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MEASUREMENT OF ELECTRICAL CONDUCTIVITY OF THIN SEMICONDUCTING FILMS BY A REFLECTION METHOD

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A thin semiconducting film has been placed transversely in a rectangular waveguide with a matched termination. A formula for the standing wave of such a system has been derived. Measurement of the conductivity σ consists in the measurement of VSWR, film thickness d, and the frequency. The range of changes (σ, d) for which the described method is applicable is given. The method has been verified experimentally for thin Cd_3As_2 films.

A thin semiconducting film placed transversely to the axis of a waveguide, filling its cross-section tightly, constitutes an element which disturbs the propagation of an electromagnetic wave in the waveguide. Let us assume that an electromagnetic wave of a dominant mode TE_{10} and a propagation coefficient γ_0 travels in a hollow waveguide of characteristic impedance Z_0. In a waveguide filled with semiconducting material the corresponding values will be designated by Z and γ (Fig. 1). A wave reflection will take place at the plane boundary, the reflection coefficient for the unbounded regions being

$$\varrho = \frac{Z-Z_0}{Z+Z_0} = \frac{\gamma_0-\gamma}{\gamma_0+\gamma} \tag{1}$$

assuming that $\mu = \mu_0$. For a film of thickness d placed in the waveguide, the load impedance of which is equal to the characteristic wave impedance, the reflection coefficient of the film r is equal to

$$r = \varrho \frac{1-e^{-2\gamma d}}{1-\varrho^2 \cdot e^{-2\gamma d}} = \frac{e^{\gamma d}-e^{-\gamma d}}{\varrho^{-1}e^{\gamma d}-\varrho e^{-\gamma d}}. \tag{2}$$

The reflection coefficient ϱ changes within the limits from 0 to -1, and it can be expressed, therefore, in the exponential form $\varrho = -e^{-\varphi}$. The reflection coefficient of the film can be written in a different form,

$$r = -\frac{e^{\gamma d}-e^{-\gamma d}}{e^{\gamma d+\varphi}-e^{-(\gamma d+\varphi)}} = -\frac{\sin\text{h}\,\gamma d}{\sin\text{h}(\gamma d+\varphi)}. \tag{3}$$

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The square modulus of the reflection coefficient is

$$|r|^2 = \frac{\sinh \gamma d \cdot \sinh \gamma^* d}{\sinh (\gamma d+\varphi) \cdot \sinh (\gamma^* d+\varphi^*)} = \frac{\cosh (\gamma+\gamma^*)d - \cosh (\gamma-\gamma^*)d}{\cosh (\gamma d+\varphi+\gamma^* d+\varphi^*) - \cosh (\gamma d+\varphi-\gamma^* d-\varphi^*)}. \tag{4}$$

The above formula refers to films of arbitrary thickness and arbitrary material constants. Further considerations will refer to thin films of good conductivity, for which the density

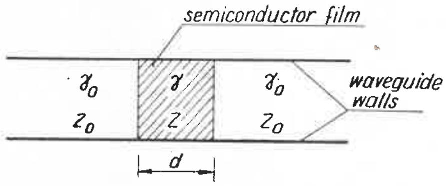

Fig. 1. Waveguide configuration

of the conduction current is much higher than the density of the dislacement current. These conditions can be written in the form of inequalities:

$$\text{a)} \quad |\gamma d| \ll 1 \qquad \text{b)} \quad \left|\frac{\gamma_0}{\gamma}\right| \ll 1. \tag{5}$$

Taking into account condition b), the reflection coefficient (1) can be written as follows,

$$\varrho = \frac{\frac{\gamma_0}{\gamma} - 1}{\frac{\gamma_0}{\gamma} + 1} = \left(\frac{\gamma_0}{\gamma} - 1\right)\left(1 - \frac{\gamma_0}{\gamma}\right) = -1 + \frac{2\gamma_0}{\gamma}. \tag{6}$$

On the other hand,

$$\varrho = -e^{-\varphi} = -(1-\varphi) \quad \text{whence} \quad \varphi = \frac{2\gamma_0}{\gamma}. \tag{7}$$

Since both φ and γd are much less than unity, by expanding the function $\cosh x$ into a series one can take into account only the first two terms of the series, $\cosh x = 1+x^2/2$. For thin films, after simple mathematical transformations, formula (4) can take the form

$$|r|^2 = \frac{1}{1 + \frac{2\gamma_0}{\gamma^2 d} + \frac{2\gamma_0^*}{\gamma^{*2} d} + \frac{4|\gamma_0|^2}{|\gamma|^4 d^2}}. \tag{8}$$

The square coefficient of wave propagation in a waveguide filled with semiconductor of electrical permittivity ε and conductivity σ is expressed by the following formula:

$$\gamma^2 = k_c^2 - \omega^2 \varepsilon \mu_0 + j\omega\mu_0\sigma.$$

Similarly, one can write for vacuum,

$$\gamma_0^2 = k_c^2 - \omega^2 \varepsilon_0 \mu_0 = \left(j\,\frac{2\pi}{\lambda_g}\right)^2 \tag{9}$$

where

$$k_c = \frac{2\pi}{\lambda_c}; \quad \omega = 2\pi f; \quad j = \sqrt{-1}$$

λ_c is cutoff wavelength, λ_g is hollow guide wavelength, and f is wave frequency.

