INFLUENCE OF EXCHANGE INTERACTIONS ON THE ENERGY SPECTRUM AND MOBILITY OF THE SMALL POLARON IN FERROMAGNETIC SEMICONDUCTORS

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(Received April 22, 1971)

The influence of magnetic ordering on the dynamics and mobility of the small polaron in a crystal lattice is discussed in ferromagnetic semiconductors, in the spin waves range.

The energy level structure of an electron due to the splitting of an atomic energy level by $s-d$ exchange interaction is shown to be asymmetric, and the temperature dependence of the widths of subbands of a charge carrier varies according to the direction of its spin; the narrowing of the subbands resulting from the interaction between carrier and magnons is small, because the electron-magnon interaction is weak for physically real cases.

The mobility of the small polaron interacting with magnons is calculated by the Kohn-Luttinger density matrix formalism in Born's approximation. The conductivity of the polaron interacting with magnons exhibits band character in the temperature range discussed here.

Introduction

In recent years, considerable interest has been devoted to the study of the physical properties of a large group of chemical compounds having a magnetic ordering and exhibiting semiconducting properties *i. e.* magnetic semiconductors (for a review, see [1]).

The existence of uncompensated magnetic moments localized at crystal lattice sites of a magnetic semiconductor modifies the dynamics as well as the kinetics of the electric charge carriers.

The interaction of a carrier's spin with the magnetic moments localized at crystal sites is responsible for this modification. Until recently, only few papers have been devoted to the theoretical investigation of the physical properties of magnetic semiconductors. Thus, in the papers [1, 2], the methods [4, 5] previously derived for the small polaron have been used for studying the influence of carrier-magnon interaction on its energetic spectrum in a ferromagnetic semiconductor in the spin wave range neglecting, however, the Ising part of $s-d$ exchange interaction which, here, plays an important role. The influence

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of interaction between the carrier and the localized magnetic moments on the shift of the optical absorption edge is discussed in Ref. [6]. Haas *et al.* [7] considered the influence of exchange interactions as well as an applied magnetic field $\boldsymbol{H}$ on the broad-band electronic conductivity in systems with ferro- and antiferromagnetic orderings. Woolsey and White [8] derived contributions from exchange interactions to the energy of an electron and the energy and specific heat of magnons in a broad-band degenerate ferromagnetic semiconductor, in the spin-wave range.

Until recently, little interest has been devoted to the influence of magnetic ordering on the dynamics and conductivity of the small polaron [9, 10], which can be formed in magnetic semiconductors considering the type of their chemical bonding.

It is the aim of the present paper to discuss the influence of magnetic ordering on the energy spectrum and mobility of the small polaron in the Born approximation for a ferromagnetic semiconductor, in the range of low temperatures ($T \ll T_c$, where T_c is the Curie temperature).

1. *Hamiltonian of the system*

Let us proceed to consider a system of ferromagnetically ordered magnetic moments localized at the sites of an ionic crystal lattice with an electron (or a hole) interacting with lattice vibrations as well as with localized magnetic moments. It is our main interest to discuss the motion of the electron modified by its interaction with the localized magnetic moments of the lattice, at strong interaction with phonons. This is the case of motion of a carrier in a narrow band; thus, the state of the electron can be described by Wannier's function $|\boldsymbol{g}\rangle = \varphi(\boldsymbol{r}-\boldsymbol{R}_g)$. Let us assume, moreover, that a weak constant magnetic field $\boldsymbol{H}$, directed along the z-axis, is applied to the system; in the ground state, the localized spins are antiparallel to the z-axis.

The hamiltonian of the system takes the form:

$$\mathscr{H} = \mathscr{H}_e + \mathscr{H}_{ph} + \mathscr{H}_{eph} + \mathscr{H}_{dd} + \mathscr{H}_{sd} + \mathscr{H}_{\text{field}}, \tag{1.1}$$

where

$$\mathscr{H}_e = \frac{\boldsymbol{p}^2}{2m} + \sum_{g=1}^{\mathscr{N}} U(\boldsymbol{r}-\boldsymbol{R}_g) \tag{1.2}$$

is the sum of the kinetic and potential energy operators of the electron, $U(\boldsymbol{r}-\boldsymbol{R}_g)$ its potential energy in the field of the $\boldsymbol{g}$-th unit cell; $\mathscr{N}$ the number of unit cells of the crystal, $\boldsymbol{R}_g$ the radius vector of the position of the appropriate cation in the unit cell $\boldsymbol{g}$. The second term in Eq. (1.1) is of the form:

$$\mathscr{H}_{ph} = \sum_q \hbar\omega_q (b_q^+ b_q + \tfrac{1}{2}) \tag{1.3}$$

and is the Hamiltonian of the system of mutually non-interacting phonons; $q \equiv \boldsymbol{q}, j$, where $\boldsymbol{q}$ is the wave vector of the phonon; j labels branches of the crystal phonon spectrum, $b_q^+(b_q)$ is the creation (annihilation) operator of a phonon in the state q.

The third term in Eq. (1.1) describes the interaction between the carrier and lattice vibration, and is of the following form [11, 12]:

$$\mathscr{H}_{eph} = \sum_g \sum_q \sum_\sigma \hbar\omega_q(\gamma^*_{gq} b_q + \text{h. c.})\, a^+_{g\sigma} a_{g\sigma}; \tag{1.4}$$

$$\gamma_{gq} \equiv (2\mathscr{N})^{-1/2} \frac{V^*_q}{\hbar\omega_q} e^{i\boldsymbol{g}\cdot\boldsymbol{R}_g}, \tag{1.5}$$

γ_{gq} determines the deformation of the neighbourhood of the crystal site $\boldsymbol{g}$ due to the carrier localized there, V_q is a parameter of interaction between the carrier and the crystal lattice.

The Hamiltonian of a system of localized spins, in the approximation of nearest neighbours, can be written as follows:

$$\mathscr{H}_{dd} = L \sum_g S^z_g - 2I \sum_{g,\delta} S_g \cdot S_{g+\delta};\ (I > 0), \tag{1.6}$$

where $L \equiv g|\mu_B|H$, $\delta \equiv \boldsymbol{R}_{g+\delta} - \boldsymbol{R}_g$, g is the spectroscopic splitting factor, S_g the spin operator localized at site $\boldsymbol{g}$, μ_B the Bohr's magneton. Let us now proceed to express the Hamiltonian in terms of spin wave operators making use of the Holstein-Primakoff transformation [13]

$$S^+_g = \sqrt{2S}\, c^+_g,\ S^-_g = \sqrt{2S}\, c_g,$$

$$S^z_g = -S + c^+_g c_g; \qquad c_f = \frac{1}{\sqrt{\mathscr{N}}} \sum_\lambda e^{i\boldsymbol{\lambda}\cdot\boldsymbol{R}_f} c_\lambda. \tag{1.7}$$

It results that

$$\mathscr{H}_{dd} = E_0 + \sum_\lambda \hbar\omega^{(0)}_\lambda c^+_\lambda c_\lambda, \tag{1.8}$$

$\boldsymbol{\lambda}$ is the magnon wave vector, $E_0 \equiv -LS\mathscr{N} - z\mathscr{N} IS^2$, z the coordination number, S denotes values of a localized spin, I is the indirect exchange integral.

$$\hbar\omega^{(0)}_\lambda \equiv L + \varepsilon_\lambda, \qquad \varepsilon_\lambda = 2ISz\left(1 - \frac{1}{z}\sum_\delta e^{i\boldsymbol{\lambda}\cdot\boldsymbol{\delta}}\right). \tag{1.9}$$

The hamiltonian (1.8) describes a set of noninteracting magnons (moreover, interactions between the phonons and localized spins are also neglected [14]).

A carrier moving in the crystal lattice interacts with the magnetic moments localized at the lattice sites. Let us assume that the interaction can be expressed by an hamiltonian of the form [15]:

$$\mathscr{H}_{sd} = -2\sum_g A(\boldsymbol{r} - \boldsymbol{R}_g)\, s \cdot S_g \approx A \sum_{g,\sigma} \sigma \left(S - \frac{1}{\mathscr{N}} \sum_\lambda c^+_\lambda c_\lambda\right) a^+_{g\sigma} a_{g\sigma} -$$

$$- \sum_{g,\lambda} \hbar\omega^{(1)}_\lambda (\gamma_{g\lambda} c^+_\lambda a^+_{g\downarrow} a_{g\uparrow} + \text{h. c.}), \tag{1.10}$$

where

$$\gamma_{g\lambda} \approx \frac{A}{\hbar\omega^{(1)}_\lambda} \sqrt{\frac{2S}{\mathscr{N}}}\, e^{i\boldsymbol{\lambda}\cdot\boldsymbol{R}_g}, \qquad A = \langle 0|A(\boldsymbol{r})|0\rangle, \tag{1.10a}$$

$A(\boldsymbol{r}) = \sum_g A(\boldsymbol{r}-\boldsymbol{R}_g)$; $\sigma = +1$ for a spin oriented along the magnetization direction (↑), and $\sigma = -1$ for a spin directed oppositely to the magnetization of the system (↓).

The last term in the Hamiltonian (1.1) corresponds to interaction between the electron and the external magnetic field. This is of the form:

$$\mathscr{H}_{\text{field}} = \tfrac{1}{2}\, g \sum_{g,\sigma} |\mu_B| H\sigma a^+_{g\sigma} a_{g\sigma}. \tag{1.11}$$

Putting $E_0 = 0$, we obtain after simple algebra the Hamiltonian of the system under consideration, in the form:

$$\mathscr{H} = \sum_{g,\sigma} \varepsilon_\sigma a^+_{g\sigma} a_{g\sigma} - \sum_{g,\delta} \sum_{\sigma} V(\delta) a^+_{g+\delta,\sigma} a_{g\sigma} + \sum_q \hbar\omega_q (b^+_q b_q + \tfrac{1}{2}) +$$
$$+ \sum_\lambda \hbar\omega^{(1)}_\lambda c^+_\lambda c_\lambda - \sum_{g,\lambda} \hbar\omega^{(1)}_\lambda (\gamma_{g\lambda} c^+_\lambda a^+_{g\downarrow} a_{g\uparrow} + \text{h. c.}) + \sum_{g,q,\sigma} \hbar\omega_q (\gamma^*_{gq} b_q + \text{h. c.})\, a^+_{g\sigma} a_{g\sigma}, \tag{1.12}$$

where

$$\varepsilon_\sigma = \varepsilon + |\mu_B| H\sigma + \sigma A \left(S - \frac{1}{\mathscr{N}} \sum_\lambda c^+_\lambda c_\lambda \right), \tag{1.13}$$

$$\hbar\omega^{(1)}_\lambda = \hbar\omega^{(0)}_\lambda + A\mathscr{P}, \qquad \mathscr{P} \equiv \frac{1}{\mathscr{N}} \sum_{g,\sigma} \sigma n_{g\sigma}, \qquad \varepsilon \equiv \langle \boldsymbol{g} | H_e | \boldsymbol{g} \rangle. \tag{1.14}$$

The Hamiltonian (1.12) describes the carrier with spin at interaction with the phonons and magnons as well as with the applied magnetic field. In the present case, this interaction is not weak and, therefore, the last two terms in Eq. (1.12) cannot be treated as a perturbation, as is usually done in the theory of metals. In order to find the perturbation, we shall recur to a unitary transformation of the Hamiltonian (1.12). This will be performed in two steps. First, we shall eliminate terms containing single phonon operators; next, we shall eliminate the magnon operators. The transformation eliminating the single phonon operators is of the form [11, 12, 16]:

$$\tilde{\mathscr{H}} = \exp(-\hat{S}_1)\, \mathscr{H} \exp(\hat{S}_1) \tag{1.15}$$

where

$$\hat{S}_1 = \sum_{g,q,\sigma} (\gamma_{gq} b^+_q - \text{h. c.}) a^+_{g\sigma} a_{g\sigma}.$$

