CALCULATION OF SURFACE IMPEDANCE IN THE INFRARED REGION FOR A FERROMAGNET

By Z. Bajorski

Institute of Mathematics and Theoretical Physics, Technical University of Wrocław*

(Received January 15, 1970)

Optical properties of ferromagnetic metals are considered in terms of Landau's theory of the Fermi liquid. The results obtained are different from those known for nonferromagnetic metals.

1. Introduction

The problem was considered by Kaganov and Slezov [1] with the application of the theory of electron gas. Then Silin [2] took into consideration the electron-electron interaction and the problem for the common metals ($\mu = 1$), using the methods of Landau's theory of Fermi liquid. In this paper we consider the surface impedance of ferromagnetic metals employing the same methods.

Due to the small depth of the electromagnetic wave penetration ($\delta \sim 10^{-8}$ m) we can neglect the domain structure of the examined metal.

Let us assume that the surface of the ferromagnetic metal is perpendicular to the direction of the incident wave. In our coordinate system the z-axis is parallel to the direction of magnetization, while the x-axis is perpendicular to the surface and goes into the interior of the metal. The vector of magnetic induction $\boldsymbol{B}$ is directed along the axis, hence, no spin waves are excited in this geometry. All characteristic functions depend only on the variable x.

We consider the infrared region, because the corresponding frequencies are small compared with those of plasma (in the latter case there is no intrinsic photoeffect), simultaneously being much larger than the collision frequencies:

$$\omega_0 \gg \omega \gg \tau^{-1} \tag{1.1}$$

where $\omega_0 \sim 10^{16}\ \mathrm{sec}^{-1}$ is the plasma frequency,

$\omega \sim 10^{12}\ \mathrm{sec}^{-1}$—$10^{15}\ \mathrm{sec}^{-1}$ is the frequency of infrared radiation,

$\tau \sim 10^{14}$ sec in the relaxation time of the electron at room temperature.

---

* Address: Instytut Matematyki i Fizyki Teoretycznej Politechniki Wrocławskiej, Wrocław, plac Grunwaldzki, Polska.

The conductivity of the ferromagnetic metals ($\sigma \sim 10^{18}\ \mathrm{sec}^{-1}$) is large compared with the frequency of infrared radiation and therefore we can neglect the displacement current in the Maxwell equations.

The electrodynamic properties of the surface of ferromagnetic metals are fully described by a function called the surface impedance:

$$Z_a = R_a - iX_a = \frac{E_a(0)}{J_a} = -i\omega\mu\mu_0 \frac{E_a(0)}{E_a'(0)}. \tag{1.2}$$

Where $E_a(0)$ is the a-component of the electric field vector $\boldsymbol{E}(x)$ on the surface of the ferromagnetic metal,

$$E_a'(0) = \left.\frac{\partial E_a(x)}{\partial x}\right|_{x=0},$$

J_a is the total current, $J_a = \int\limits_0^\infty j_a(x)\,dx$, μ is the magnetic permeability of the metal, and μ_0 is the magnetic permeability of vacum.

We use the transport equation given by Czerwonko [3]:

$$-i\omega g_\alpha + V_x^{k\alpha} \frac{\partial}{\partial x} \bar{g}_\alpha + k\mathrm{Nn}\varepsilon_{abz} V_b^{k\alpha} \frac{\partial}{\partial k_a} \bar{g}_\alpha$$

$$= eV_a^{k\alpha} E_a - ieP \frac{\alpha}{\omega} V_x^{k\alpha} \frac{\partial^2}{\partial x^2} E_y - \frac{\bar{g}_\alpha}{\tau_\alpha} \tag{1.3}$$

where $\boldsymbol{V}^{k\alpha}$ is the velocity at the Fermi surface for spin α and wave vector $\boldsymbol{k}$,

$$K = \frac{e}{\hbar} \mu_B \mu_0 = 8\pi\, 10^{-15}\mathrm{m},$$

μ_β is Bohr's magneton, N is the number atoms per unit volume, n is the mean number of Bohr's magnetons per atom, $P = \frac{\hbar}{2m} = 5\times 10^{-5}\ \mathrm{m^2\,sec^{-1}}$ and $\alpha = \pm 1$. Also

