1, where $s_1 = -\frac{1}{2}K_\dagger i$ and $s_2 = \frac{1}{2}K_\dagger(2+i)$.

In the case where $K_\dagger < 0$ we have the following possibilities:

2.1. γ_1 is arbitrary such that $\gamma_1^2+\gamma_2^2 \neq 1$ for $\mathrm{Re}\, s = 0$, while $\gamma_1 = 0$ for $\mathrm{Re}\, s \neq 0$; $\gamma_2 = -\frac{1}{2}K_\dagger/\mathrm{Im}\, s \neq 1, -1$, $\overline{E}_{\max} = E_{21} \equiv s_0 - \frac{1}{4}K_\dagger^2/\mathrm{Im}\, s$. This possibility may hold in the case where $|\mathrm{Im}\, s| > -\frac{1}{2}K_\dagger$ only.

2.2. $\gamma_1^2+\gamma_2^2=1$; $\gamma_2=\frac{1}{2}K_{\dagger}/(\mathrm{Re}\,s-\mathrm{Im}\,s)\neq 1,-1$; $\overline{E}_{\max}=E_{22}\equiv s_0+\mathrm{Re}\,s+\frac{1}{4}K_{\dagger}^2/$ $/(\mathrm{Re}\,s-\mathrm{Im}\,s)$. This possibility may hold in the case where $|\mathrm{Re}\,s-\mathrm{Im}\,s|>-\frac{1}{2}K_{\dagger}$ only.

2.3. $\gamma_1=0$; $\gamma_2=-1$; $\overline{E}_{\max}=E_{23}\equiv s_0+\mathrm{Im}\,s-K_{\dagger}$.

Obviously the analogue of Fig. 1 is obtainable by formal replacing of $\frac{1}{2}K_{\dagger}$ with $-\frac{1}{2}K_{\dagger}$ in the formulae for s_1 and s_2, and of 1β with 2β in $E_{1\beta}$, $\beta=1,2,3$.

5. *Films without the unidirectional anisotropy*

Finally, we consider the case where the unidirectional anisotropy constant $K_{\dagger}$ vanishes and no restrictions for the cubic anisotropy constant K_0 are made. Then the energy E of the system of spins is given by formula (5), where

$$\overline{E}=s_0+(\mathrm{Re}\,s-2K_0)\overline{\gamma}_1+(\mathrm{Im}\,s-2K_0)\overline{\gamma}_2+2K_0(\overline{\gamma}_1^2+\overline{\gamma}_1\overline{\gamma}_2+\overline{\gamma}_2^2),$$

$$s_0=3I+M_3+K_0+K_{\perp},\quad s=M_1-M_3-K_{\perp}+i(M_2-M_3+K_{\parallel}-K_{\perp}),$$

$$\overline{\gamma}_1=\gamma_1^2,\ \overline{\gamma}_2=\gamma_2^2.$$

Let

$$\overline{K}_0=1/24K_0.$$

We shall consider, separately, two cases: $K_0>0$ and $K_0<0$. The case $K_0=0$ has been discussed in Section 4.

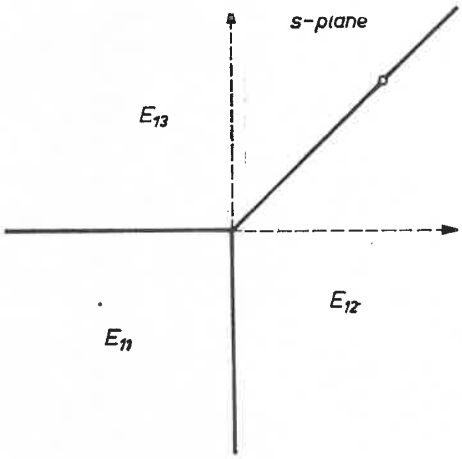

Fig. 2

In the case where $K_0>0$ we have the following possibilities:

1.1. $\overline{\gamma}_1=0$, $\overline{\gamma}_2=0$, $\overline{E}_{\max}=E_{11}\equiv s_0$.

1.2. $\overline{\gamma}_1=1$, $\overline{\gamma}_2=0$, $\overline{E}_{\max}=E_{12}\equiv s_0+\mathrm{Re}\,s$.

1.3. $\overline{\gamma}_1=0$, $\overline{\gamma}_2=1$, $\overline{E}_{\max}=E_{13}\equiv s_0+\mathrm{Im}\,s$.