After this transformation, the Hamiltonian (1.12) takes the following form:

$$\tilde{\mathscr{H}} = \sum_{g,\sigma} \varepsilon^{(1)}_\sigma a^+_{g\sigma} a_{g\sigma} - \sum_{g,\delta,\sigma} V(\delta) \hat{B}_{g\delta} a^+_{g+\delta,\sigma} a_{g\sigma} + \sum_q \hbar\omega_q (b^+_q b_q + \tfrac{1}{2}) +$$
$$+ \sum_\lambda \hbar\omega^{(1)}_\lambda c^+_\lambda c_\lambda - \sum_{g,\lambda} \hbar\omega^{(1)}_\lambda (\gamma_{g\lambda} c^+_\lambda a^+_{g\downarrow} a^+_{g\uparrow} + \text{h. c.}) +$$
$$+ \sum_{\substack{g,f \\ (g\neq f)}} \sum_{\substack{\sigma,\sigma' \\ (\sigma\neq\sigma')}} \sum_q \hbar\omega_q \gamma^*_{fq} \gamma_{gq} a^+_{g\sigma} a^+_{f\sigma'} a_{g\sigma} a_{f\sigma'}, \tag{1.16}$$

where

$$\varepsilon_\sigma^{(1)} \equiv \varepsilon_\sigma - \varepsilon_p,\ \varepsilon_p \equiv \sum_q \hbar\omega_q |\gamma_q|^2,\ V(\delta) \equiv -\langle \boldsymbol{g}+\delta|\mathscr{H}_e|\boldsymbol{g}\rangle.$$

$$\hat{B}_{\boldsymbol{g}\delta} \equiv \exp\{\sum_q [\Delta_q(\boldsymbol{g}, \boldsymbol{g}+\delta) - \text{h. c.}]\}, \tag{1.17}$$

$$\Delta_q(\boldsymbol{g}, \boldsymbol{g}+\delta) \equiv \gamma_{\boldsymbol{g}q} - \gamma_{\boldsymbol{g}+\delta,q}. \tag{1.18}$$

The last term in Eq. (1.16) accounts for interaction between electrons localized at different lattice sites. This interaction will be neglected because we in general omit here interactions between the carriers. ε_p is the polaron shift of an atomic energy level of the carrier resulting from its interaction with the phonons: $a_{g\sigma}^+(a_{g\sigma})$ is the creation (annihilation) operator of a polaron formed by dynamical interaction between the carrier and the crystal lattice. Thus, the second term in Eq. (1.16) describes the interaction between the polaron and the phonons.

In order to diagonalize the terms with magnon operators, we shall perform the following transformation:

$$\tilde{\tilde{\mathscr{H}}} = \exp(-\hat{S}_2)\, \tilde{\mathscr{H}} \exp(\hat{S}_2), \tag{1.19}$$

where

$$\hat{S}_2 = \sum_{g,\lambda} (\Phi_{g\lambda} c_\lambda^+ a_{g\downarrow}^+ a_{g\uparrow} - \text{h. c.}) \tag{1.20}$$

and

$$\Phi_{g\lambda} = \frac{\hbar\omega_\lambda^{(1)}}{\varepsilon_\downarrow^{(1)} - \varepsilon_\uparrow^{(1)} + \hbar\omega_\lambda^{(1)}}\, \gamma_{g\lambda}. \tag{1.21}$$

The choice of the operator $\hat{S}_2$ in the form (1.20) permits the approximate diagonalization of the magnon terms in Eq. (1.16) (with accuracy to normal products of four polaron operators which are neglected), without naglecting the σ-dependence of ε_σ. Zilichikhis and Irkhin [3], when following a similar procedure with regard to the present problem, choose the operator $\hat{S}_2$ in a different form. Namely, they assumed that $\varepsilon_\sigma \simeq \varepsilon$. This assumption leads to the omission of terms by one order of magnitude larger than the beforelast term in Eq. (1.16).

In order to perform the transformation (1.19), it is most convenient to transform one by one the terms of the Hamiltonian (1.16). Thus, we see that the terms of Eq. (1.16) take the following form:

$$\sum_{g,\sigma} \varepsilon_\sigma^{(1)} e^{-\hat{S}_2} a_{g\sigma}^+ a_{g\sigma} e^{\hat{S}_2} = \sum_{g,\sigma} \varepsilon_\sigma^{(1)} a_{g\sigma}^+ a_{g\sigma} +$$
$$+\tfrac{1}{2}(\varepsilon_\downarrow^{(1)} - \varepsilon_\uparrow^{(1)}) \sum_{g,\lambda,\sigma} |\Phi_\lambda|^2 a_{g\sigma}^+ a_{g\sigma} + (\varepsilon_\downarrow^{(1)} - \varepsilon_\uparrow^{(1)}) \sum_{g,\lambda} (\Phi_{g\lambda} c_\lambda^+ a_{g\downarrow}^+ a_{g\uparrow} + \text{h. c.}) +$$
$$+\tfrac{1}{2}(\varepsilon_\downarrow^{(1)} - \varepsilon_\uparrow^{(1)}) \sum_{f,g,\lambda} (\Phi_{f\lambda}^* \Phi_{g\lambda} a_{g\downarrow}^+ a_{g\uparrow}^+ a_{f\downarrow} a_{g\uparrow} + \text{h. c.}) +$$
$$+\tfrac{1}{2}(\varepsilon_\downarrow^{(1)} - \varepsilon_\uparrow^{(1)}) \sum_{g,\sigma} \sum_{\lambda,\kappa} \sigma(\Phi_{g\lambda}^* \Phi_{g\kappa} c_\kappa^+ c_\lambda + \Phi_{g\lambda} \Phi_{g\kappa}^* c_\kappa c_\lambda^+) a_{g\sigma}^+ a_{g\sigma} + \ldots; \tag{1.22}$$

$$e^{-\hat{S}_2} \sum_{g,\delta,\sigma} V(\delta)\, \hat{B}_{g\delta} a^+_{g+\delta,\sigma} a_{g\sigma} e^{\hat{S}_2}$$

$$= \sum_{g,\delta} V(\delta)\, \hat{B}_{g\delta} \{ \hat{D}^{++}_{g\delta} a^+_{g+\delta,\uparrow} a_{g\uparrow} + \hat{D}^{--}_{g\delta} a^+_{g+\delta,\downarrow} a_{g\downarrow} +$$

$$+ \hat{D}^{+-}_{g\delta} a^+_{g+\delta,\uparrow} a_{g\downarrow} + \hat{D}^{-+}_{g\delta} a^+_{g+\delta,\downarrow} a_{g\uparrow} + \hat{Z}_{g\delta} \}, \qquad (1.23)$$

where

$$\hat{D}^{++}_{g\delta} \approx \tfrac{1}{2} \{ \mathrm{ch} \sum_{\lambda} [(e^{i\lambda\cdot\delta} - 1)\, \Phi_{g+\delta,\lambda} c^+_{\lambda} + \Phi^*_{g+\delta,\lambda} c_{\lambda}] +$$

$$+ \mathrm{ch} \sum_{\lambda} [(e^{i\lambda\cdot\delta} - 1)\, \Phi^*_{g\lambda} c_{\lambda} + \Phi_{g\lambda} c^+_{\lambda}] \} \cdot \exp[-\Delta(\delta)], \qquad (1.24a)$$

$$\hat{D}^{--}_{g\delta} = (\hat{D}^{++}_{g\delta})^+ \exp[2\Delta(\delta)], \qquad (1.24b)$$

$$\hat{D}^{+-}_{g\delta} \approx \sum_{\lambda} (e^{i\lambda\cdot\delta} - 1)\, \Phi^*_{g\lambda} c_{\lambda}, \qquad (1.24c)$$

$$\hat{D}^{-+}_{g\delta} = -(\hat{D}^{+-}_{g\delta})^+, \qquad (1.24d)$$

$$\Delta(\delta) \equiv \tfrac{1}{2} \sum_{\lambda} |\Phi_{\lambda}|^2 (1 - \cos \lambda \cdot \delta).$$

$$\hat{Z}_{g\delta} \equiv \tfrac{1}{2} \sum_{g,\lambda} [(\Phi^*_{f+\delta,\lambda} - \Phi^*_{f,\lambda})\, \Phi_{g\lambda} a^+_{g\downarrow} a^+_{f+\delta,\uparrow} a_{f\uparrow} a_{g\uparrow} + (\Phi_{f\lambda} - \Phi_{f+\delta,\lambda})\, \Phi^*_{g\lambda} a^+_{g\uparrow} a^+_{g+\delta,\downarrow} a_{f\uparrow} a_{g\downarrow}] +$$

$$+ \textit{ terms containing more than four polaron operators.} \qquad (1.25)$$

The third term of the Hamiltonian (1.16) remains unchanged in form, because

$$[\hat{S}_2, b^+_q] = [\hat{S}_2, b_q] = 0.$$

On transformation, the fourth term in Eq. (1.16) becomes:

$$\sum_{\lambda} \hbar\omega^{(1)}_{\lambda} e^{-\hat{S}_2} c^+_{\lambda} c_{\lambda} e^{\hat{S}_2} = \sum_{\lambda} \hbar\omega^{(1)}_{\lambda} c^+_{\lambda} c_{\lambda} + \sum_{g,\lambda} \hbar\omega^{(1)}_{\lambda} (\Phi_{g\lambda} c^+_{\lambda} a^+_{g\downarrow} a_{g\uparrow} + \text{h. c.}) +$$

$$+ \tfrac{1}{2} \sum_{g,\lambda} \sum_{\sigma} \hbar\omega^{(1)}_{\lambda} |\Phi_{\lambda}|^2\, a^+_{g\sigma} a_{g\sigma} +$$

$$+ \tfrac{1}{2} \sum_{g,\sigma} \sum_{\lambda,\kappa} \sigma \hbar\omega^{(1)}_{\lambda} [\Phi^*_{g\lambda} \Phi_{g\kappa} c^+_{\kappa} c_{\lambda} + \Phi_{g\lambda} \Phi^*_{g\kappa} c_{\kappa} c^+_{\lambda}]\, a^+_{g\sigma} a_{g\sigma} +$$

$$+ \tfrac{1}{2} \sum_{f,g,\lambda} \hbar\omega^{(1)}_{\lambda} [\Phi^*_{f\lambda} \Phi_{g\lambda} a^+_{g\downarrow} a^+_{f\uparrow} a_{f\downarrow} a_{g\uparrow} + \text{h. c.}] + \ldots \qquad (1.26)$$

Finally, the last term in Eq. (1.16) takes the form:

$$- \sum_{g\lambda} \hbar\omega^{(1)}_{\lambda} e^{-\hat{S}_2} (\gamma_{g\lambda} c^+_{\lambda} a^+_{g\downarrow} a_{g\uparrow} + \text{h. c.})\, e^{\hat{S}_2} = - \sum_{g,\lambda,\sigma} \hbar\omega^{(1)}_{\lambda} \gamma^*_{g\lambda} \Phi_{g\lambda} a^+_{g\sigma} a_{g\sigma} -$$

$$- \sum_{g,\lambda} \hbar\omega^{(1)}_{\lambda} (\gamma_{g\lambda} c^+_{\lambda} a^+_{g\downarrow} a_{g\uparrow} + \text{h. c.}) -$$

$$- \sum_{g,\sigma} \sum_{\kappa,\lambda} \sigma \hbar\omega^{(1)}_{\lambda} (\gamma_{g\lambda} \Phi^*_{g\kappa} c_{\kappa} c^+_{\lambda} + \gamma^*_{g\lambda} \Phi_{g\kappa} c^+_{\kappa} c_{\lambda})\, a^+_{g\sigma} a_{g\sigma} -$$

$$- \sum_{f,g,\lambda} \hbar\omega^{(1)}_{\lambda} (\gamma_{f\lambda} \Phi^*_{g\lambda} a^+_{g\uparrow} a^+_{f\downarrow} a_{f\uparrow} a_{g\downarrow} + \text{h. c.}) + \ldots \qquad (1.27)$$

Having transformed the Hamiltonian (1.16), let us now proceed to the reduction of similar terms. The sum of the following terms: the second in Eq. (1.22), the third in Eq. (1.26) and the first in Eq. (1.27), is equal to:

$$-\tfrac{1}{2}\sum_{g,\lambda}\sum_{\sigma}\hbar\omega_\lambda^{(1)}\gamma_{g\lambda}^*\Phi_{g\lambda}a_{g\sigma}^+a_{g\sigma} = -\sum_{g,\sigma}\varepsilon_m a_{g\sigma}^+a_{g\sigma};$$

$$\varepsilon_m \equiv \tfrac{1}{2}\sum_\lambda \hbar\omega_\lambda^{(1)}\gamma_{g\lambda}^*\Phi_{g\lambda} \tag{1.28}$$