$$\bar{g}_\alpha(\boldsymbol{k}, x) = g_\alpha(\boldsymbol{k}, x) + \frac{1}{(2\pi)^3\hbar} \sum_\beta \int d^3k' F_{\boldsymbol{kk}'}^{\alpha\beta} \delta(E_{\beta\boldsymbol{k}'}^0 - c) g_\beta(\boldsymbol{k}', x) \tag{1.4}$$

where $\delta(E_{\alpha\boldsymbol{k}}^0 - c)$ denotes the variation of the occupation number of the local equilibrium value, $E_{\alpha\boldsymbol{k}}^0$ denotes the excitation energy for the particle with momentum $\boldsymbol{k}$ and spin α, c is chemical potential, $F_{\boldsymbol{kk}'}^{\alpha\beta}$ is the effective interaction of quasiparticles.

A current of electrons is expressed as follows:

$$j_a(x) = \frac{e\hbar}{(2\pi)^3} \sum_\alpha \int d^3k V_a^{k\alpha} \delta(E_{\beta k}^0 - c) \bar{g}_\alpha(\boldsymbol{k}, x). \tag{1.5}$$

The velocity of the electron in metals is about 4×10^6 m sec^{-1}. $N\sim 9\times10^{28}$, $n\sim 2.2$ (for iron) and $N\sim 3\times10^{28}$, $n\sim 7.1$ (for gadolinium). The relaxation time of the electron is approximately 4×10^{-14} sec at room temperature. For iron and gadolinium the magnetization term $kNn\varepsilon_{abz} V_b^{k\alpha} \dfrac{\partial}{\partial k_a}$ is of the order of 5×10^{11} sec^{-1}. We can neglect this term at room temperature (see condition (1.1)). However, at low temperatures the relaxation time is about 10^{-10} to 10^{-11} sec, so it is important in this case. Terms determining collisions and magnetization shall be considered separately as the first and second order of approximation. We shall consider those ferromagnetic metals for which the depth of penetration in the infrared region is of the order of 10^{-8} m. The electron mean free path is

$$\Lambda = v\tau = 10^{-5}\text{—}10^{-7}\ \text{m}. \tag{1.6}$$

Then we shall consider the region ($\Lambda\sim\delta$) of the anomalous skin effect.

Two terms in (1.3) contain electric field $\boldsymbol{E}$. These terms are comparable, if the condition

$$\omega\delta^2 \sim P \cong 5\times10^{-5}\ \text{m}^2\ \text{sec}^{-1} \tag{1.7}$$

is fulfilled. In our problem the condition (1.7) is fulfilled and this causes some differences between our results and those of Silin.

In the next Section we shall calculate the O-th approximation. In the third section we shall consider the complete expression for the surface impedance of ferromagnetic metals. In the fourth section we consider several particular Fermi surfaces and in last section we present our conclusions.

2. *The electric field*

In O-th approximation we shall consider only the electric field. Then,

$$-i\omega g_\alpha = eV_a^{k\beta}E_a - ieP\frac{\alpha}{\omega}V_x^{k\alpha}\frac{\partial^2}{\partial x^2}E_y,$$

and we have

$$\bar{g}_\alpha = \frac{ie}{\omega}E_b\sum_\beta\int d^3k' Q_{kk'}^{\alpha\beta}V_b^{k'\beta} + \frac{eP}{\omega^2}\frac{\partial^2 E_y}{\partial x^2}\sum_\beta\int d^2k' Q_{kk'}^{\alpha\beta}V_x^{k'\beta} \tag{2.1}$$

where

$$Q_{kk'}^{\alpha\beta} = \delta_{\alpha\beta}\delta(\boldsymbol{k}-\boldsymbol{k}') + \frac{1}{(2\pi)^3}F_{kk'}^{\alpha\beta}\delta(E_{\beta k'}^0 - c). \tag{2.2}$$

Using (1.5) and the relation $i_a = \sigma_{ab}E_b$, we obtain the conductance tensor in the null approximation:

$$\sigma_{ab}^{(0)} = \frac{e^2}{(2\pi)^3\omega}\left\{i\Gamma_{ab} + \frac{2P}{\omega}R_{ax}\delta_{by}\frac{\partial^2}{\partial x^2} - \frac{P}{\omega}\Gamma_{ax}\delta_{by}\frac{\partial^2}{\partial x^2}\right\} \tag{2.3}$$

where

$$\Gamma_{ab} = \Gamma_{ba} = \tilde{\mathrm{Tr}}\,\{\delta(E^0_{\alpha k}-c)Q^{\alpha\beta}_{kk'}V^{k\alpha}_a V^{k'\beta}_b\} \tag{2.4}$$