The regions in the s-plane, corresponding to the cases $\overline{E}_{\max}=E_{1\beta}$, $\beta=1,2,3$, are shown in Fig. 2, where $s_1=2K_0(1+i)$.

In the case where $K_0 < 0$ we have the following possibilities:

2.1. $\overline{\gamma}_1 = 0,\ \overline{\gamma}_2 = 0,\ \overline{E}_{\max} = E_{21} \equiv s_0.$

2.2. $\overline{\gamma}_1 = 1,\ \overline{\gamma}_2 = 0,\ \overline{E}_{\max} = E_{22} \equiv s_0 + \mathrm{Re}\, s.$

2.3. $\overline{\gamma}_1 = 0,\ \overline{\gamma}_2 = 1,\ \overline{E}_{\max} = E_{23} = s_0 + \mathrm{Im}\, s.$

2.4. $\overline{\gamma}_1 = \frac{1}{2} - 6\overline{K}_0\, (\mathrm{Re}\, s - \mathrm{Im}\, s) \neq 0,\ \overline{\gamma}_2 = \frac{1}{2} + 6\overline{K}_0\, (\mathrm{Re}\, s - \mathrm{Im}\, s) \neq 0,\ \overline{E}_{\max} = E_{24} \equiv \equiv s_0 - \frac{1}{2} K_0 + \frac{1}{2} (\mathrm{Re}\, s + \mathrm{Im}\, s) - 3\overline{K}_0\, (\mathrm{Re}\, s - \mathrm{Im}\, s)^2$. This possibility may hold in the case where $|\mathrm{Re}\, s - \mathrm{Im}\, s| < -2K_0$ only.

2.5. $\overline{\gamma}_1 = 0,\ \overline{\gamma}_2 = \frac{1}{2} - 6\overline{K}_0\, \mathrm{Im}\, s \neq 0, 1,\ \overline{E}_{\max} = E_{25} \equiv s_0 - 3\overline{K}_0\, (\mathrm{Im}\, s - 2K_0)^2$. This possibility may hold in the case where $|\mathrm{Im}\, s| < -2K_0$ only.

2.6. $\overline{\gamma}_1 = \frac{1}{2} - 6\overline{K}_0\, \mathrm{Re}\, s \neq 0, 1,\ \ \overline{\gamma}_2 = 0,\ \overline{E}_{\max} = E_{26} \equiv s_0 - 3\overline{K}_0 (\mathrm{Re}\ \ s - 2K_0)^2$. This possibility may hold in the case where $|\mathrm{Re}\, s| < -2K_0$ only.

2.7. $\gamma_1 = \frac{1}{3} - 4\overline{K}_0\, (2\mathrm{Re}\, s - \mathrm{Im}\, s) \neq 0, \overline{\gamma}_2 = \frac{1}{3} + 4\overline{K}_0\, (\mathrm{Re}\, s - 2\mathrm{Im}\, s) \neq 0,\ \overline{\gamma}_1 + \overline{\gamma}_2 \neq 1,$ $\overline{E}_{\max} = E_{27} \equiv s_0 - 4\overline{K}_0\, [(\mathrm{Re}\ \ s - 2K_0)^2 - (\mathrm{Re}\ \ s - 2K_0)(\mathrm{Im}\ \ s - 2K_0) + (\mathrm{Im}\ \ s - 2K_0)^2]$. This possibility may only hold in the case where s lies in the interior of the triangle with vertices at $2K_0(1+i)$, $-2K_0$ and $2K_0 i$.