The sum consisting of the third term in Eq. (1.22), the second term in Eq. (1.26) and the second term in Eq. (1.27) vanishes. The terms in Eqs (1.22), (1.23), (1.26) and (1.27) containing normal fourth-order products of Fermi operators are proportional to $1/2S$ and can be neglected for $2S \gg 1$. The sum consisting of the fifth term in Eq. (1.22), the fourth term in Eq. (1.26) and the third term in Eq. (1.27) is equal to:

$$\eta \equiv -\tfrac{1}{2}\sum_{g,\sigma}\sum_{\lambda,\kappa}\sigma\hbar\omega_\lambda^{(1)}(\gamma_{g\lambda}\Phi_{g\kappa}^*c_\kappa c_\lambda^+ + \gamma_{g\lambda}^*\Phi_{g\kappa}c_\kappa^+c_\lambda)a_{g\sigma}^+a_{g\sigma} \tag{1.29}$$

The expression (1.29) can be rewritten on the basis of the RPA approximation in the following form:

$$\eta = -\sum_{g,\sigma}\varepsilon_{m\sigma}a_{g\sigma}^+a_{g\sigma} - \sum_{g,\lambda,\sigma}\sigma\hbar\omega_\lambda^{(1)}\gamma_{g\lambda}\Phi_{g\lambda}^*\langle n_\lambda\rangle a_{g\sigma}^+a_{g\sigma} -$$
$$-\sum_{g,\lambda,\sigma}\sigma\hbar\omega_\lambda^{(1)}\gamma_{g\lambda}\Phi_{g\lambda}^*\langle n_{g\sigma}\rangle c_\lambda^+c_\lambda, \tag{1.30}$$

$$\varepsilon_{m\sigma} \equiv \tfrac{1}{2}\sum_\lambda \sigma\hbar\omega_\lambda^{(1)}\gamma_{g\lambda}^*\Phi_{g\lambda}. \tag{1.31}$$

It should be noted that the products $\gamma_{g\lambda}^*\Phi_{g\lambda} = \gamma_{g\lambda}\Phi_{g\lambda}^*$, appearing in the above equations do not depend on g. Finally, assembling all the terms derived by the canonical transformation, we obtain

$$\tilde{\tilde{\mathscr{H}}} = \mathscr{H}_0 + \mathscr{H}_{\text{int}}, \tag{1.32}$$

where

$$\mathscr{H}_0 = \sum_{g,\sigma}\mathscr{E}_\sigma a_{g\sigma}^+a_{g\sigma} + \sum_q \hbar\omega_q(b_q^+b_q + \tfrac{1}{2}) + \sum_\lambda \hbar\omega_\lambda^{(2)}c_\lambda^+c_\lambda, \tag{1.33}$$

$$\mathscr{H}_{\text{int}} = -\sum_{g,\delta}V(\delta)\hat{B}_{g\delta}\{\hat{D}_{g\delta}^{++}a_{g+\delta,\uparrow}^+a_{g\uparrow} + \hat{D}_{g\delta}^{--}a_{g+\delta,\downarrow}^+a_{g\downarrow} +$$
$$+\hat{D}_{g\delta}^{+-}a_{g+\delta,\uparrow}^+a_{g\downarrow} + \hat{D}_{g\delta}^{-+}a_{g+\delta,\downarrow}^+a_{g\uparrow}\}. \tag{1.34}$$

$$\mathscr{E}_\sigma \equiv \varepsilon_\sigma^{(1)} - \varepsilon_m - \varepsilon_{m\sigma} - \sum_\lambda \sigma\hbar\omega_\lambda^{(1)}\gamma_{g\lambda}^*\Phi_{g\lambda}\langle n_\lambda\rangle,\ \varepsilon_\sigma^{(1)} = \varepsilon_\sigma - \varepsilon_p, \tag{1.35}$$

$$\hbar\omega_\lambda^{(2)} = \hbar\omega_\lambda^{(1)}(1 - \sum_{g,\sigma}\sigma\gamma_{g\lambda}\Phi_{g\lambda}^*\langle n_{g\sigma}\rangle) = \hbar\omega_\lambda^{(1)}\left(1 + \frac{\mathscr{P}}{\hbar\omega_\lambda^{(1)}}\frac{2SA^2}{\varepsilon_\downarrow - \varepsilon_\uparrow + \hbar\omega_\lambda^{(0)}}\right). \tag{1.36}$$

Taking into account Eqs (1.28) and (1.31) one notes easily that

$$\varepsilon_m + \varepsilon_{m\sigma} = \varepsilon_m(1+\sigma); \quad \varepsilon_m \approx -\frac{SA}{2\langle S^z\rangle}, \tag{1.37}$$

$$\langle S^z\rangle = S - \frac{1}{\mathscr{N}}\sum_\lambda \langle n_\lambda\rangle.$$

Eq. (1.35) now takes the following form:

$$\mathscr{E}_\sigma = \varepsilon_\sigma^{(1)} - \varepsilon_m(1+\sigma) - \sum_\lambda \sigma\hbar\omega_\lambda^{(1)}\gamma_{g\lambda}\Phi_{g\lambda}^* \langle n_\lambda\rangle. \tag{1.38}$$

The second and third terms of the above equation are associated with the shifts in the carrier's energy levels resulting from its interaction with the magnons. The last term in Eq. (1.38) is of order of $\langle n_\lambda\rangle/\mathscr{N}$ and can be neglected in the temperature range considered. Thus, the possible energy levels of the carrier are determined by the expression:

$$\mathscr{E}_\sigma = \varepsilon_\sigma^{(1)} - 2\varepsilon_m\delta_{\sigma,\uparrow}. \tag{1.39}$$

It can be concluded from Eq. (1.39) that interaction with magnons shifts solely the energy level of the carrier with spin parallel to the magnetization of the system, by a value of $2|\varepsilon_m|$, irrespective of the sign of the $s-d$ exchange integral. It seems worth to note that the above shift takes place also for $T=0°\mathrm{K}$. At the temperature $T=0°\mathrm{K}$, a carrier with spin parallel to the localized spins does not excite the magnons and, hence, does not change its energy level. In the opposite case it excites the magnons changing the x, y-components of the localized spins at constant z-component; thus, it interacts with them, thereby changing its energy.

The spacing between the energy levels determined by Eq. (1.39) is given as:

$$|\mathscr{E}_\uparrow - \mathscr{E}_\downarrow| \approx |A|\left(2\langle S^z\rangle + \frac{S}{\langle S^z\rangle}\right). \tag{1.40}$$

2. *Energy spectrum of carriers, band narrowing, and effective mass*

Let us now proceed to transform the Hamiltonian (1.32). Adding and subtracting simultaneously the following expression [3, 11, 16]:

$$\sum_{g,\delta} V(\delta)\,\langle \hat{B}_\delta \hat{D}_\delta^{\sigma\sigma}\rangle\, a_{g+\delta,\sigma}^+ a_{g\sigma} \tag{2.1}$$

where $\langle\ldots\rangle$ denotes an averege value over the states of the Hamiltonian (1.33), we obtain

$$H = H_0 + W,$$

$$H_0 = \mathscr{H}_0 - \sum_{g,\delta}\sum_\sigma V(\delta)\,\langle\hat{B}_\delta\rangle\,\{\langle\hat{D}_\delta^{++}\rangle\delta_{\sigma,\uparrow} + \langle\hat{D}_\delta^{--}\rangle\delta_{\sigma,\downarrow}\}\, a_{g+\delta,\sigma}^+ a_{g\sigma}, \tag{2.2}$$

$$W = -\sum_{g,\delta} V(\delta)\,\{(\hat{B}_{g\delta}\hat{D}_{g\delta}^{++} - \langle\hat{B}_\delta\hat{D}_\delta^{++}\rangle)\, a_{g+\delta,\uparrow}^+ a_{g\uparrow} +$$

$$+ (\hat{B}_{g\delta}\hat{D}_{g\delta}^{--} - \langle\hat{B}_\delta\hat{D}_\delta^{--}\rangle)\, a_{g+\delta,\downarrow}^+ a_{g\downarrow} + \hat{B}_{g\delta}(\hat{D}_{g\delta}^{+-} a_{g+\delta,\uparrow}^+ a_{g\downarrow} + \hat{D}_{g\delta}^{-+} a_{g+\delta,\downarrow}^+ a_{g\uparrow})\}. \tag{2.3}$$

The average values of the operators $\hat{B}_{g\delta}$ and $\hat{D}_{g\delta}^{\sigma\sigma}$ are derived easily [11, 17]. They are:

$$\langle\hat{B}_{g\delta}\rangle = \langle\hat{B}_\delta\rangle = \exp\left[-S_T(\delta)\right]$$

$$S_T(\boldsymbol{\delta}) = \sum_q |\gamma_q|^2 (1 - \cos \boldsymbol{q}\cdot\boldsymbol{\delta})\, \mathrm{cth}\,\frac{\beta\hbar\omega_q}{2}, \quad \beta \equiv (kT)^{-1}; \tag{2.4}$$

$$\langle \hat{D}_{g\delta}^{\sigma\sigma} \rangle = \langle \hat{D}_{\delta}^{\sigma\sigma} \rangle = \exp\left[-S_T^{\sigma}(\delta)\right]$$

$$S_T^{\sigma}(\boldsymbol{\delta}) \equiv \tfrac{1}{2} \sum_{\lambda} |\Phi_\lambda|^2 (1-\cos \boldsymbol{\lambda}\cdot\boldsymbol{\delta}) \left[\sigma + \operatorname{cth} \frac{\beta\hbar\omega_\lambda^{(2)}}{2}\right]. \tag{2.5}$$

In momentum space, the Hamiltonians (2.2) and (2.3) take the forms:

$$H_0 = \sum_{\boldsymbol{k},\sigma} \mathscr{E}_\sigma(\boldsymbol{k})\, a_{\boldsymbol{k}\sigma}^{+} a_{\boldsymbol{k}\sigma} + \sum_{q} \hbar\omega_q (b_q^{+} b_q + \tfrac{1}{2}) + \sum_{\lambda} \hbar\omega_\lambda^{(2)} c_\lambda^{+} c_\lambda, \tag{2.6}$$

$$W = -\frac{1}{\mathscr{N}} \sum_{g,\lambda} \sum_{\boldsymbol{k},\boldsymbol{k}'} V(\boldsymbol{\delta}) e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot \boldsymbol{R}_g + i\boldsymbol{k}\cdot\boldsymbol{\delta}} \cdot \{(\hat{B}_{g\lambda}\hat{D}_{g\delta}^{++} - \langle \hat{B}_\delta \hat{D}_\delta^{++}\rangle) a_{\boldsymbol{k}\uparrow}^{+} a_{\boldsymbol{k}'\uparrow} +$$

$$+ (\hat{B}_{g\delta}\hat{D}_{g\delta}^{--} - \langle \hat{B}_\delta \hat{D}_\delta^{--}\rangle)\, a_{\boldsymbol{k}\downarrow}^{+} a_{\boldsymbol{k}'\downarrow} + \hat{B}_{g\delta}(\hat{D}_{g\delta}^{+-} a_{\boldsymbol{k}\uparrow}^{+} a_{\boldsymbol{k}'\downarrow} + \hat{D}_{g\delta}^{-+} a_{\boldsymbol{k}\downarrow}^{+} a_{\boldsymbol{k}'\uparrow})\}. \tag{2.7}$$

where

$$\mathscr{E}_\sigma(\boldsymbol{k}) \equiv \mathscr{E}_\sigma - \sum_{\boldsymbol{\delta}} V(\boldsymbol{\delta}) \langle \hat{B}_\delta \hat{D}_\delta^{\sigma\sigma} \rangle e^{i\boldsymbol{k}\cdot\boldsymbol{\delta}}. \tag{2.8}$$

The Hamiltonian (2.7) can now be dealt with as a perturbation in the $\boldsymbol{k}$-representation of our problem.

The scalar effective mass of the carrier is determined by the expression:

$$(m_\sigma^*)^{-1} = \hbar^{-2} \left(\frac{\partial^2}{\partial k_x^2} \mathscr{E}_\sigma(\boldsymbol{k}) \right)_{\boldsymbol{k}=0}$$

$$= \hbar^{-2} \sum_{\boldsymbol{\delta}} \delta_x^2 V(\delta) \exp\left[-S_T(\delta) - S_T^{\sigma}(\delta)\right]. \tag{2.9}$$

As seen from Eqs (2.8) and (2.9), the energy and effective mass of a carrier depend on the direction of its spin. The energy band splits into two subbands, their widths are described by the equation:

$$\Gamma(\sigma, T) = 2 \sum_{\boldsymbol{\delta}} V(\delta) \exp\left[-S_T(\delta) - S_T^{\sigma}(\delta)\right]. \tag{2.10}$$

For longitudinal optical phonons [4, 11, 12, 16]

$$S_T(\delta_z) = S_T(a) = \gamma' \operatorname{cth} \frac{\Theta}{2T}, \qquad \gamma' \equiv \frac{V_0^2}{2\hbar^2\omega_0^2}, \tag{2.11}$$

where $\Theta \equiv \dfrac{\hbar\omega_0}{k}$ — the Einstein temperature and k — the Boltzmann constant.