$$R_{ab} = \tilde{\mathrm{Tr}}\,\{\delta(E^0_{\alpha k}-c)Q^{\alpha 1}_{kk'}V^{k\alpha}_a V^{k'1}_b\} \tag{2.5}$$

$$\tilde{\mathrm{Tr}}\,\{A(\alpha, \beta, \ldots, \boldsymbol{k}, \boldsymbol{k}', \ldots)\} = \sum_{\alpha\beta\ldots}\int d^3k\, d^3k' \ldots A. \tag{2.6}$$

We have made use here of the relation

$$\tilde{\mathrm{Tr}}\,\{\beta\delta(E^0_{\alpha k}-c)Q^{\alpha\beta}_{kk'}V^{k\alpha}_a V^{k'\beta}_b\} = 2R_{ab}-\Gamma_{ab}.$$

Knowledge of the conductance tensor enables us to calculate the dielectric tensor from the equation $\varepsilon_{ab} = -\dfrac{4\pi i}{\omega}\sigma_{ab}$. We emphasize that the conductance tensor is an operator. This is in agreement with the theory of the anomalous skin effect.

Disregarding changes of charge, we have div $\boldsymbol{j} = 0$. Hence

$$j^{(0)}_x(x) = \sigma^{(0)}_{xx}E^{(0)}_x + \sigma^{(0)}_{xy}E^{(0)}_y = 0$$

and

$$E^{(0)}_x(x) = -\frac{\sigma^{(0)}_{xy}}{\sigma^{(0)}_{xx}}E^{(0)}_y(x) = \Gamma^{-1}_{xx}\left\{\Gamma_{xx} - \frac{iP}{\omega}R_{xx}\frac{\partial^2}{\partial x^2} + \frac{iP}{\omega}\Gamma_{xx}\frac{\partial^2}{\partial x^2}\right\}E_y(x). \tag{2.7}$$

From the Maxwell equations and (2.6) it follows that

$$\left\{\Gamma_{xx} + \frac{2i\mu_0\mu e^2P}{(2\pi)^3\omega}[\Gamma_{xx}R_{yx} - \Gamma_{xy}R_{xx}]\right\}\frac{\partial^2}{\partial x^2}E_y(x)$$

$$= \frac{\mu\mu_0 e^2}{(2\pi)^3}\{\Gamma_{xx}\Gamma_{yy} - \Gamma^2_{xy}\}E_y(x). \tag{2.8}$$

The solution of the above equation must fulfill the boundary conditions

$$E^{(0)}_y(x)|_{x=0} = E_y(0), \qquad \lim_{x\to\infty} E^{(0)}_y(x) = 0. \tag{2.9}$$

A solution of this type is

$$E^{(0)}_y(x) = E_y(0)e^{-r\left(\cos\frac{\varphi}{2}+i\sin\frac{\varphi}{2}\right)x} \tag{2.10}$$

where

$$r = \frac{e}{2\pi}\sqrt{\frac{\mu\mu_0}{2\pi}}\,\frac{\sqrt{\Gamma_{xx}\Gamma_{yy}-\Gamma^2_{xy}}}{\sqrt[4]{\Gamma^2_{xx} + \dfrac{4\mu^2\mu_0^2e^2P^2}{(2\pi)^6\omega^2}(\Gamma_{xx}R_{yx}-\Gamma_{xy}R_{xx})^2}} \tag{2.11}$$

$$\varphi = \mathrm{arc\,tg}\,\frac{2\mu\mu_0e^2P}{(2\pi)^3\omega}\,\frac{\Gamma_{xx}R_{yx}-\Gamma_{xy}R_{xx}}{\Gamma} \tag{2.12}$$

$$E_x^{(0)}(x) = -\Gamma_{xx}^{-1}\left\{\Gamma_{xy} - \frac{iP}{\omega} r^2(\cos\varphi + i\sin\varphi)\,(R_{xx} - \Gamma_{xx})\right\} E_y^{(0)}(x). \tag{2.13}$$

Now we can easily calculate the surface impedance in the null approximation:

$$Z_y^{(0)} = i\omega\mu\mu_0 r^{-1}\left(\cos\frac{\varphi}{2} - i\sin\frac{\varphi}{2}\right). \tag{2.14}$$

If in (1.3) we neglect the Pauli term, $ieP\frac{\alpha}{\omega}V_x^{k\alpha}\frac{\partial}{\partial x^2}E_y$ we have

$$\left.\begin{aligned} r &= \frac{e}{2\pi}\sqrt{\frac{\mu\mu_0}{2\pi}}\sqrt{\frac{\Gamma_{xx}\Gamma_{yy} - \Gamma_{xy}^2}{\Gamma_{xx}}} \\ \varphi &= 0 \end{aligned}\right\} \tag{2.15}$$

and the surface impedance is an imaginary function like in Silin's paper.