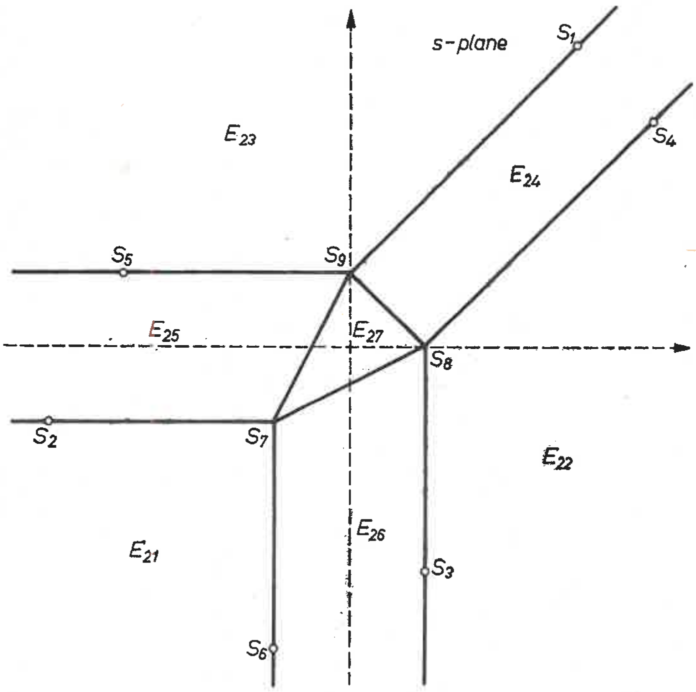

Fig. 3

It is not difficult to verify that the regions in the s-plane, corresponding to the cases $\bar{E}_{\max} = E_{2\beta}$, $\beta = 1, \ldots, 7$, are such as shown in Fig. 3, where

$$s_1 = -2K_0(3+4i),\ s_2 = 2K_0(4+i),\ s_3 = -2K_0(1-3i),$$
$$s_4 = -2K_0(4+3i),\ s_5 = 2K_0(3-i),\ s_6 = 2K_0(1+4i),$$
$$s_7 = 2K_0(1+i),\ s_8 = -2K_0,\ s_9 = 2K_0 i.$$

6. An example: influence of an applied magnetic field

To the best of our knowledge, in papers treating the domain structure of ferromagnets the possible directions of magnetization were preassigned (*cf. e.g.* [4]). Our method gives approximate but effective results concerning the problem in question, predicting not only the simple possibilities discussed in Section 3 (cases (a)–(c)), but also more complicated situations which may be observed in case of various impurities.

It is worth-while to notice that our method enables us to discuss the influence of an applied magnetic field. For instance, taking into account the results of Section 4 with $K_\dagger$ replaced by some magnetic field K, $K > 0$, applied along the easy axis of magnetization, we see that in the case where $K_{||}$ is sufficiently large (namely for $K_{||} > \max(M_1, -M_2, M_3-M_2+K_\perp)$), the vector of magnetization is parallel with the easy axis of magnetization, and the magnetic field K does not change the direction of magnetization.

Similarly, in the case where $-K_\perp$ is sufficiently large (namely for $K_\perp < \max(M_1-M_3, M_2-M_3+K_{||})$), the vector of magnetization is perpendicular to the easy axis in the direction parallel with the z-plane, and the magnetic field K turns the vector of magnetization over a normal to the z-plane to the direction of the easy axis, as shown on the following scheme: the vector of magnetization lies in the z-plane and $\gamma_2 = \frac{1}{2}K/(M_1-M_2-K_{||})$ for $K < 2(M_1-M_2-K_{||})$, while $\gamma_2 = 1$ for $K \geqslant 2(M_1-M_2-K_{||})$.

The last case corresponds to the homogeneous magnetization stimulated by an external field. Here the vector of magnetization lies in a plane perpendicular to the z-plane. This vector is not parallel with the z-plane provided the perpendicular anisotropy constant $K_\perp$ is sufficiently large (namely for $K_\perp > \max(M_1-M_3, M_2-M_3+K_{||}+\frac{1}{2}K)$). Applying some sufficiently strong field K (namely $K > 2(M_3-M_2-K_{||}+K_\perp)$) we obtain the situation where the vector of magnetization is parallel with the z-plane. If, in addition $K > 2(M_1-M_2-K_{||})$, this vector is directed along the easy axis of magnetization.

7. Configurations of domains within the sample

In Section 3 we have given an effective method of an approximate determination of the possible directions of magnetization. If $L(s, s^*, s_0, s_0^*) = 0$ is the equation of a line separating some regions of these directions in the s-plane, then

$$L(s(z, z^*),\ s^*(z, z^*),\ s_0(z, z^*),\ s_0^*(z, z^*)) = 0 \tag{6}$$

is the equation of the corresponding line in the z-plane, *i.e.* the plane of a layer in the sample. Curves of the form $z^* = f(z)$, determined as solutions of Eq. (6), represent boundaries of magnetic domains effectively. In this way one can characterize theoretically various configurations of domains that may be observed in experiments.

Eq. (6) depends on the demagnetizing factors which themselves are functions of z and z^*. The shape of these functions is connected with the distribution of magnetization, *i.e.* with the boundaries of domains, given by Eq. (6). The relation between the demagnetizing factors and the distribution of magnetization may be found *e.g.* in [2]. In this way Eq. (6) is a self-consistent equation for determining the curves $z^* = f(z)$.