Let us now proceed to estimate $S_T^{\sigma}(\delta)$. We assume $\mathscr{P} \cong 0$; then $\omega_\lambda^{(2)} \cong \omega_\lambda^{(0)}$. Moreover, let us put $\boldsymbol{H} = 0$. On performing volumme integration over the first Brillouin zone instead of summation over $\boldsymbol{\lambda}$ in Eq. (2.5), and taking $|A|/2I = 10$ as a typical example we obtain the following result:

$$S_T^{\sigma}(\delta) \equiv S_T^{\sigma} = \frac{1+\sigma}{4S} \left\{ 2.61 + 8.92 \frac{\zeta(3/2)}{8S} \left(\frac{\chi T}{T_c} \right)^{3/2} + \right.$$

$$\left. + 8.92 \frac{3\pi\zeta(5/2)}{128S} \left(\frac{\chi T}{T_c} \right)^{5/2} \right\} + \frac{35\pi\zeta(5/2)}{64S} \left(\frac{\chi T}{T_c} \right)^{5/2} + 0\left(\left(\frac{I}{T_c} \right)^3 \right), \tag{2.12}$$

$$\chi \equiv (2\pi S C(S))^{-1}, \quad C(S) \equiv |I| \beta_c \text{ (see [10])},$$

$\zeta(\alpha)$ — Riemann's ζ—function. As seen from Eq. (2.12), the widths of the subbands exhibit a different shape of their temperature dependences. For the lower subband ($\downarrow$), the first term in Eq. (2.12) disappears and $S_T^{\downarrow} \sim T^{5/2}$, whereas for the higher subband ($\uparrow$), $S_T^{\uparrow} \sim$ const $+ T^{3/2} + 0\,(T^{5/2})$. The interaction between the carrier and magnons is responsible for the difference in temperature dependence of the subband widths, because the factor resulting from carrier-phonon interaction is the same for both subbands. For $S > 1$ we have $S_T^{\sigma} < 1$ and carrier-magnon interaction slightly narrows the conduction band; therefore, in this aspect, this is not a strong interaction, although $|A|/I \gg 1$, and cannot be responsible

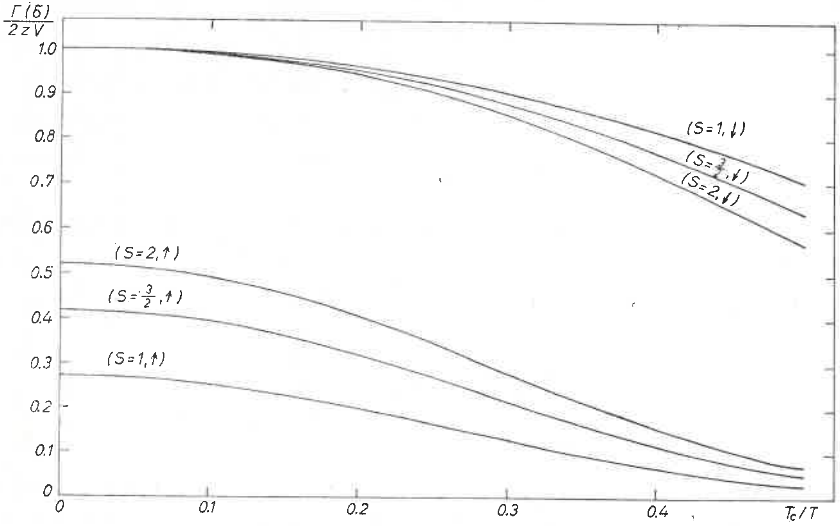

Fig. 1. Temperature-dependence of the relative subband widths for $S = 1$, 3/2 and 2 ($S_T = 0$)

for self-trapping of an electron (or a hole) and formation of a small magnetic polaron at low temperatures. The temperature dependence of the relative width of the subbands is shown in Fig. 1 for $S = 1,3/2$ and 2, where interaction with phonons is omitted.

3. *Equation of motion for the density matrix, and Boltzmann equation*

Taking into account an external electric field $\boldsymbol{F}$, we can rewrite the Hamiltonian of the system in the following form:

$$\mathcal{H}_T = \mathcal{H}_0 + \mathcal{H}_{\text{int}} + \mathcal{H}_F e^{+st}, \quad -\infty < t \leqslant 0, \tag{3.1}$$

where $\mathcal{H}_0$ and $\mathcal{H}_{\text{int}}$ are given by Eqs (1.33) and (1.34), respectively;

$$\mathcal{H}_F = e \sum_{\alpha} F_\alpha R_g^\alpha, \; (\alpha = x, y, z) \tag{3.1a}$$

stands for the carrier's energy operator in the external electric field; s — adiabatic parameter ($s \to +0$).

Let us now define a density matrix

$$\varrho_T = Z^{-1}\, e^{-\beta \mathcal{H}_T}, \; Z = \text{Tr}\, e^{-\beta \mathcal{H}_T}$$

fulfilling the following equation of motion

$$i\hbar \frac{\partial \varrho_T}{\partial t} = [\mathscr{H}_T, \varrho_T]. \tag{3.2}$$

The Hamiltonian $\mathscr{H}_{\text{int}}$ is non-diagonal in the representation of eigenstates of the operator $\mathscr{H}_0$ which we shall apply in our further considerations

$$\langle \vartheta | \mathscr{H}_{\text{int}} | \vartheta \rangle = 0, \tag{3.3}$$

$$\mathscr{H}_0 | \vartheta \rangle = E_\vartheta | \vartheta \rangle, \tag{3.4}$$

$$E_\vartheta = \mathscr{E}_\sigma + \sum_q \hbar\omega_q (N_q + \tfrac{1}{2}) + \sum_\lambda \hbar\omega_\lambda^{(2)} n_\lambda \tag{3.5}$$

$$|\vartheta\rangle \equiv |\sigma, \boldsymbol{g}, \{n\}, \{N\}\rangle = |\sigma, \boldsymbol{g}, \ldots n_\lambda \ldots, \ldots N_q \ldots\rangle = |\sigma \boldsymbol{g} n N\rangle \tag{3.4a}$$

where N_q and n_λ stand for occupation numbers of phonons and magnons in the states q and $\boldsymbol{\lambda}$, respectively. Since we are interested only in effects linear in the electric field, we can write ϱ_T, in an approximation linear with respect to this field, in the following form:

$$\varrho_T = \varrho + f e^{st} \tag{3.6}$$

where f is a time-independent correction to the density matrix linear with respect to the electric field and resulting from its action at $t = 0$ [18]. In our further considerations, f will be termed the linear reaction operator or simply, the reaction operator. On inserting Eqs (3.6) and (3.1) into Eq. (3.2) and taking into account the equation:

$$i\hbar \frac{\partial \varrho}{\partial t} = [\mathscr{H}_0 + \mathscr{H}_{\text{int}}, \varrho];$$

we obtain:

$$i\hbar s f - [\mathscr{H}_0, f] = [\mathscr{H}_{\text{int}}, f] + [\mathscr{H}_F, \varrho]. \tag{3.7}$$

Equation (3.7) determines the reaction operator. In deriving Eq. (3.7), terms of the second order with respect to the electric field $\boldsymbol{F}$ were neglected. Eq. (3.7), which holds in an arbitrary representation, will be solved in the eigen-representation of the operator $\mathscr{H}_0$. In the basis eigen-functions of the operator $\mathscr{H}_0$, Eq. (3.7) takes the form:

$$\begin{aligned} -\hbar(\omega_{\vartheta'\vartheta} - is) f_{\vartheta',\vartheta} &= eF_x (R_g^x - R_{g'}^x)\, \varrho_{\vartheta',\vartheta} + \\ &+ \sum_{\vartheta''} \{\mathscr{H}_{\text{int}\,\vartheta'',\vartheta} f_{\vartheta',\vartheta''} - f_{\vartheta'',\vartheta} \mathscr{H}_{\text{int}\,\vartheta',\vartheta''}\}, \\ \langle \vartheta | \hat{A} | \vartheta' \rangle &\equiv \hat{A}_{\vartheta'\vartheta};\ \hbar\omega_{\vartheta'\vartheta} \equiv E_\vartheta - E_{\vartheta'}. \end{aligned} \tag{3.8}$$

Let us assume that the external electric field is directed along the x-axis. The matrix elements of the perturbation operator $\mathscr{H}_{\text{int}}$ take the forms [5,3]:

$$\begin{aligned} \mathscr{H}_{\text{int}} = -\sum_{\boldsymbol{\delta}} V(\boldsymbol{\delta})\, \delta_{\boldsymbol{g},\boldsymbol{g}'+\boldsymbol{\delta}} \langle N | \hat{B}_{\boldsymbol{g}'\boldsymbol{\delta}} | N' \rangle \langle n | \hat{D}^{\sigma\sigma'}_{\boldsymbol{g}'\boldsymbol{\delta}} (\delta_{\sigma\sigma'} \delta_{\sigma,\uparrow} + \\ + \delta_{\sigma\sigma'} \delta_{\sigma,\uparrow} + \delta_{\sigma,\downarrow} \delta_{\sigma',\downarrow} + \delta_{\sigma,\downarrow} \delta_{\sigma',\uparrow}) | n \rangle, \end{aligned} \tag{3.9}$$

The matrix element (3.9), diagonal with respect to n, N and σ, after statistical averaging becomes:

$$\langle \sigma g n N | \mathscr{H}_{\text{int}} | \sigma g' n N \rangle_{\text{Av}} = -\sum_{\delta} V(\delta)\, \delta_{g,g'+\delta} \langle \hat{B}_\delta \rangle \left\{ \frac{\langle \hat{D}_\delta^{++} \rangle}{1+e^{-\beta\hbar\omega}} + \frac{\langle \hat{D}_\delta^{--} \rangle}{1+e^{+\beta\hbar\omega}} \right\}, \quad (3.10)$$

$$\hbar\omega \equiv \mathscr{E}_\downarrow - \mathscr{E}_\uparrow .$$

Eq. (3.8) will be split into two parts: a diagonal part ($n = n'$, $N = N'$, $\sigma = \sigma'$) and a non-diagonal part ($n \neq n'$, $N \neq N'$, $\sigma \neq \sigma'$).

The diagonal part of Eq. (3.8) is of the form:

$$\begin{aligned} i\hbar s f_{g'g}(\sigma n N) &= e F_x (R_g^x - R_{g'}^x)\, \varrho_{g'g}(\sigma n N) + \\ &+ \sum_{g''} \{ \mathscr{H}_{\text{int}\, \sigma g'' n N, \sigma g n N} f_{g'g''}(\sigma n N) - \\ &- f_{g''g}(\sigma n N)\, \mathscr{H}_{\text{int}\, \sigma g' n N, \sigma g'' n N} \} + \\ &+ \sideset{}{'}\sum_{\sigma'', g'', n'', N''} \{ \mathscr{H}_{\text{int}\, \sigma'' g'' n'' N'', \sigma g n N} f_{\sigma g' n N, \sigma'' g'' n'' N''} - \\ &- f_{\sigma'' g'' n'' N'', \sigma g n N} \mathscr{H}_{\text{int}\, \sigma g' n N, \sigma'' g'' n'' N''} \} . \end{aligned} \quad (3.11)$$