3. *Full calculation of the surface impedance*

First we shall consider the approximation in which the collision integral and the local derivative appear. Both are of the same order, therefore, in the transport equation (1.3) they can be considered together. We shall call this approximation the first approximation:

$$-i\omega g_\alpha = -V_x^{k\alpha}\frac{\partial}{\partial x}\bar{g}_\alpha - \bar{g}_\alpha\frac{1}{\tau^\alpha}.$$

Now we can substitute $\bar{g}_\alpha$ in the right-hand side of the above equation by (2.1) and employ the solutions (2.10) and (2.13). Furthermore we have:

$$\begin{aligned} \bar{g}_\alpha = \frac{e}{\omega}\Bigg\{&\frac{\partial E_b}{\partial x}\sum_{\beta\gamma}\int d^3k'\,d^3k''\,Q_{kk''}^{\alpha\gamma}Q_{k''k'}^{\gamma\beta}\,V_x^{k''\gamma}V_b^{k'\beta} + \\ &+ E_b\sum_{\beta\gamma}\int d^3k'd^3k''\,Q_{k'k''}^{\alpha\gamma}\,Q_{k''k'}^{\gamma\beta}V_b^{k'\beta}\frac{1}{\tau^\gamma} - \\ &- \frac{iP}{\omega}\frac{\partial^3 E_y}{\partial x^3}\sum_{\beta\gamma}\beta\int d^3k'd^3k''\,Q_{kk''}^{\alpha\gamma}Q_{k''k'}^{\gamma\beta}V_x^{k''\gamma}V_x^{k'\beta} - \\ &- \frac{iP}{\omega}\frac{\partial^2 E_y}{\partial x^2}\sum_{\beta\gamma}\beta\int d^3k'd^3k''\,Q_{kk''}^{\alpha\gamma}Q_{k''k'}^{\gamma\beta}\,V_x^{k'\beta}\frac{1}{\tau^\gamma}\Bigg\}. \end{aligned} \tag{3.1}$$

Inserting the last expression into (1.5) we obtain

$$\begin{aligned} j_a(x) = \frac{e^2}{(2\pi)^3\,\omega^2}\Bigg\{&\frac{\partial E_b}{\partial x}K_{ab} + E_b L_{ab} - \\ &- \frac{i\beta}{\omega}\left(\frac{\partial^3 E_y}{\partial x^3}S_a + \frac{\partial^2 E_y}{\partial x^2}T_a\right)\Bigg\} \end{aligned} \tag{3.2}$$

where

$$K_{ab} = K_{ba} = \tilde{\mathrm{T}}\mathrm{r}' \{\delta(E^0_{\alpha k}-c) Q^{\alpha\gamma}_{kk''} Q^{\gamma\beta}_{k''k'} V^{k\alpha}_a V^{k'\beta}_b V^{k''\gamma}_x\} \tag{3.3}$$

$$L_{ab} = L_{ba} = \tilde{\mathrm{T}}\mathrm{r}' \left\{ \delta(E^0_{\alpha k}-c)\, Q^{\alpha\gamma}_{kk''} Q^{\gamma\beta}_{k''k'} \frac{1}{\tau\gamma}\, V^{k\alpha}_a V^{k'\beta}_b \right\} \tag{3.4}$$

$$S_a = \tilde{\mathrm{T}}\mathrm{r}' \{\delta(E^0_{\alpha k}-c)\beta Q^{\alpha\gamma}_{kk''} Q^{\gamma\beta}_{k''k'} V^{k\alpha}_a V^{k'\beta}_x V^{k''\gamma}_x\} \tag{3.5}$$

$$T_a = \tilde{\mathrm{T}}\mathrm{r}' \left\{ \delta(E^0_{\alpha k}-c)\, \beta\, \frac{1}{\tau\gamma}\, Q^{\alpha\gamma}_{k'k'} Q^{\gamma\beta}_{k''k'} V^{k\alpha}_a V^{k'\beta}_x \right\}. \tag{3.6}$$