In order to illustrate Step (*ii*) we present Eqs (6) in their explicit form in the case where $K_0 = 0$ and $K_\dagger = 0$; this corresponds to the example discussed in Section 3. Denote by (ab) the situation where the lines in question separate the regions corresponding to (a) and (b). Further let (bc) and (ca) have the analogous meaning. Then, by (A)–(C), we have

$$M_1(z, z^*) - M_3(z, z^*) = K_\perp, \tag{ab}$$

$$M_2(z, z^*) - M_1(z, z^*) = -K_{||}, \tag{bc}$$

$$M_3(z, z^*) - M_2(z, z^*) = K_{||} - K_\perp. \tag{ca}$$

Given some concrete values of the parameters appearing in the Hamiltonian (1), Eqs (ab), (bc), and (ca) can be solved effectively with the help of standard numerical computations. For better clarity this is illustrated in the forthcoming section by a simple example.

8. *An example: influence of the uniaxial anisotropy on the shape of the closure domains*

Consider a square thin film with the dimensions D, D, and na, where $D \approx 1.5 \cdot 10^{-2}$ cm and $na \approx 10^{-5}$ cm, characterized by the Hamiltonian (1) with

$$H_a = -n \sum_j K_{||} S_{j,2}^2.$$

Here $K_{||}$ is the parameter of the problem in question, $I \approx 10^{-14}$ erg, $g\mu_B^2/v_0 \approx 10^3$ erg/cm^3, where g is the gyromagnetic factor, μ_B is the Bohr magneton, and v_0 is the volume of an elementary cell. In this case, according to Sections 3 and 4, the only possible directions of magnetization within a magnetic domain are:

(b) perpendicular to the easy axis of magnetization in the direction parallel or antiparallel with the section of the sample by a plane parallel to the surface (*e.g.* the z-plane),

(c) parallel or antiparallel with the same axis.

According to Section 4 the lines that separate the regions corresponding to (b) and (c) satisfy Eq. (bc).

Since the numerical calculations in this case are rather complicated, we take into account the empirical fact that the shape of domain structure is such as shown in Fig. 4, where $K_{||}$ on C_1, C_2, C_3, and C_4 is a function of b. Under this hypothesis we will calculate this $K_{||}$ and show that Eq. (bc) is self-consistent.

By [2], pp. 56–59, the demagnetizing factors M_1, M_2, and M_3 at the point z of the layer $x_3 = \nu a$ are given by the formulae

$$M_1^{(k)} = c_k \int_{t_k}^{t_k'} \int_0^{na} \frac{\mathrm{Re}(z_k - z)\mathfrak{n}_1(z_k, z_k^*)}{(|z_k - z|^2 + |\tau - \nu a|^2)^{3/2}} \left| \frac{d}{dt} z_k \right| d\tau dt,$$

$$M_2^{(k)} = c_k \int_{t_k}^{t_k'} \int_0^{na} \frac{\mathrm{Im}\,(z_k - z)\mathfrak{n}_2(z_k, z_k^*)}{(|z_k - z|^2 + |\tau - \nu a|^2)^{3/2}} \left| \frac{d}{dt} z_k \right| d\tau dt,$$

$$M_3^{(k)} \approx 0.$$

Here $z = z_k(t)$, $t_k \leqslant t \leqslant t'_k$, $k = 1, 2, 3, 4$, are the boundary curves corresponding to the four magnetic domains shown in Fig. 4, and $\mathfrak{n}_\alpha(\bar{z}_k, \bar{z}_k^*)$, $\alpha = 1, 2, 3$, are the rectangular coordinates of the versor normal to $z = z_k(t)$, $t_k \leqslant t \leqslant t'_k$, at $\bar{z}_k$. The parametric equations