The sum over $N''(n'')$ denotes the product of sums with one sum for each q (λ),

$$\sideset{}{'}\sum_{\sigma'', g'', n'', N''} (\ldots) \equiv \sum_{\sigma'', g'', n'', N''} (\ldots)\, (1 - \delta_{\sigma\sigma''} \delta_{nn''} \delta_{NN''}),$$

$$\varrho_{g'g}(\sigma n N) \equiv \varrho_{\sigma g' n N, \sigma g n N};\; f_{g'g}(\sigma n N) \equiv f_{\sigma g' n N, \sigma g n N},$$

$$\omega_{\vartheta'\vartheta}(g') = \omega_{\vartheta'\vartheta}(g).$$

On insertion of Eq. (3.10) into (3.11), one obtains:

$$\begin{aligned} i\hbar s f_{g'g}(\sigma n N) &= e F_x (R_g^x - R_{g'}^x)\, \varrho_{g'g}(\sigma n N) - \\ &- \sum_{\delta} V(\delta) \langle \hat{B}_\delta \rangle \left(\frac{\langle \hat{D}_\delta^{++} \rangle}{1+e^{-\beta\hbar\omega}} + \frac{\langle \hat{D}_\delta^{--} \rangle}{1+e^{+\beta\hbar\omega}} \right) [f_{g'g-\delta}(\sigma n N) - f_{g'+\delta,g}(\sigma n N)] + \\ &+ \sideset{}{'}\sum_{\sigma'', g'', n'', N''} \{ \mathscr{H}_{\text{int}\, \sigma'' g'' n'' N'', \sigma g n N} f_{\sigma g' n N, \sigma'' g'' n'' N''} - \\ &- f_{\sigma'' g'' n'' N'', \sigma g n N} \mathscr{H}_{\text{int}\, \sigma g' n N, \sigma'' g'' n'' N''} \} . \end{aligned} \quad (3.12)$$

Analogously, after simple algebra, we obtain the following equation for the non-diagonal part of Eq. (3.10):

$$\begin{aligned} -\hbar(\omega_{\vartheta'\vartheta} - is) f_{\vartheta',\vartheta} &= e F_x (R_g^x - R_{g'}^x)\, \varrho_{\vartheta',\vartheta} + \\ &+ \sum_{g''} \{ \mathscr{H}_{\text{int}\, \sigma' g'' n' N'', \sigma g n N} f_{g'g''}(\sigma' n' N') - f_{g''g}(\sigma n N)\, H_{\text{int}\, \sigma' g' n' N', \sigma g'' n N} \} + \\ &+ \sum_{g''} \{ \mathscr{H}_{\text{int}\, \sigma g'' n N, \sigma g n N} f_{\sigma' g' n' N', \sigma g'' n N} - \\ &- f_{\sigma g'' n' N', \sigma g n N} \mathscr{H}_{\text{int}\, \sigma' g' n' N', \sigma' g'' n' N'} \} + \\ &+ \sum_{\substack{\sigma'', g'', n'' N'' \\ (\sigma'' \neq \sigma;\, n'' \neq n, n'; \\ N'' \neq N, N')}} \{ \mathscr{H}_{\text{int}\, \vartheta'', \vartheta} f_{\vartheta', \vartheta''} - f_{\vartheta'', \vartheta} \mathscr{H}_{\text{int}\, \vartheta', \vartheta''} \} . \end{aligned} \quad (3.13)$$

Kohn and Luttinger have shown [18] (see also: [5]), when solving Eq. (3.13) by the iteration method, that in the lowest approximation with respect to $\mathscr{H}_{\text{int}}$, the main part in this equation is played by the second term on its right-hand side.

Thus, we can write:

$$f_{\vartheta',\vartheta} = -\frac{1}{\hbar}\frac{1}{\omega_{\vartheta'\vartheta}-is}\sum_{g'''}\{\langle\sigma\boldsymbol{g}nN|\mathscr{H}_{\text{int}}|\sigma'\boldsymbol{g}'''n'N'\rangle f_{g'g'''}(\sigma'n'N')- -f_{g'''g}(\sigma nN)\langle\sigma\boldsymbol{g}'''nN|\mathscr{H}_{\text{int}}|\sigma'\boldsymbol{g}'n'N'\rangle\}. \quad (3.14)$$

Inserting Eq. (3.14) (on appropriately changing the indices) into Eq. (3.12), we obtain the kinetic equation — a counterpart of Boltzmann's equation, in the form [3, 5]:

$$i\hbar s f_{g'g}(\sigma nN) = eF_x(R_g^x - R_{g'}^x)\,\varrho_{g'g}(\varrho nN) - -\sum_{\delta}V(\delta)\langle\hat{B}_\delta\rangle\left\{\frac{\langle\hat{D}_\delta^{++}\rangle}{1+e^{-\beta\hbar\omega}}+\frac{\langle\hat{D}_\delta^{--}\rangle}{1+e^{\beta\hbar\omega}}\right\}[f_{g',g-\delta}(\sigma nN)-f_{g'+\delta,g}(\sigma nN)]+P-D, \quad (3.15)$$

where

$$D = D_1 + D_2;$$

$$P \equiv \frac{1}{\hbar}\sum_{\sigma'',g'',g''',n'',N''}{}' \langle\sigma\boldsymbol{g}nN|\mathscr{H}_{\text{int}}|\sigma''\boldsymbol{g}''n''N''\rangle\,\langle\sigma''\boldsymbol{g}'''n''N''|\mathscr{H}_{\text{int}}|\sigma\boldsymbol{g}'nN\rangle+\times \times\left(\frac{1}{\omega_{\vartheta\vartheta''}-is}-\frac{1}{\omega_{\vartheta\vartheta''}+is}\right)f_{g'''g''}(\sigma''n''N''), \quad (3.16)$$

$$D_1 \equiv \frac{1}{\hbar}\sum_{\sigma'',g'',g''',n'',N''}{}'\left(\frac{1}{\omega_{\vartheta''\vartheta}-is}\right)\langle\sigma\boldsymbol{g}'''nN|\mathscr{H}_{\text{int}}|\sigma''\boldsymbol{g}''n''N''\rangle\times \times\langle\sigma''\boldsymbol{g}''n''N''|\mathscr{H}_{\text{int}}|\sigma\boldsymbol{g}'nN\rangle f_{g'''g}(\sigma nN), \quad (3.17)$$

$$D_2 \equiv \frac{1}{\hbar}\sum_{\sigma'',g'',g''',n'',N''}{}'\left(\frac{1}{\omega_{\vartheta\vartheta''}-is}\right)\langle\sigma\boldsymbol{g}nN|\mathscr{H}_{\text{int}}|\sigma''\boldsymbol{g}''n''N''\rangle\times \times\langle\sigma''\boldsymbol{g}''n''N''|\mathscr{H}_{\text{int}}|\sigma\boldsymbol{g}'''nN\rangle f_{g'g'''}(\sigma nN). \quad (3.18)$$

In order to solve Eq. (3.15), we have first to calculate the expressions: (3.16), (3.17) and (3.18). Let us now proceed to obtain P. Inserting (3.9) into (3.16), we obtain

$$P = \frac{1}{\hbar}\sum_{\sigma'',n'',N''}{}'\sum_{\delta,\delta'}V(\delta)V(\delta')\langle N|\hat{B}_{g-\delta,\delta}|N''\rangle\,\langle N''|\hat{B}_{g',\delta'}|N\rangle\times \times\langle n|\hat{D}_{g-\delta,\delta}^{\sigma\sigma''}[\delta_{\sigma\sigma''}+\delta_{\sigma,\uparrow}\delta_{\sigma'',\downarrow}+\delta_{\sigma,\downarrow}\delta_{\sigma'',\uparrow}]|\,n''\rangle\times \times\langle n''|\hat{D}_{g',\delta'}^{\sigma''\sigma}[\delta_{\sigma\sigma''}+\delta_{\sigma'',\uparrow}\delta_{\sigma,\downarrow}+\delta_{\sigma'',\downarrow}\delta_{\sigma,\uparrow}]|\,n\rangle\times \times\left(\frac{1}{\omega_{\vartheta\vartheta''}-is}-\frac{1}{\omega_{\vartheta\vartheta''}+is}\right)f_{g'+\delta',g-\delta}(\sigma''n''N''). \quad (3.19)$$

The products of matrix elements appearing in Eq. (3.19) are of the form:

$$\langle N|\hat{B}_{g-\delta,\delta}|N''\rangle\langle N''|\hat{B}_{g'\delta'}|N\rangle$$

$$=\prod_q\{[1-(|\Delta_q(\boldsymbol{g}-\boldsymbol{\delta},\boldsymbol{g})|^2+|\Delta_q(\boldsymbol{g}',\boldsymbol{g}'+\boldsymbol{\delta})|^2)(N_q+\tfrac{1}{2})]\,\delta_{N_q'',N_q}-$$

$$-\Delta_q(\boldsymbol{g}-\boldsymbol{\delta},\boldsymbol{g})\,\Delta_q^*(\boldsymbol{g}',\boldsymbol{g}'+\boldsymbol{\delta}')\,N_q\delta_{N_q'',N_q-1}-\Delta_q^*(\boldsymbol{g}-\boldsymbol{\delta},\boldsymbol{g})\,\Delta_q(\boldsymbol{g}',\boldsymbol{g}'+\boldsymbol{\delta}')\,(N_q+1)\delta_{N_q'',N_q+1}\},\tag{3.20}$$

where

$$\Delta_q(\boldsymbol{g}-\boldsymbol{\delta},\boldsymbol{g})\,\Delta_q^*(\boldsymbol{g}',\boldsymbol{g}'+\boldsymbol{\delta}')=-|\gamma_q|^2\,I_p^q(\boldsymbol{g}-\boldsymbol{g}',\boldsymbol{\delta},\boldsymbol{\delta}'),$$

$$I_p^q(\boldsymbol{g}-\boldsymbol{g}',\boldsymbol{\delta},\boldsymbol{\delta}')\equiv e^{i\boldsymbol{q}\cdot(\boldsymbol{g}-\boldsymbol{g}')}(1-e^{-i\boldsymbol{q}\cdot\boldsymbol{\delta}})(1-e^{-i\boldsymbol{q}\cdot\boldsymbol{\delta}'}).\tag{3.21}$$

For the various possible cases, Eq. (3.21) takes the form:

$$I_p^q(\pm\boldsymbol{\delta},\pm\boldsymbol{\delta},\mp\boldsymbol{\delta})=2(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\cos\boldsymbol{q}\cdot\boldsymbol{\delta},$$

$$I_p^q(\mp\boldsymbol{\delta},\pm\boldsymbol{\delta},\pm\boldsymbol{\delta})=-2(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\cos 2\boldsymbol{q}\cdot\boldsymbol{\delta},$$

$$I_p^q(\pm\boldsymbol{\delta},\pm\boldsymbol{\delta},\pm\boldsymbol{\delta})=-2(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta}),$$

$$I_p^q(0,\pm\boldsymbol{\delta},\pm\boldsymbol{\delta})=-I_p^q(\boldsymbol{\delta},\boldsymbol{\delta},-\boldsymbol{\delta})$$

$$I_p^q(0,\pm\boldsymbol{\delta},\mp\boldsymbol{\delta})=-I_p^q(\boldsymbol{\delta},\boldsymbol{\delta},\boldsymbol{\delta})\tag{3.21a}$$

With regard to the one-dimensional system, Friedman [5] showed that for $\boldsymbol{g}\neq\boldsymbol{g}'$ we have $P=0$ and, therefore, a finite contribution to P stems the last two cases of (3.21a). P attains its maximum value in the last case (3.21a), because the terms in P containing $I_p^q(0,\pm\boldsymbol{\delta},\pm\boldsymbol{\delta})$ account for processes of a higher order; their contribution to P is small and will be neglected in our further considerations [5, 11]. Putting in Eq. (3.20) $\boldsymbol{\delta}'=-\boldsymbol{\delta}$ one obtains:

$$\langle N|\hat{B}_{g-\delta,\delta}|N''\rangle\langle N''|\hat{B}_{g,-\delta}|N\rangle$$

$$=\prod_q\{[1-2|\gamma_q|^2(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})(2N_q+1)]\,\delta_{N_q'',N_q}+$$

$$+2|\gamma_q|^2(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\,N_q\delta_{N_q'',N_q-1}+2|\gamma_q|^2(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\,(N_q+1)\,\delta_{N_q'',N_q+1}\}.\tag{3.22}$$