The prime denotes integration for $V_x \geqslant 0$, in the space $\boldsymbol{k}$. Also, we have

$$J_a = \int_0^\infty j_a(x)dx = \frac{e^2}{(2\pi)^3\omega^2} \left\{ E_b(0) \left[-K_{ab} + r^{-1} L_{ab} \left(\cos\frac{\varphi}{2} - i\sin\frac{\varphi}{2} \right) + \right.\right.$$

$$\left.\left. + E_y(0) r^2 (\cos\varphi + i\sin\varphi)\, S_a - r\left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\right) T_a \right] \right\}. \tag{3.7}$$

All of these calculations are carried out under the so-called diffusion reflection conditions on the metal-vacuum boundary (see Appendix).

Putting (3.7) into (1.2) we get an expression for the surface impedance in the first approximation:

$$Z^{(1)}_y = (2\pi)^3\, \omega^2\, e^{-2} \left\{ -K_{yy} + L_{yy}\, r^{-1} \left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2} \right) + \right.$$

$$+ (L_{yx} - K_{yx})\, \Gamma^{-1}_{xx} \left[\Gamma_{xy} - \frac{iP}{\omega} r^2 (\cos\varphi + i\sin\varphi)\, (R_{xx} - \Gamma_{xx}) \right] +$$

$$\left. + \frac{iP}{\omega} \left[r^2 (\cos\varphi + i\sin\varphi)\, Sy - r\left(\cos\frac{\varphi}{2} + i\sin\frac{\varphi}{2}\right) T_y \right] \right\}^{-1}. \tag{3.8}$$

Let us consider the second approximation. Now we have to take into account the magnetization term in the transport equation (1.3) which was neglected before. The transport equation now has the form

$$-i\,\omega g_\alpha + S\varepsilon_{abz} V \frac{\boldsymbol{k}_\alpha}{a} \frac{\partial}{\partial k_a} g_\alpha = 0 \tag{3.9}$$

where

$$S = K \cdot N \cdot n = 5\times 10^{15}\mathrm{m}^{-2} \text{ (for iron)}.$$

We use the same method as for the calculation of (3.2) and we have:

$$j_a(x) = \frac{e^2}{(2\pi)^3\,\omega^2}\, S \left\{ E_b A_{ab} + i\,\frac{P}{\omega} \frac{\partial^2 E_y}{\partial x^2} B_a \right\} \tag{3.10}$$

where

$$A_{ab} = \varepsilon_{cdz} \tilde{\mathrm{Tr}}' \left\{ \delta(E^0_{\alpha k} - c)\, V^{k\alpha}_a Q^{\alpha\gamma}_{kk''} V^{k''\gamma}_d \frac{\partial}{\partial k''_c} Q^{\gamma\beta}_{k''k'} V^{k'\beta}_b \right\} \tag{3.11}$$

$$B_a = \varepsilon_{cdz} \tilde{\mathrm{Tr}}' \left\{ \beta\, \delta(E^0_{\alpha k} - c)\, V^{k\alpha}_a Q^{\alpha\gamma}_{kk'} V^{k'\gamma}_d \frac{\partial}{\partial k'_c} Q^{\gamma\beta}_{k'k'} V^{k'\beta}_x \right\}. \tag{3.12}$$

Moreover,

$$J_a = \frac{e^2}{(2\pi)^3 \omega^2} S \left\{ E_b(0)\, r^{-1} A_{ab} \left(\cos\frac{\varphi}{2} - i \sin\frac{\varphi}{2} \right) + \right.$$

$$\left. + i \frac{P}{\omega} r \left(\cos\frac{\varphi}{2} + i \sin\frac{\varphi}{2} \right) E_y(0) B_a \right\}. \tag{3.13}$$

Finally, we obtain the surface impedance in the second approximation:

$$Z^{(2)}_y = \frac{(2\pi)^3 \omega^2}{e^2 S} \left\{ i \frac{P}{\omega} r \left(\cos\frac{\varphi}{2} + i \sin\frac{\varphi}{2} \right) B_y + \right.$$

$$+ r^{-1} \left(\cos\frac{\varphi}{2} - i \sin\frac{\varphi}{2} \right) \left[A_{yy} + A_{yx} \Gamma^{-1}_{xx} \left\{ \Gamma_{yx} - \right. \right. \tag{3.14}$$

$$\left. \left. \left. - \frac{iP}{\omega} r^2 (\cos\varphi + i \sin\varphi)\, (R_{xx} - \Gamma_{xx}) \right\} \right] \right\}^{-1}.$$