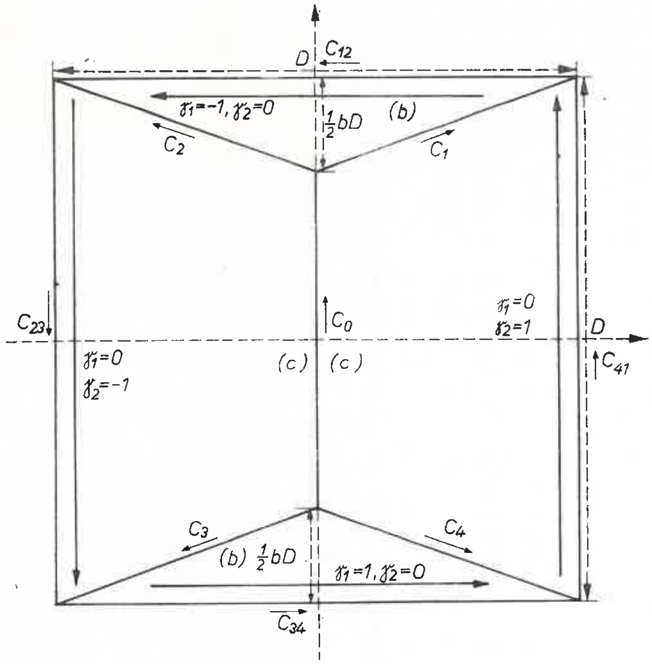

Fig. 4

of the curves in question are chosen so that these curves are oriented positively with respect to the corresponding domains. The constants c_k are determined by the conditions

$$M_1^{(k)} + M_2^{(k)} + M_3^{(k)} = g\mu_B^2/v_0.$$

After an easy calculation we get

$$M_1^{(k)} = c_k \int_{t_k}^{t'_k} \overline{M}_k \operatorname{Re}(z_k - z) \operatorname{Im} \frac{d}{dt} z_k dt$$

and

$$M_2^{(k)} = c_k \int_{s_k}^{t'_k} \overline{M}_k \operatorname{Im}(z_k - z) \operatorname{Re} \frac{d}{dt} z_k dt,$$

where

$$\overline{M}_k = a|z_k - z|^{-2}[\nu(|z_k - z|^2 + \nu^2 a^2)^{-\frac{1}{2}} + (n-\nu)(|z_k - z|^2 + (n-\nu^2)a^2)^{-\frac{1}{2}}] \approx$$

$$\approx na|z_k - z|^{-2}(|z_k - z|^2 + \tfrac{1}{4} n^2 a^2)^{\frac{1}{2}}.$$

Let now

$$I(u, v) = \log \left| \frac{(u^2+v^2+n^2a^2)^{1/2}-na}{(u^2+v^2+n^2a^2)^{1/2}+na} \right|,$$

$$J(u, v) = 2 \arctan \left(\frac{na}{u} \cdot \frac{v}{(u^2+v^2+n^2a^2)^{1/2}} \right).$$

By the symmetry with respect to the axes $\operatorname{Im} z = 0$ and $\operatorname{Re} z = 0$, we may, without any loss of generality, confine ourselves to the case where z is situated on C_1. Hence, for $z = x+iy$ of C_1, we get

$$M_1^{(1)} = c_1 \Big\{ J(D-2x, D-2y)+J(D-2x, D+2y)-J(-2x, D-bD-2y)-$$

$$-J(-2x, D-bD+2y) - \frac{b}{1+b^2} [I(D-2x, D+2y)+I(D-2x, D-2y)-$$

$$-I(-2x, D-bD+2y)-I(-2x, D-bD-2y)] + \frac{b^2}{1+b^2} \Big[J \Big(\frac{D+2y-b(D+2x)}{(1+b^2)^{1/2}},$$

$$\frac{D-2x+b(D+2y)}{(1+b^2)^{1/2}} \Big) - J \Big(\frac{D-bD+2y+2bx}{(1+b^2)^{1/2}}, \frac{-2x+b(D-bD+2y)}{(1+b)^{1/2}} \Big) \Big] \Big\},$$

$$M_2^{(1)} = c_1 \Big\{ -\frac{b}{1+b^2} [I(D-2x, D+2y)+I(D-2x, D-2y)-I(-2x, D-bD+2y) -$$

$$-I(-2x, D-bD-2y)] - \frac{1}{1+b^2} \Big[J \Big(\frac{D+2y-b(D-2x)}{(1+b^2)^{1/2}}, \frac{D-2x+b(D+2y)}{(1+b^2)^{1/2}} \Big) -$$

$$-J \Big(\frac{D-bD+2y+2bx}{(1+b^2)^{1/2}}, \frac{-2x+b(D-bD+2y)}{(1+b^2)^{1/2}} \Big) \Big] \Big\},$$

$$M_1^{(2)} = c_2 \Big\{ \frac{b}{1+b^2} [I(D-2x, D-2y)+I(D+2x, D-2y)-I(-2x, D-bD-2y)-$$