Analogously,

$$\langle n|\hat{D}^{\sigma\sigma''}_{g-\delta,\delta}[\delta_{\sigma,\sigma''}+\delta_{\sigma,\uparrow}\delta_{\sigma'',\downarrow}+\delta_{\sigma,\downarrow}\delta_{\sigma'',\uparrow}]|n''\rangle\times$$

$$\times\langle n''|\hat{D}^{\sigma''\sigma}_{g,-\delta}[\delta_{\sigma\sigma''}+\delta_{\sigma'',\uparrow}\delta_{\sigma,\downarrow}+\delta_{\sigma'',\downarrow}\delta_{\sigma,\uparrow}]|\,n\rangle$$

$$=\langle n|\hat{D}'^{++}_{g-\delta,\delta}|n''\rangle\langle n''|\hat{D}'^{++}_{g,-\delta}|n\rangle\,(e^{-\Delta(\delta)}\delta_{\sigma,\uparrow}+e^{\Delta(\delta)}\delta_{\sigma,\downarrow})\delta_{\sigma,\sigma''}+\tag{3.23}$$

$$+\langle n|\hat{D}|^{+-}_{g-\delta,\delta}|n''\rangle\langle n''|\hat{D}^{-+}_{g,-\delta}|n\rangle\,\delta_{\sigma'',\downarrow}\delta_{\sigma,\downarrow}+\tag{3.24}$$

$$+\langle n|\hat{D}^{-+}_{g-\delta,\delta}|n''\rangle\langle n''|\hat{D}^{+-}_{g,-\delta}|n\rangle\,\delta_{\sigma'',\uparrow}\delta_{\sigma,\downarrow},\tag{3.25}$$

$$\langle n|\hat{D}'^{++}_{g-\delta,\delta}|n''\rangle\langle n''|\hat{D}'^{++}_{g,-\delta}|n\rangle$$

$$=\tfrac{1}{4}\prod_\lambda\{[1-|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})(2n_\lambda+1)]\,\delta_{n_\lambda'',n_\lambda}+|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})n_\lambda\delta_{n_\lambda'',n_\lambda-1}+$$

$$+|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(n_\lambda+1)\,\delta_{n''_\lambda,n_\lambda+1}\}+$$

$$+\tfrac{1}{4}\prod_\lambda\{[1-|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(2n_\lambda+1)]\,\delta_{n''_\lambda,n_\lambda}-|\Phi_\lambda|^2\,(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,n_\lambda\delta_{n''_\lambda,n_\lambda-1}-$$

$$-|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(n_\lambda+1)\,\delta_{n''_\lambda,n_\lambda+1}\}+$$

$$+\tfrac{1}{8}\prod_\lambda\{[1-|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(2n_\lambda+1)]\delta_{n''_\lambda,n_\lambda}+2|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})n_\lambda\delta_{n''_\lambda,n_\lambda-1}+$$

$$+|\Phi_\lambda|^2(n_\lambda+1)\,\delta_{n''_\lambda,n_\lambda+1}\}+$$

$$+\tfrac{1}{8}\prod_\lambda\{[1-|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(2n_\lambda+1)]\,\delta_{n''_\lambda,n_\lambda}-2|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})n_\lambda\delta_{n''_\lambda,n_\lambda-1}-$$

$$-|\Phi_\lambda|^2(n_\lambda+1)\delta_{n''_\lambda,n_\lambda+1}\}+\tfrac{1}{8}\prod_\lambda\{[1-|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(2n_\lambda+1)]\,\delta_{n''_\lambda,n_\lambda}+$$

$$+|\Phi_\lambda|^2n_\lambda\delta_{n''_\lambda,n_\lambda-1}+2|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(n_\lambda+1)\,\delta_{n''_\lambda,n_\lambda+1}\}+$$

$$+\tfrac{1}{8}\prod_\lambda\{[1-|\Phi_\lambda|^2\,(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(2n_\lambda+1)]\delta_{n''_\lambda,n_\lambda}-|\Phi_\lambda|^2\,n_\lambda\delta_{n''_\lambda,n_\lambda-1}-$$

$$-2|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(n_\lambda+1)\,\delta_{n''_\lambda,n_\lambda+1}\}, \tag{3.23a}$$

$$\langle|\hat{D}^{+-}_{g-\delta,\,\delta}|n''\rangle\,\langle n''|\hat{D}^{-+}_{g,\,-\delta}|n\rangle=\tfrac{1}{2}\prod_\lambda\{\delta_{n''_\lambda,n_\lambda}+2|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(n_\lambda+1)\,\delta_{n''_\lambda,n_\lambda+1}\}-$$

$$-\tfrac{1}{2}\prod_\lambda\{\delta_{n''_\lambda,n_\lambda}-2|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(n_\lambda+1)\,\delta_{n''_\lambda,n_\lambda+1}\}, \tag{3.24a}$$

$$\langle n|\hat{D}^{-+}_{g-\delta,\,\delta}|n''\rangle\,\langle n''|\hat{D}^{+-}_{g,\,-\delta}|n\rangle=\tfrac{1}{2}\prod_\lambda\{\delta_{n''_\lambda,n_\lambda}+2|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,n_\lambda\delta_{n''_\lambda,n_\lambda-1}\}-$$

$$-\tfrac{1}{2}\prod_\lambda\{\delta_{n''_\lambda,n_\lambda}-2|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,n_\lambda\delta_{n''_\lambda,n_\lambda-1}\}. \tag{3.25a}$$

Let us now assume, after Friedman [5], that the matrix elements of the reaction operator $f_{g-\delta,\,g-\delta}\,(\sigma''n''N'')$ diagonal in σ, n and N can be written in the factorized form:

$$f_{g-\delta,\,g-\delta}\,(\sigma''n''N'')=f_{g-\delta,\,g-\delta}g(\sigma''n''N''), \tag{3.26}$$

where

$$g(\sigma''n''N'')=g(\sigma'')\,g(n'')\,g(N'');\,g(x'')=Z^{-1}\,e^{-\beta E_{x''}},$$
$$Z=\sum_x e^{-\beta E_x} \tag{3.27}$$

are the partition functions for the system at equilibrium. The assumption (3.26) accounts for the fact that the thermal excitations of the phonons, magnons and the carrier's spin are not associated with localization of a single extra carrier. In the non-perturbed state, with constant occupation numbers of magnons (the assumption (3.26) deals only with diagonal matrix elements), a change in direction of the carrier's spin is not possible. As regards the assumed form of the partition function (3.27), for a system at equilibrium, it should be noted that this assumption is necessary in any theory describing irreversible processes and constructed on the basis of the time-invariant equation of motion for the density matrix [5].

On resorting to the relation:

$$\frac{1}{x-is}-\frac{1}{x-is}=i\int_{-\infty}^{+\infty} dte^{ixt-s|t|} \tag{3.28}$$

we can rewrite Eq. (3.19) in the following form:

$$P=\frac{1}{\hbar}\sum_{\sigma''}\sum_{n'',N''}\sum_{\delta}V^2(\delta)f_{g-\delta,g-\delta}\int_{-\infty}^{+\infty}dte^{i\omega_{\vartheta\vartheta''}t-s|t|}\times$$

$$\times g(\sigma''n''N'')\,\langle N|\hat{B}_{g-\delta,\delta}|N''\rangle\,\langle N''|\hat{B}_{g,-\delta}|N\rangle\times$$

$$\times[\textit{matrix elements determined by (3.23)–(3.25)}]\,(1-\delta_{\vartheta'',\vartheta}), \tag{3.29}$$

where

$\delta_{\vartheta'',\vartheta}\equiv\delta_{\sigma,\sigma''}\delta_{n,n''}\delta_{N,N''}$; $\omega_{\vartheta,\vartheta''}$ does not depend on the position of a site, since it represents the difference in energies, independent of position. On insertion of (3.23a), (3.24a), (3.25a) and (3.22) into (3.29) and statistical averaging over n, N as well as resorting to the transformation: $t=\tau-\dfrac{i\beta\hbar}{2}$, one obtains

$$P=i\hbar\sum_{\delta}f_{g-\delta,\,g-\delta}W_T(\delta), \tag{3.30}$$

where

$$W_T(\delta)=\sum_{\sigma}p(\sigma)\;W_T(\delta,\sigma), \tag{3.31}$$

$$W_T(\delta,\sigma)=W_T^d(\delta,\sigma)+W_T^{nd}(\delta,\sigma), \tag{3.32}$$

$$p(\sigma)=Z_\sigma^{-1}\,e^{-\beta\mathscr{E}_\sigma},\;Z_\sigma=\sum_{\delta}e^{-\beta\mathscr{E}_\sigma}; \tag{3.33}$$

$$W_T^d(\delta,\sigma)=\frac{V^2(\delta)}{2\hbar^2}\,(e^{-2\Delta(\delta)}\delta_{\sigma,\uparrow}+e^{2\Delta(\delta)}\delta_{\sigma,\downarrow})\times$$

$$\times\exp\left\{-2\sum_q|\gamma_q|^2\,(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\,\mathrm{cth}\,\frac{\beta\hbar\omega_q}{2}-\sum_\lambda|\Phi_\lambda|^2\,(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,\mathrm{cth}\,\frac{\beta\hbar\omega_\lambda^{(2)}}{2}\right\}\times$$

$$\times\int_{-\infty+i\beta\hbar/2}^{+\infty+i\beta\hbar/2}d\tau e^{-s|t|}e^{i\beta s\hbar/2}\{e^{2\sum_q F(q)(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\cos\omega_q\tau}(\mathrm{ch}[2\sum_\lambda(F(\boldsymbol{\lambda})(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\cos\omega_\lambda\tau]+$$

$$+\mathrm{ch}\,[\sum_\lambda F(\boldsymbol{\lambda})(2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})+1)\cos\omega_\lambda\tau]\cos\,[\sum_\lambda F(\boldsymbol{\lambda})\,(2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})-1)\sin\omega_\lambda\tau])-2\}, \tag{3.34}$$

$$W_T^{nd}(\boldsymbol{\lambda},\sigma)\equiv\frac{V^2(\delta)}{\hbar^2}\exp\left\{-2\sum_q|\gamma_q|^2\,(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\,\mathrm{cth}\,\frac{\beta\hbar\omega_q}{2}\right\}\times$$

$$\times\int_{-\infty+i\beta t/2}^{+\infty+i\beta t/2}d\tau e^{-s|t|}e^{i\beta s\hbar/2}e^{2\sum_q F(q)(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\cos\omega_q\tau}\{\delta_{\sigma,\uparrow}e^{i\omega\tau+\frac{\beta\hbar\omega}{2}}\times$$

$$\times \operatorname{sh}[2\sum_{\lambda} F(\boldsymbol{\lambda})(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})e^{i\omega_\lambda\tau}] + \delta_{\sigma,\downarrow} e^{-i\omega\tau-\frac{\beta\hbar\omega}{2}} \operatorname{sh}[2\sum_{\lambda} F(\boldsymbol{\lambda})(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})e^{i\omega_\lambda\tau}]\};$$

(3.35)

$$\hbar\omega \equiv \mathscr{E}_\downarrow - \mathscr{E}_\uparrow; \qquad F(\boldsymbol{\lambda}) \equiv \frac{|\Phi_\lambda|^2}{2\operatorname{sh}\frac{\beta\hbar\omega_\lambda^{(2)}}{2}}; \qquad F(q) = \frac{|\gamma_q|^2}{\operatorname{sh}\frac{\beta\hbar\omega_q}{2}}. \tag{3.36}$$

After averaging Eqs (3.34) and (3.35) over spins, we obtain

$$W_T(\delta) = W_T^d(\delta) + W_T^{nd}(\delta), \tag{3.37}$$

where

$$W_T^d(\boldsymbol{\delta}) = \frac{1}{2}\frac{V^2(\boldsymbol{\delta})}{\hbar^2}\left(\frac{e^{-2\Delta(\boldsymbol{\lambda})}}{1+e^{-\beta\hbar\omega}} + \frac{e^{2\Delta(\boldsymbol{\delta})}}{1+e^{\beta\hbar\omega}}\right)\exp\left\{-2\sum_q |\gamma_q|^2(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\times\right.$$

$$\left.\times \operatorname{cth}\frac{\beta\hbar\omega_2}{2} - \sum_{\lambda}|\Phi_\lambda|^2(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\operatorname{cth}\frac{\beta\hbar\omega_\lambda^{(2)}}{2}\right\}(I_1(\boldsymbol{\delta})+I_2(\boldsymbol{\delta})); \tag{3.38}$$

$$I_1(\delta) = \int_{-\infty+i\beta\hbar/2}^{+\infty+i\beta\hbar/2} d\tau e^{-s|\tau|+i\beta s\hbar/2}\{e^{2\sum_q F(q)(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\cos\omega_q\tau}\operatorname{ch}[\sum_\lambda F(\lambda)(2(1-\cos\lambda\cdot\delta)+$$

$$+1)\cos\omega_\lambda^{(2)}\tau]-1\}, \tag{3.39}$$

$$I_2(\delta) = \int_{-\infty+i\beta\hbar/2}^{+\infty+i\beta\hbar/2} d\tau e^{-s|\tau|+i\beta s\hbar/2}\{e^{2\sum_q F(q)(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\cos\omega_q\tau}\operatorname{ch}[\sum_\lambda F(\lambda)(2(1-\cos\lambda\cdot\boldsymbol{\delta})-1)\cos\omega_\lambda^{(2)}\tau]\times$$

$$\times\cos[\sum_\lambda F(\lambda)(2(1-\cos\lambda\cdot\boldsymbol{\delta})-1)\sin\omega_\lambda^{(2)}\tau]-1\}. \tag{3.40}$$

$$W_T^{nd}(\boldsymbol{\delta}) = \frac{V^2(\boldsymbol{\delta})}{\hbar^2}\exp\left\{-2\sum_q|\gamma_q|^2(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\operatorname{cth}\frac{\beta\hbar\omega_q}{2}\right\}\cdot I_n(\boldsymbol{\delta}); \tag{3.41}$$

$$I_n(\boldsymbol{\delta}) = \frac{\exp(\frac{1}{2}\beta\hbar\omega)}{1+e^{-\beta t\omega}}I_3(\boldsymbol{\delta}) + \frac{\exp(-\frac{1}{2}\beta\hbar\omega)}{1+e^{\beta t\omega}}I_4(\boldsymbol{\delta}),$$

$$I_3(\delta) = \int_{-\infty+i\beta\hbar/2}^{+\infty+i\beta\hbar/2} d\tau e^{-s|\tau|+i\beta s\hbar/2}e^{2\sum_q F(q)(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\cos\omega_q\tau+i\omega\tau}\operatorname{sh}[2\sum_\lambda F(\lambda)(1-\cos\lambda\cdot\boldsymbol{\delta})e^{i\omega_\lambda^{(2)}\tau}],$$