Hence, the complete expression for the surface impedance of the ferromagnetic metals will be the following:

$$Z_y = Z^{(0)}_y + Z^{(1)}_y + Z^{(2)}_y. \tag{3.15}$$

4. *Particulai forms of the Fermi surface*

In this paragraph we shall consider two different shapes of Fermi surface. First we take the spherical Fermi surface. In this case the effective interaction of quasiparticles can be expanded into Legendre polynomials,

$$F^{\alpha\beta}_{kk'} = \sum_l (2l+1) F^{\alpha\beta}_l P_l(\hat{k}_a \hat{k}'_b) \tag{4.1}$$

where

$$\hat{k}_a = \frac{k_a}{\sqrt{k_b k_b}}. \tag{4.2}$$

Became of the spherical symmetry we have

$$V\frac{k\alpha}{a} = V_a \hat{k}_a \tag{4.3}$$

where V_α is the modulus of the velocity at the Fermi surface.

We must remember that the integration is given for $V_x \geqslant 0$ in the space $\boldsymbol{k}$, therefore, we have $0 \leqslant \vartheta \leqslant \pi$, $-\frac{\pi}{2} \leqslant \varphi \leqslant \frac{\pi}{2}$. Now we can express the functions in Sections 2 and 3 in the much simpler form

$$\Gamma_{ab} = \frac{4\pi}{3} G\delta_{ab} \tag{4.4}$$

where

$$G = \frac{1}{\hbar} \sum_{\alpha\beta} k_\alpha^2 \left(V_\alpha + \frac{1}{2\pi^2\hbar} k_\beta^2 F_1^{\alpha\beta} \right) \tag{4.5}$$

$$R_{ab} = \frac{4\pi}{3} R\delta_{ab} \tag{4.6}$$

where

$$R = \frac{1}{\hbar} k_1^2 \left(V_1 + \frac{1}{2\pi^2\hbar} \sum_\alpha k_\alpha^2 F_1^{\alpha 1} \right). \tag{4.7}$$

Note, that in all quantities in the null approximation only the nondiagonal components appear of the tensor R_{ab}. Because the tensor R_{ab} is diagonal for the spherical Fermi surface, it does not occur in formulas for the surface impedance. We have

$$r = \frac{e}{2\pi} \sqrt{\frac{2\mu\mu_0 G}{3}} \tag{4.8}$$

$$\varphi = 0 \tag{4.9}$$

$$E_x(x) = (G-R) \frac{i\, e^2 P \mu\mu_0}{8\pi^3 \omega} E_y(x) \tag{4.10}$$

$$E_y(x) = E_y(0)e^{-rx}. \tag{4.11}$$

In this case the surface impedance, in the null approximation, has only the imaginary part

$$X_y^{(0)} = \frac{2\pi\omega}{e} \sqrt{\frac{3\mu\mu_0}{2G}}. \tag{4.12}$$

Further we have:

$$K_{yy} = \frac{\pi}{4} K \tag{4.13}$$

where

$$K = \frac{1}{\hbar} \sum_{\alpha\beta} K_\alpha^2 v_\alpha \left(v_\alpha + \frac{1}{2\pi^2\hbar} K_\beta^2 F_1^{\alpha\beta} \right) \tag{4.14}$$

$$L_{yy} = \frac{2}{3} \pi L \tag{4.15}$$

where

$$L = \frac{1}{\hbar} \sum_{\alpha\beta} K_\alpha^2 \left\{ \frac{v_\alpha}{\tau_\alpha} + \frac{1}{2\pi^2} K_\beta^2 \left(\frac{1}{\tau^\alpha} + \frac{1}{\tau^\beta} \right) F_1^{\alpha\beta} \right\} \tag{4.16}$$

$$K_{yx} = S_y = T_y = L_{yx} = 0 \tag{4.17}$$

Now the surface impedance, in first approximation, is given by the real function:

$$R_y^{(1)} = \frac{(2\pi^3 w^2}{\pi e^2 \left(\frac{2}{3} L r^{-1} - \frac{1}{4} K \right)} \tag{4.18}$$