$$-I(2x, D-bD-2y)] - \frac{b^2}{1+b^2} \Big[J \Big(\frac{D-2y-b(D+2x)}{(1+b^2)^{1/2}}, \frac{D+2x+b(D-2y)}{(1+b^2)^{1/2}} \Big) -$$

$$-J \Big(\frac{D-bD-2y-2bx}{(1+b^2)^{1/2}}, \frac{2x+b(D-bD-2y)}{(1+b^2)^{1/2}} \Big) \Big] \Big\},$$

$$M_2^{(2)} = c_2 \Big\{ -J(D-2y, D+2x)-J(D-2y, D-2x) + \frac{b}{1+b^2} [I(D-2x, D-2y)+$$

$$+I(D+2x, D-2y)-I(-2x, D-bD-2y)-I(2x, D-bD-2y)]+$$

$$+\frac{1}{1+b^2} \Big[J \Big(\frac{D-2y-b(D+2x)}{(1+b^2)^{1/2}}, \frac{D+2x+b(D-2y)}{(1+b^2)^{1/2}} \Big) -$$

$$-J \Big(\frac{D-bD-2y-2bx}{(1+b^2)^{1/2}}, \frac{2x+b(D-bD-2y)}{(1+b^2)^{1/2}} \Big) \Big] \Big\};$$

$$M_1^{(3)} = c_3 \Big\{ J(D+2x,\, D+2y) + J(D+2x,\, D-2y) + J(-2x,\, D-bD-2y) +$$

$$+ J(-2x,\, D-bD+2y) - \frac{b}{1+b^2} [I(D+2x,\, D-2y) + I(D+2x,\, D+2y) -$$

$$- I(2x,\, D-bD-2y) - I(2x,\, D-bD+2y)] + \frac{b^2}{1+b^2} \Big[J\Big(\frac{D-2y-b(D+2x)}{(1+b^2)^{1/2}},$$

$$\frac{D+2x+b(D-2y)}{(1+b^2)^{1/2}} \Big) + J\Big(\frac{D+2y-b(D+2x)}{(1+b^2)^{1/2}},\, \frac{D+2x+b(D+2y)}{(1+b^2)^{1/2}} \Big) -$$

$$- J\Big(\frac{D-bD-2y-2bx)}{(1+b^2)^{1/2}},\, \frac{x+b(D-bD-2y)}{(1+b^2)^{1/2}} \Big) -$$

$$- J\Big(\frac{D-bD+2y-2bx}{(1+b^2)^{1/2}},\, \frac{x+b(D-bD+2y)}{(1+b^2)^{1/2}} \Big) \Big] \Big\},$$

$$M_2^{(3)} = c_3 \Big\{ - \frac{b}{1+b^2} [I(D+2x,\, D-2y) + I(D+2x,\, D+2y) - I(2x,\, D-bD-2y) -$$

$$- I(2x,\, D-bD+2y)] - \frac{1}{1+b^2} \Big[J\Big(\frac{D-2y-b(D+2x)}{(1+b^2)^{1/2}},\, \frac{D+2x+b(D-2y)}{(1+b^2)^{1/2}} \Big) +$$

$$+ J\Big(\frac{D+2y-b(D+2x)}{(1+b^2)^{1/2}},\, \frac{D+2x+b(D-2y)}{(1+b^2)^{1/2}} \Big) - J\Big(\frac{D-bD-2y-2bx}{(1+b^2)^{1/2}},$$

$$\frac{2x+b(D-bD-2y)}{(1+b^2)^{1/2}} \Big) - J\Big(\frac{D-bD+2y-2bx}{(1+b^2)^{1/2}},\, \frac{2x+b(D-bD+2y)}{(1+b^2)^{1/2}} \Big) \Big] \Big\},$$

$$M_1^{(4)} = c_4 \Big\{ \frac{b}{1+b^2} [I(D+2x,\, D+2y) + I(D-2x,\, D+2y) - I(2x,\, D-bD+2y) -$$

$$- I(-2x,\, D-bD+2y)] - \frac{b^2}{1+b^2} \Big[J\Big(\frac{D+2y-b(D+2x)}{(1+b^2)^{1/2}},\, \frac{D+2x+b(D+2y)}{(1+b^2)^{1/2}} \Big) +$$

$$+ J\Big(\frac{D+2y-b(D-2x)}{(1+b^2)^{1/2}},\, \frac{D-2x+b(D+2y)}{(1+b^2)^{1/2}} \Big) - J\Big(\frac{D-bD+2y-2bx}{(1+b^2)^{1/2}},$$