(3.42)

$$I_4(\delta) = \int_{-\infty+i\beta\hbar/2}^{+\infty+i\beta\hbar/2} d\tau e^{-s|\tau|+i\beta s\hbar/2}e^{2\sum_q F(q)(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\cos\omega_q\tau-i\omega\tau}\operatorname{sh}[2\sum_\lambda F(\lambda)(1-\cos\lambda\cdot\boldsymbol{\delta})e^{-i\omega_\lambda^{(2)}\tau}],$$

(3.43)

The physical meaning of Eq. (3.37) is obvious $\sum_{\boldsymbol{\delta}} W_T(\delta)$ is the reciprocal relaxation time of a carrier. The reciprocal relaxation time is equal to the sum of the reciprocal relaxation times for transitions with inversion and without inversion of spin, respectively. $W_T(\delta)$ stands for the sum of probabilities of transitions without inversion of spin ($W_T^d(\delta)$) and transitions with inversion of spin ($W_T^{nd}(\delta)$).

The expressions:

$$f(\tau) = 2 \sum_q F(q)\,(1-\cos \boldsymbol{q}\cdot\boldsymbol{\delta})\cos\omega_q\tau, \text{ and}$$

$$\varphi(\tau) \equiv 2\sum_\lambda F(\lambda)\,(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\cos\omega_\lambda^{(2)}\tau \tag{3.44}$$

stand for oscillating functions of τ decreasing with growing τ as $\tau^{-\frac{3}{2}}$[4b]. In the considered temperature range $kT \ll 2IS$ we have $f(0)$ and $\varphi(0) \ll 1$, and thus, in order to calculate the integrals (3.39), (3.40), (3.42) and (3.43), we expand the sub-integral terms in power series.

Then, taking the real part in the integral (3.39) and performing integration from $-\infty$ to $+\infty$, we obtain

$$I_1(\delta) = 4\pi\sum_q F(q)\,(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\,\delta(\omega_q) + 2\pi\sum_{q,q_1} F(q)\,F(q_1)\,(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\times$$
$$\times(1-\cos\boldsymbol{q}_1\cdot\boldsymbol{\delta})\,[\delta(\omega_q-\omega_{q_1})+\delta(\omega_q+\omega_{q_1})]+$$
$$+2\pi\sum_{\lambda,\lambda_1} F(\lambda)\,F(\lambda_1)\,(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,(1-\cos\boldsymbol{\lambda}_1\cdot\boldsymbol{\delta})\,[\delta(\omega_\lambda-\omega_{\lambda_1})+\delta(\omega_\lambda+\omega_{\lambda_1})]+$$
$$+ \textit{ terms accounting for higher order processes.} \tag{3.45}$$

The first term in Eq. (3.45) is associated with a one-phonon process and vanishes by the law of energy conservation. The terms containing $\delta(\omega_\alpha+\omega_\beta)$ also vanish for the same reason (an emission or an absorption of two quanta). Thus, for the lowest non-vanishing and giving basic contribution processes we obtain the following expressions:

$$I_1(\delta)+I_2(\delta) = 2\pi\,\{2\sum_{q_1,q_2}\delta(\omega_{q_1}-\omega_{q_2})\prod_{l=1,2}F(q_l)\,(1-\cos\boldsymbol{q}_l\cdot\boldsymbol{\delta})+$$
$$+\sum_{\lambda_1\lambda_2}\delta(\omega_{\lambda_1}-\omega_{\lambda_2})\,F(\lambda_1)\,F(\lambda_2)\,[3-2\cos\boldsymbol{\lambda}_1\cdot\boldsymbol{\delta}-2\cos\boldsymbol{\lambda}_2\cdot\boldsymbol{\delta}+\cos\boldsymbol{\lambda}_1\cdot\boldsymbol{\delta}\cos\boldsymbol{\lambda}_2\cdot\boldsymbol{\delta}]\}. \tag{3.46}$$

Similarly, we calculate $I_3(\boldsymbol{\delta})$ and $I_4(\boldsymbol{\delta})$, with $|\beta\hbar\omega| \gg 1$ and $I_n(\delta) \simeq e^{\beta\hbar|\omega|/2}I_3(\delta)$ irrespective of the sign of the $s-d$ exchange integral:

$$I_3(\boldsymbol{\delta}) = 4\pi\,\{\sum_\lambda \delta(\omega_\lambda+\omega)\,F(\lambda)\,(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})+$$
$$+\sum_{q,\lambda} F(\lambda)\,F(\boldsymbol{q})\,(1-\cos\boldsymbol{q}\cdot\boldsymbol{\delta})\,(1-\cos\boldsymbol{\lambda}\cdot\boldsymbol{\delta})\,\delta(\omega_\lambda-\omega_q+\omega)\}. \tag{3.47}$$

Making use of the same method, we can derive D (see [5]). We obtain the following result:

$$D = i\hbar\sum_{\boldsymbol{\delta}}\{f_{\boldsymbol{g}'\boldsymbol{g}}(\sigma nN)\,W_T(\boldsymbol{\delta})+f_{\boldsymbol{g}+2\boldsymbol{\delta},\boldsymbol{g}'}(\sigma nN)\,W_T(-\boldsymbol{\delta})\}. \tag{3.48}$$

The second term in Eq. (3.48) stands for processes of the type: $\boldsymbol{g}nN\to\boldsymbol{g}''n''N''\to\boldsymbol{g}+2\delta$, $n''N''$ *i. e.* transitions to the second coordination zone; its contribution is small and thus can be neglected. We finally obtain:

$$D = i\hbar\sum_{\boldsymbol{\delta}} f_{\boldsymbol{g}'\boldsymbol{g}}(\sigma nN)\,W_T(\boldsymbol{\delta}). \tag{3.49}$$

Inserting (3.30) and (3.49) into (3.15), we finally obtain the Boltzmann equation determining the diagonal matrix element of the reaction operator in the following form:

$$eF_x(R_g^x - R_{g'}^x)\varrho_{g'g}(\sigma nN) - \sum_{\delta} V(\delta)\langle\hat{B}_\delta\rangle\left\{\frac{\langle\hat{D}_\delta^{++}\rangle}{1+e^{-\beta\hbar\omega}} + \frac{\langle\hat{D}_\delta^{--}\rangle}{1+e^{\beta\hbar\omega}}\right\}\times$$

$$\times\{f_{g',g-\delta}(\sigma nN) - f_{g'+\delta,g}(\sigma nN)\} + i\hbar\sum_{\delta} f_{g-\delta,g-\delta}(\sigma nN)\, W_T(\delta) -$$

$$-i\hbar\sum_{\delta} f_{g'g}(\sigma nN)\, W_T(\delta) = 0. \tag{3.50}$$

Keeping in mind the assumption (3.26), we can rewrite Eq. (3.50) in the form:

$$eF_x(R_g^x - R_{g'}^x)\varrho_{g'g} - \sum_{\delta} V(\delta)\langle\hat{B}_\delta\rangle\left(\frac{\langle\hat{D}_\delta^{++}\rangle}{1+e^{-\beta\hbar\omega}} + \frac{\langle\hat{D}_\delta^{--}\rangle}{1+e^{\beta\hbar\omega}}\right)\times$$

$$\times(f_{g',g-\delta} - f_{g'+\delta,g}) + i\hbar\sum_{\delta} f_{g-\delta,g-\delta}W_T(\delta) - i\hbar\sum_{\delta} f_{g'g}W_T(\delta) = 0. \tag{3.51}$$

In order to solve Eq. (3.51), one has to know the matrix elements $\varrho_{g'g}$. ϱ is the density matrix of the system unperturbed by an external electric field. At $t = -\infty$ $(\varrho_T(-\infty) = \varrho)$, the electric field is zero, but the matrix elements of the operator $\mathscr{H}_{\text{int}}$ do not vanish; consequently, the density matrix $\varrho_{g'g}$ depends on $\mathscr{H}_{\text{int}}$. This density matrix is of the form:

$$\varrho = Z^{-1}\exp\{-\beta(\mathscr{H}_0 + \mathscr{H}_{\text{int}})\};\ Z = \operatorname{Tr}\exp[-\beta(\mathscr{H}_0 + \mathscr{H}_{\text{int}})]. \tag{3.52}$$

Let us now expand the matrix element ϱ in a series of powers of $\mathscr{H}_{\text{int}}$ [18, 22]:

$$\langle\vartheta|\varrho|\vartheta'\rangle = \varrho^{(0)}_{\vartheta',\vartheta} + \varrho^{(1)}_{\vartheta',\vartheta} + \varrho^{(2)}_{\vartheta',\vartheta} + \ldots \tag{3.53}$$

It is easily verified that the terms of this expansion are of the forms:

$$\varrho^{(0)}_{\vartheta',\vartheta} = Z^{-1}\mathscr{N}^{-1}e^{-\beta E_\vartheta}\delta_{\vartheta,\vartheta'}, \tag{3.53a}$$

$$\varrho^{(1)}_{\vartheta,\vartheta'} = Z^{-1}\mathscr{N}^{-1}\frac{e^{-\beta E_\vartheta} - e^{-\beta E_{\vartheta'}}}{\hbar\omega_{\vartheta',\vartheta}}\mathscr{H}_{\text{int}\,\vartheta',\vartheta}, \tag{3.53b}$$

$$\varrho^{(2)}_{\vartheta',\vartheta} = Z^{-1}\mathscr{N}^{-1}\sum_{\vartheta''}\frac{\mathscr{H}_{\text{int}\,\vartheta'',\vartheta}\mathscr{H}_{\text{int}\,\vartheta',\vartheta''}}{\hbar\omega_{\vartheta'\vartheta''}}\left[\frac{e^{-\beta E_{\vartheta'}} - e^{-\beta E_\vartheta}}{\hbar\omega_{\vartheta'\vartheta}} - \frac{e^{-\beta E_{\vartheta''}} - e^{-\beta E_\vartheta}}{\hbar\omega_{\vartheta''\vartheta}}\right]. \tag{3.53c}$$

The first term of the expansion (3.53) is diagonal in all stage state indices. This fact expresses the translational symmetry of the problem and leads to the same probability for finding a carrier at any lattice site and on the appropriate energy level determined by σ. It should be noted here that the probability is different for the two possible energy levels resulting from removal of spin degeneration at a site by $s-d$ exchange interaction. $\varrho^0_{g'g}$ is diagonal also with respect to site indices and, therefore, does not contribute to the field term of Eq. (3.51). A finite contribution to this term is given by the second term of the expansion (5.53). Its diagonal part (with respect to σnN) is of the form:

$$\varrho^{(1)}_{g'g}(\sigma nN) = -Z^{-1}\mathscr{N}^{-1}\beta\, e^{-\beta E_\vartheta}\langle\sigma g nN|\mathscr{H}_{\text{int}}|\sigma g' nN\rangle. \tag{3.54}$$

In our further considerations, we restrict the expansion (3.53) to the term lowest with respect to the perturbation and neglect the remaining terms as small and accounting for previously neglected processes. Taking into account (3.26) and inserting (3.9) into (3.54), we obtain

$$\varrho^{(1)}_{g'g} = \beta \sum_{\delta} V(\delta)\delta_{g,g'+\delta}\,\hat{B}_{g\delta}\hat{D}^{\sigma\sigma}_{g\delta}. \tag{3.55}$$

Let us now insert (3.55) into (3.51), to obtain:

$$f_{g'g} = -\frac{i}{\hbar}\frac{eF_x(R^x_g - R^x_{g'})\varrho^{(1)}_{g'g}}{\sum\limits_{\delta} W_T(\boldsymbol{\delta})}, \tag{3.56}$$

or, on statistical averaging over σ, n, N,

$$f_{g'g} = -\frac{i}{\hbar}\frac{\beta eF_x(R^x_g - R^x_{g'})}{\mathcal{N}\sum\limits_{\delta} W_T(\boldsymbol{\delta})}\sum_{\delta} V(\boldsymbol{\delta})\delta_{g,g'+\delta}\langle\hat{B}_\delta\rangle\left(\frac{\langle\hat{D}^{++}_\delta\rangle}{1+e^{-\beta\hbar\omega}} + \frac{\langle\hat{D}^{--}_\delta\rangle}{1+e^{\beta\hbar\omega}}\right). \tag{3.57}$$

This is the diagonal matrix element of the linear reaction operator in the linear lowest approximation with respect to $\mathcal{H}_{\text{int}}$.