Hereafter let us examine the second approximation in the case of the spherical Fermi surface. First we can write:

$$\varepsilon_{cdz} V_d^{k\alpha} \frac{\partial}{\partial K_c} = \frac{v_\alpha}{K_\alpha} \frac{\partial}{\partial \varphi} \tag{4.19}$$

where φ denotes the azimuthal angle in X the momentum space. We obtain,

$$A_{yx} = -\frac{\pi}{4} A \tag{4.20}$$

where

$$A = \frac{1}{\hbar} \sum_{\alpha\beta} K_\alpha \left(v_\alpha^2 + \frac{1}{2\pi^2 \hbar} K_\beta (K_\alpha v_\beta + K_\beta v_\alpha) F_1^{\alpha\beta} \right) \tag{4.21}$$

$$B_y = -\frac{\pi}{4} B \tag{4.22}$$

where

$$B = \frac{1}{\hbar} \sum_{\alpha\beta\gamma} K_\alpha \left(\alpha v_\alpha^2 + \frac{1}{2\pi^2 \hbar} \beta K_\beta (v_\alpha K_\beta + v_\beta K_\alpha) F_1^{\alpha\beta} \right) \tag{4.23}$$

$$A_{yy} = 0 \tag{4.24}$$

The surface impedance in the second approximation is now an imaginary function,

$$X_y^{(2)} = \frac{4(2\pi)^3 \omega^3}{\pi SP e^2 r} \left[B + \frac{A}{G} (R - G) \right]^{-1}. \tag{4.25}$$

The general expression for the surface impedance of the spherical Fermi surface now has the form:

$$Z_y = R_y^{(1)} - i\{X_y^{(0)} + X_y^{(2)}\}. \tag{4.26}$$

We consider the following form of the Fermi surface:

$$F_{kk'}^{\alpha\beta} = F(\boldsymbol{k}, \boldsymbol{k}') + G(\boldsymbol{k}, \boldsymbol{k}')\alpha\beta. \tag{4.27}$$

We also assume

$$k_\alpha = k_{\bar{\alpha}}. \tag{4.28}$$

We can easily see that in (2.4) the part which contains $G(k, k')\alpha\beta$ vanishes. Now the tensor Γ_{ab} can be written as follows:

$$\Gamma_{ab} = 4 \int d^3k d^3k' \delta(E^0_{k'} - c) Q(k, k') V^k_a V^{k'}_b. \tag{4.29}$$

Similarly we have

$$2R_{ab} = \Gamma_{ab}. \tag{4.30}$$

It is easily seen that the conductance tensor in the null approximation is the same as in the paper by Silin [2].

In like manner we rewrite the other formulas:

$$K_{ab} = 4 \int' d^3k d^3k' d^3k'' \delta(E^0_k - c) Q(k, k'') Q(k'', k') V^k_a V^{k'}_b V^{k''}_x \tag{4.31}$$

$$L_{ab} = 4 \int' d^3k d^3k' \, d^3k'' \, \delta(E^0_k - c) \, Q(k, k'') \, Q(k'', k') \frac{1}{\tau} V^k_b V^k_b \tag{4.32}$$

$$S_a = T_a = B_a = 0 \tag{4.33}$$

$$A_{ab} = 4\varepsilon_{cdz} \int d^3k d^3k' \, d^3k'' \delta(E^0_k - c) \, V^k_a V^{k''}_d Q(k, k'') \frac{\partial}{\partial k''_c} Q(k'', k') \, V^{k'}_b \tag{4.34}$$

Here

$$Q(k, k') = \delta(k - k') + \frac{1}{(2\pi)^3} F(k, k') \, \delta(E^0 k' - c). \tag{4.35}$$

These formulas can be treated as ones for metals which have $\mu = 1$. Now we can substitute (4.29–34) into (2.14), (3.9) and (3.14) in order to obtain the expression for the surface impedance.