$$\frac{2x+b(D-bD+2y)}{(1+b^2)^{1/2}} \Big) - J\Big(\frac{D-bD+2y+2bx}{(1+b^2)^{1/2}},\, \frac{-2x+b(D-bD+2y)}{(1+b^2)^{1/2}} \Big) \Big] \Big\},$$

$$M_2^{(4)} = c_4 \Big\{ -J(D+2y,\, D-2x) - J(D+2y,\, D+2x) + \frac{b}{1+b^2} [I(D+2x,\, D+2y) +$$

$$+ I(D-2x,\, D+2y) - I(2x,\, D-bD+2y) - I(-2x,\, D-bD+2y)] +$$

$$+ \frac{1}{1+b^2} \Big[J\Big(\frac{D+2y-b(D+2x)}{(1+b^2)^{1/2}},\, \frac{D+2x+b(D+2y)}{(1+b^2)^{1/2}} \Big) + J\Big(\frac{D+2y-b(D-2x)}{(1+b^2)^{1/2}},$$

$$\frac{D-2x+b(D+2y)}{(1+b^2)^{1/2}} \Big) - J\Big(\frac{D-bD+2y-2bx}{(1+b^2)^{1/2}},\, \frac{2x+b(D-bD+2y)}{(1+b^2)^{1/2}} \Big) -$$

$$- J\Big(\frac{D-bD+2y+2bx}{(1+b^2)^{1/2}},\, \frac{-2x+b(D-bD+2y)}{(1+b^2)^{1/2}} \Big) \Big] \Big\}.$$

Analogously, for z' of the domain bounded by the curve $C_4+C_{41}-C_1-C_0$ we get

$$\lim_{z'\to z} M_1^{(1)} = (\lim_{z'\to z} c_1)\left\{\frac{1}{c_1} M_1^{(1)} + \frac{b^2}{1+b^2}\left[J\left(\frac{D-2y-b(D-2x)}{(1+b^2)^{1/2}},\right.\right.\right.$$

$$\left.\left.\left.\frac{D-2x+b(D-2y)}{(1+b^2)^{1/2}}\right) - J\left(\frac{D-bD-2y+2bx}{(1+b^2)^{1/2}}, \frac{-2x+b(D-bD-2y)}{(1+b^2)^{1/2}}\right)\right]\right\} =$$

$$= (\lim_{z'\to z} c_1)\left\{\frac{1}{c_1} M_1^{(1)} + \frac{2\pi b^2}{1+b^2}\right\}$$

and

$$\lim_{z'\to z} M_2^{(1)} = (\lim_{z'\to z} c_1)\left\{\frac{1}{c_1} M_2^{(1)} - \frac{2\pi}{1+b^2}\right\},$$

while for z' of the domain bounded by the curve $C_1+C_{12}-C_2$ we get

$$\lim_{z'\to z} M_1^{(2)} = (\lim_{z'\to z} c_2)\left\{\frac{1}{c_2} M_1^{(2)} + \frac{2\pi b^2}{1+b^2}\right\}$$

and

$$\lim_{z'\to z} M_2^{(2)} = (\lim_{z'\to z} c_2)\left\{\frac{1}{c_2} M_2^{(2)} - \frac{2\pi}{1+b^2}\right\}.$$

Here we recall that, since z belongs to C_1, we have

$$y = bx + \tfrac{1}{2}(1-b)D. \tag{7}$$

Next, since we evaluate $K_{||}$ for z of C_1 which, in fact, is a domain wall, in order to get the more appropriate value we have to take the average value of the demagnetizing factors for which C_1 is a line of discontinuity:

$$M_{\alpha\mathrm{av}}^{(k)}(z) = \lim_{z'\to z}\left\{\left[\frac{M_\alpha^{(k)}(z)}{c_k(z)} + \frac{M_\alpha^{(k)}(z')}{c_k(z')}\right] \Big/ \left[\frac{1}{c_k(z)} + \frac{1}{c_k(z')}\right]\right\},$$

where $\alpha = 1, 2$; $k = 1, 2$, and z' tends to z within the domain bounded by the curve

$$C_4+C_{41}-C_1-C_0 \quad \text{for} \quad k=1$$

and

$$C_1+C_{12}-C_2 \quad \text{for} \quad k=2.$$

Finally we can check by direct calculation that the contribution of $M_1^{(3)}$, $M_2^{(3)}$, $M_1^{(4)}$, and $M_2^{(4)}$ to the $K_{||}$, evaluated at some z of C_1, is very small. Consequently,