In order to evaluate the non-diagonal matrix element of the linear reaction operator we insert Eqs (3.56), (3.53b) into (3.13). Leaving in Eq. (3.13) only the first two terms, we obtain:

$$f_{\vartheta',\vartheta} = f^{(1)}_{\vartheta',\vartheta} + f^{(2)}_{\vartheta',\vartheta} \tag{3.58}$$

where

$$f^{(1)}_{\vartheta',\vartheta} = -\frac{1}{\hbar}\frac{1}{\omega_{\vartheta'\vartheta} - is}\,eF_x(R^x_g - R^x_{g'})\varrho^{(1)}_{\vartheta',\vartheta}, \tag{3.59}$$

$$f^{(2)}_{\vartheta',\vartheta} = -\frac{1}{\hbar}\frac{1}{\omega_{\vartheta'\vartheta} - is}\sum_{g''}[\langle\sigma\boldsymbol{g}nN|\mathcal{H}_{\text{int}}|\sigma'\boldsymbol{g}''n'N'\rangle f_{g'g''}(\sigma'n'N') - \langle\sigma\boldsymbol{g}''nN|\mathcal{H}_{\text{int}}|\sigma'\boldsymbol{g}'n'N'\rangle f_{g''g}(\sigma nN)]. \tag{3.60}$$

The diagonal matrix elements of the reaction operator appearing in Eq. (3.60) are determined by (3.56) or (3.57).

4. *Mobility of the carriers*

The mean velocity of a carrier in an approximation linear with respect to the electric field is determined by:

$$\langle\boldsymbol{v}\rangle = \text{Tr}\,[\varrho_T, \boldsymbol{v}] = \text{Tr}\,[f, \boldsymbol{v}] = \sum_{\vartheta,\vartheta'} f_{\vartheta',\vartheta}\cdot\boldsymbol{v}_{\vartheta,\vartheta'}. \tag{4.1}$$

The explicit form of the velocity operator in the appropriate representation is obtained by making use of its definition by the equation of motion:

$$v^x = \frac{dR^x_g}{dt} = \frac{i}{\hbar}[\mathcal{H}_T, R^x_g] = \frac{i}{\hbar}[\mathcal{H}_{\text{int}}, R^x_g], \tag{4.2}$$

where $[\mathcal{H}_0 + \mathcal{H}_F, R^x_g] = 0$ is taken into account.

The matrix element of the operator of the velocity component is of the form:

$$v^x_{\vartheta,\vartheta'} = \frac{i}{\hbar}(R^x_g - R^x_{g'})\langle\vartheta'|\mathscr{H}_{\text{int}}|\vartheta\rangle. \tag{4.3}$$

Let us now split Eq. (4.1) into two parts, the one containing the diagonal (with respect to σnN) matrix element of the reaction operator and the other — the non-diagonal matrix element, *i. e.*

$$\langle v^x\rangle = \langle v^x\rangle^{(b)} + \langle v^x\rangle^{(h)}, \tag{4.4}$$

where

$$\langle v^x\rangle^{(b)} \equiv \sum_{g,g'}\sum_{\sigma,n,N}\langle\sigma\boldsymbol{g}'nN|v^x|\sigma\boldsymbol{g}nN\rangle f_{g'g}(\sigma nN), \tag{4.5}$$

$$\langle v^x\rangle^{(h)} \equiv \sum_{g'g}\sum_{\substack{\sigma,\sigma'\\(\sigma\neq\sigma')}}\sum_{\substack{n,n'\\(n\neq n')}}\sum_{\substack{N,N'\\(N\neq N')}}\langle\sigma'\boldsymbol{g}'n'N'|v^x|\sigma\boldsymbol{g}nN\rangle f_{\vartheta',\vartheta}. \tag{4.6}$$

Considering Eq. (3.58), we can write:

$$\langle v^x\rangle^{(h)} = \langle v^x\rangle^{(h)}_{(1)} + \langle v^x\rangle^{(h)}_{(2)}, \tag{4.7}$$

where

$$\langle v^x\rangle^{(h)}_{(1)} = \sum_{g'g}\sum_{\substack{\sigma,\sigma'\\(\sigma\neq\sigma')}}\sum_{\substack{n,n'\\(n\neq n')}}\sum_{\substack{N,N'\\(N\neq N')}}\langle\sigma'\boldsymbol{g}'n'N'|v^x|\sigma\boldsymbol{g}nN\rangle f^{(1)}_{\vartheta',\vartheta}, \tag{4.8}$$

and

$$\langle v^x\rangle^{(h)}_{(2)} = \sum_{g,g'}\sum_{\substack{\sigma,\sigma'\\(\sigma\neq\sigma')}}\sum_{\substack{n,n'\\(n\neq n')}}\sum_{\substack{N,N'\\(N\neq N')}}\langle\sigma'\boldsymbol{g}'n'N'|v^x|\sigma\boldsymbol{g}nN\rangle f^{(2)}_{\vartheta',\vartheta}. \tag{4.9}$$

Inserting (3.54) into (3.57), and then the obtained result together with (4.3) into (4.5), and taking into account (3.26), we finally obtain:

$$\langle v^x\rangle^{(b)} = -F_x\mu^x_b \tag{4.10}$$

where

$$\mu^x_b = \frac{e\beta}{\hbar^2}\sum_{\delta}\delta^2_x V^2(\boldsymbol{\delta})\langle\hat{B}_\delta\rangle^2\left(\frac{\langle\hat{D}^{++}_\delta\rangle}{1+e^{-\beta\hbar\omega}} + \frac{\langle\hat{D}^{--}_\delta\rangle}{1+e^{\beta\hbar\omega}}\right)^2\tau_T, \tag{4.11}$$

$$\tau_T = \Big(\sum_{\delta} W_T(\delta)\Big)^{-1}. \tag{4.12}$$

Analogously, we insert (3.53b) into (3.59) and the obtained result together with (4.3) into (4.8):

$$\langle v^x\rangle^{(h)}_{(1)} = \frac{i}{2\hbar^2}eF_x\mathscr{N}^{-1}\sum_{\sigma\cdot\sigma'}\sum_{n,n'}\sum_{N,N'}Z^{-1}e^{\beta E_\vartheta}|\langle\sigma\boldsymbol{g}nN|\mathscr{H}_{\text{int}}|\sigma'\boldsymbol{g}'n'N'\rangle|^2\times$$

$$\times(R^x_g - R^x_{g'})^2\left(\frac{1}{\omega_{\vartheta'\vartheta}-is} - \frac{1}{\omega_{\vartheta'\vartheta}+is}\right)\frac{e^{\beta\hbar\omega_{\vartheta'\vartheta}}-1}{\hbar\omega_{\vartheta'\vartheta}}(1-\delta_{\sigma\sigma'}\delta_{nn'}\delta_{NN'}). \tag{4.13}$$

The difficulties connected with summation over $\sigma'n'N'$, caused by the beforelast unfactorizable term in Eq. (4.13) can be removed by noting that the Dirac δ-function therein guarantees conservation of energy; consequently, we can replace this term by its limit when $\omega_{\vartheta'\vartheta} \to 0$ [5]. This is equal to β. Making use of the integral form of the δ-function (3.28), we obtain after the evaluates transformations similar to those employed in evaluating P:

$$\langle v^x \rangle_{(1)}^{(h)} = -F_x \mu_{(1)h}^x, \tag{4.14}$$

where

$$\mu_{(1)h}^x = \tfrac{1}{2} e\beta \sum_{\boldsymbol{\delta}} \delta_x^2 W_T(\boldsymbol{\delta}). \tag{4.15}$$

It is easily shown, by inserting (3.60) and (4.3) into (4.9), that $\langle v^x \rangle_{(2)}^{(h)} = 0$, because the obtained expression can be rewritten in the form: $\sum_{\delta} \delta_x \Psi(\delta)$, which is equal to zero, since $\Psi(\delta)$ is an even function of δ [5].

As seen from Eq. (4.4), the mobility of carriers

$$\mu^x = \mu_b^x + \mu_{(1)h}^x \tag{4.16}$$

is determined by two competitive processes: mobility in the polaron band (μ_b^x) and mobility determined by the hopping process ($\mu_{(1)h}^x$). As seen from Eqs (3.46) and (3.47), the main contribution to mobility is due to one-magnon and two-quantum processes. Each one-magnon process is accompanied by the carrier's spin flip; in the considered temperature range, with no external fields, such processes cannot occur. At low temperatures, where the spin wave

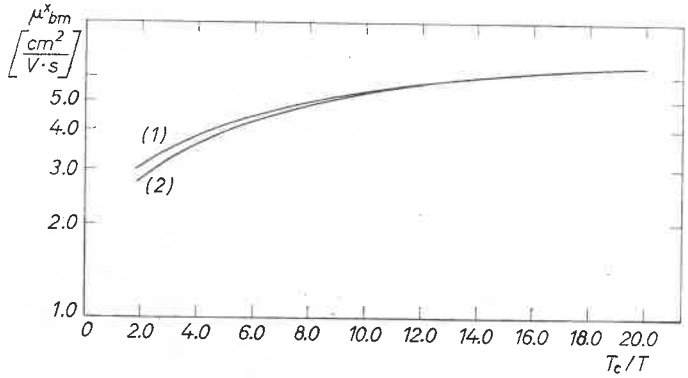

Fig. 2. Temperature-dependence of the μ_{bm}^x for $S = 2$: (1), for $A > 0$; (2), for $A < 0$

approximation is reasonable, the main conductivity mechanism is band conductivity, [11, 12], whereas interaction with magnons plays the part in the dissipative mechanism. Moreover, we do not take into account carrier scatterings on phonons [12, 13], whose role is restriced to a factor narrowing of the conductivity band. Thus, considering only scattering on the magnons we obtain

$$\mu^x \approx \mu_b^x \equiv \mu_{bm}^x, \tag{4.17}$$

From Eqs (3.38), (4.11) and (4.12), and keeping in mind that $\beta\hbar|\omega| \gg 1$, we obtain

$$\mu_{bm}^{x} \approx \frac{96S^2\mu_0}{\pi}\left(\chi\frac{T}{T_c}\right)^{-4} \times \begin{cases} \exp\left(\gamma\frac{T}{T_c}\right)^{5/2} & \text{for } A > 0, \\ \exp\left[-\left(\gamma\frac{T}{T_c}\right)^{5/2}\right] & \text{for } A < 0. \end{cases} \tag{4.18}$$

$$\gamma \equiv \chi\left(\frac{35\pi\zeta(5/2)}{64S}\right)^{5/2}, \quad \mu_0 \equiv \frac{|e|a^2}{\hbar} \cong 1\ \text{cm}^2/\text{V}\cdot\text{s},$$

The temperature dependence of μ_{bm}^{x} is shown in Fig. 2 for $S = 2$ and two possible signs of the $s-d$ exchange integral.

It seems of interest to derive the non-Born contribution to mobility; however, the calculation of transition probabilities of higher (up to the fourth) orders in the perturbation is a highly tedious procedure in the formalism adopted here. The problem is discussed in Ref. [23].

The author wishes to thank sincerely Dr J. Morkowski for suggesting the subject and for his interest and discussions.

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