5. *Concluding remarks*

In this paragraph we present some numerical estimates and conclusions. Once more let us return to the condition (1.7). In Section 1 we found that if the condition (1.7) is fulfilled both expressions including the electric field have the same magnitude and may be consider together. Now, we ask what happens when we assume that the condition is not fulfilled. The case $\omega\delta^2 \ll \beta$ must be rejected, because then the frequency is much higher than the of the infrared region and our considerations become nonphysical. On the other hand, the condition

$$\omega\delta^2 \gg P \tag{5.1}$$

is essential. Then the conductance tensor does not contain a local part and is the same as in the case of ordinary metals ($\mu = 1$).

$$\sigma^{(0)}_{ab} = \frac{e^2}{(2\pi)^3 \, \omega^2} \, i \, \Gamma_{ab}. \tag{5.2}$$

In this case the results of this paper are like those of Silin. In the formula (5.1) the frequency ω is restricted to the infrared region and we then are dealing with an increase of the depth of penetration skin effect δ in comparison with (1.7). On the other hand, we can neglect the tensor R_{ab}, bearing in mind formulas (2.10) and (2.11). The neglection of this term is due to the small value of μ. Indeed, for $\mu \to 1$ the terms containing R_{ab} in (2.11) decrease and we can assume that $\varphi \to 0$. Therefore, for $\mu \to 1$ we obtain the same results as Silin. Thus we can conclude that the depth of penetration is inversely proportional to the magnetic permeability μ. Here we consider the ferromagnetic metals, for which $\mu \sim 10^3$. Now we compare the terms in (4.23). Due to Eq. (1.1) we have

$$R_y^{(1)} \gg X_y^{(0)}. \tag{5.3}$$

Then,

$$X_y^{(0)} \gg X_y^{(2)}. \tag{5.4}$$

The author would like to express his gratitude to Dr J. Czerwonko for many useful discussions during preparation of this paper.

APPENDIX

We consider the boundary conditions (see [5, 7, 8]) for the calculation of $\bar{g}_\alpha(\boldsymbol{k}, x)$ from the expression

$$\frac{\partial \bar{g}_\alpha(\boldsymbol{k}, x)}{\partial x} = f_\alpha(\boldsymbol{k}, x) \tag{A}$$

where $f_\alpha(\boldsymbol{k}, x)$ is an arbitrary function.
For $V_x \leqslant 0$, the condition that $\bar{g}_\alpha$ must not become exponentially large as $x \to \infty$ gives:

$$\bar{g}_\alpha^{(1)} = \int\limits_x^\infty f_\alpha(\boldsymbol{k}, x)dx \tag{B}$$

where $\bar{g}_\alpha^{(1)}$ is the value of $\bar{g}_\alpha$ for all $\boldsymbol{V}$ such that $V_x \leqslant 0$. The distribution function of the electrons which are moving towards the surface of the metal thus depends on the values of the electric field at all points between infinity and x. The value $\bar{g}_\alpha$ for $V_x > 0$, which will be denoted by $\bar{g}_\alpha^{(2)}$, is determined by the nature of the scattering at the surface of the metal. It will be assumed that a fraction p of the electrons arriving at the surface is scattering specularly with reversal of the volocity component V_x, while the rest are scattered diffusely complete loss of their drift velocity. With these assumptions, the distribution function of the electrons leaving the surface ($x = 0$) is given by

$$\bar{g}_\alpha^{(2)}(V_x, V_y, V_z, x = 0) = p\bar{g}_\alpha^{(1)}(-V_x, V_y, V_z, x = 0) \tag{C}$$

In this way it is found that:

$$\bar{g}_\alpha^{(2)} = p\int\limits_{-\infty}^x f_\alpha(\boldsymbol{k}, x)dx + (1-p)\int\limits_0^x f_\alpha(\boldsymbol{k}, x)dx \tag{D}$$

REFERENCES

[1] M. J. Kaganov, V. V. Slezov, *J. Exper. Theor. Phys.* (USSR), **32**, 1496 (1957).
[2] V. P. Silin, *J. Exper. Theor. Phys.* (USSR), **34**, 707 (1958).
[3] J. Czerwonko, *Acta Phys. Polon.*, **A37**, 575 (1970).
[4] R. B. Dingle, *Physica*, **19**, 311, (1953).
[5] W. L. Ginzburg, G. P. Motulewicz, *Uspekhi Fiz. Nauk*, **55**, 469 (1955).
[6] M. J. Kaganov, M. J. Azbel, *Dokl. Akad. Nauk SSSR*, **102**, 49 (1955).
[7] G. E. H. Reuter, E. H. Sondheimer, *Proc. Roy. Soc.*, **195**, 336 (1948).
[8] R. N. Gurzhy, *J. Exper. Theor. Phys.*, **33**, 660 (1957).