$$K_{||} \approx M_{1\mathrm{av}}^{(2)} - M_{2\mathrm{av}}^{(1)} = \frac{g\mu_B^2}{v_0}\left[\frac{A_2b-B_2b^2}{B_2-C_2+2A_2b-(B_2+C_2)b^2} - \frac{B_1+A_1b}{B_1-C_1+2A_1b-(B_1+C_1)b^2}\right],$$

where

$$A_1 = I(D-2x, D-2y)+I(D-2x, D+2y) -$$
$$-I(-2x, D-bD-2y)-I(-2x, D-bD+2y),$$
$$A_2 = I(D-2x, D-2y)+I(D+2x, D-2y) -$$
$$-I(-2x, D-bD-2y)-I(2x, D-bD-2y),$$
$$B_1 = \pi+J\left(\frac{D+2y-b(D-2x)}{(1+b^2)^{1/2}}, \frac{D-2x+b(D+2y)}{(1+b^2)^{1/2}}\right)-$$
$$-J\left(\frac{D-bD+2y+2bx}{(1+b^2)^{1/2}}, \frac{-2x+b(D-bD+2y)}{(1+b^2)^{1/2}}\right),$$
$$B_2 = -\pi+J\left(\frac{D-2y-b(D+2x)}{(1+b^2)^{1/2}}, \frac{D+2x+b(D-2y)}{(1+b^2)^{1/2}}\right)-$$
$$-J\left(\frac{D-bD-2y-2bx}{(1+b^2)^{1/2}}, \frac{2x+b(D-bD-2y)}{(1+b^2)^{1/2}}\right),$$
$$C_1 = J(D-2x, D-2y)+J(D-2x, D+2y)-$$
$$-J(-2x, D-bD-2y)-J(-2x, D-bD+2y),$$
$$C_2 = J(D-2y, D-2x)+J(D-2y, D+2x),$$

where y is given by (7). In particular,

$$K_{||} = 0 \text{ for } b = 1, \; z \text{ arbitrary on } C_1,$$

$$K_{||} = \frac{g\mu_B^2}{v_0} \cdot \frac{\pi+J(2D, D)}{J(D, 2D)-J(2D, D)} \approx \frac{1}{3}\sqrt{5}\,\frac{g\pi_B^2}{v_0} \cdot \frac{\pi D}{na} \approx 3.51 \cdot 10^6 \text{ erg/cm}^3$$

$$\text{for } b = 0, \; z = \tfrac{1}{2}iD \text{ and } z = \tfrac{1}{2}(1+i)D.$$

The above results show that the structure consisting of four isosceles triangles with the vertices at the centre of the sample corresponds to $K_{||} = 0$. The growth of $K_{||}$ causes that the closure domains appear in the direction of the easy axis of magnetization, decrease and, finally, vanish for some critical value of $K_{||}$, in our case for $K_{||} \approx 3.51 \cdot 10^6$ erg/cm^3. Further growth of $K_{||}$ corresponds to the stripe domain structure. In this way the above discussed simple example points out the well-known experimental facts, qualitative and quantitative as well. For instance, in the case where $K_{||} = 4.75 \cdot 10^6$ erg/cm^3 or $K_{||} = 5.28 \cdot 10^6$ erg/cm^3 and the physical parameters of the sample were of the same order as in our calculations, the stripe domain structure has been observed in [1].

9. The second approximation

Considerations concerned with the first approximation allowed us to find the magnetic domains with boundaries having no thickness. This fact was connected with the assumption for spins in their nearest neighbourhood to have the same directions. Now, in the second approximation, we also take into account a change of spin directions in a neighbourhood $\bar{C}$ of any boundary curve C of a domain, determined above in the form $z^* = f(z)$. The boundary curves of $\bar{C}$ may be expressed in the form

$$z^* = f(z) - \varepsilon_1(z), \quad z^* = f(z) + \varepsilon_2(z). \tag{8}$$

From the physical point of view the strip bounded by the curves (8) corresponds to a domain wall. In particular, if the curves (8) are parallel with C and equidistant from C, then the segment of the normal to C at any point z, with end points on (8), may be interpreted as the thickness of the domain wall in question.

In this way the problem is reduced to a form that can be investigated with the help of a usual procedure based on the variational principle (*cf. e.g.* [6] and [4]), and determining the parameters of the preassigned structure only. In our method this structure was determined in the first approximation.

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