Because $|\gamma| \gg |\gamma_0|$ one can also write the approximate formula

$$\gamma^2 = j\omega\mu_0\sigma. \tag{10}$$

Formula (8), after taking into account equations (9) and (10), can be written in the simpler form

$$|r|^2 = \frac{1}{\left(1+\dfrac{2|\gamma_0|}{|\gamma|^2 d}\right)^2}. \tag{11}$$

Finally, the reflection coefficient modulus is

$$|r| = \frac{1}{1+\dfrac{2}{f\lambda_g\mu_0\sigma d}}. \tag{12}$$

Quite often, instead of a direct measurement of the reflection coefficient, the voltage standing wave ratio VSWR is measured,

$$\mathrm{VSWR} = \frac{1+|r|}{1-|r|} = 1+f\lambda_9\mu_0\sigma d. \tag{13}$$

From the measurements of wave parameters (VSWR, f, λ_g) and film thickness one can determine the conductivity of the semiconductor. For the WR 90 waveguide and frequency $f = 9.5$ GHz the conductivity is

$$\sigma = \frac{\mathrm{VSWR}-1}{520d}. \tag{14}$$

The range of changes σ, d for which the described method can be applied is shown in Fig. 2. The restriction on the part of small and large values of the measured VSWR is dictated by the strongly increasing measurement errors for the extreme values of VSWR.

The described method of measuring electrical conductivity of thin films at microwave frequencies has been employed for the investigation of thin films of cadmium arsenide. The thin films have been obtained through evaporation in vacuum at a pressure of 5×10^{-5} mmHg on to a mica substrate of 0.05–0.1 mm thickness [1]. The rate of evaporation was 700 mg/min, and the temperature of the substrate 140°C. The thickness of the film, determined by the shift of interference fringes, varied within the limits of 1–2 μm for different films. Using a slotted line and a precision attenuator, the measurement of VSWR has been

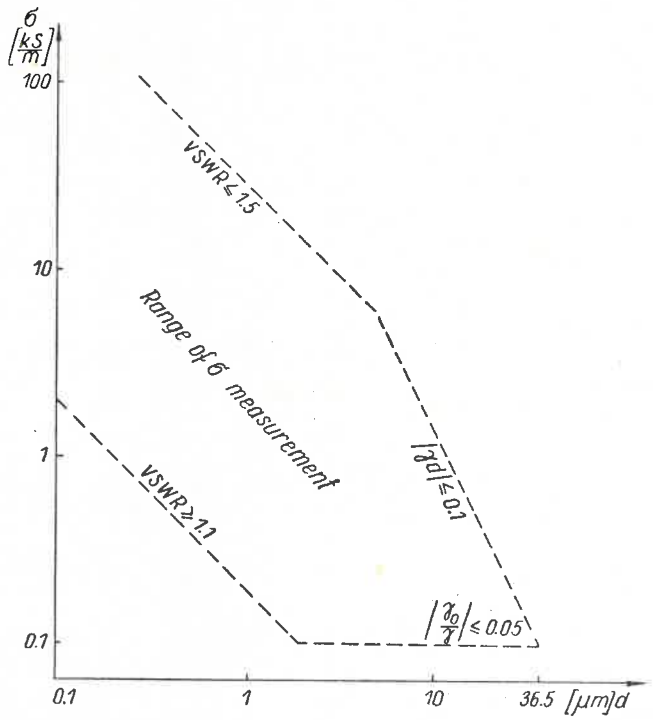

Fig. 2. The applicability range of the described method of measuring conductivity

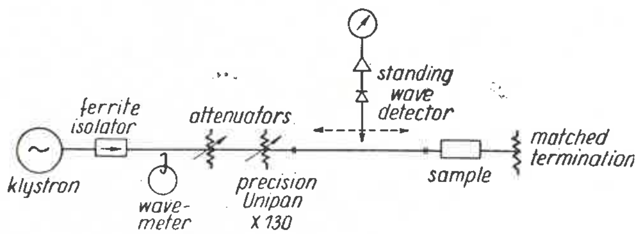

Fig. 3. Microwave test set-up

carried out on the measurement arrangement shown in Fig. 3. The results of measurements and calculations are given in Table I.

Relation $\sigma = \sigma(d)$ has been observed, what means that the conductivity of the films increases with increasing thickness. The precision of the σ measurement depends on several factors. The thickness of the film has been determined with an accuracy of 1%. The measurement of VSWR has been performed with an accuracy of 3%. This error depends, among other things, on the value of the measured VSWR. The measurement error introduced by the substrate amounts to 5% for the given value of conductivity. The influence of the power and frequency instability is sufficiently small and can be neglected. The length of the particularly important smaller side of the sample was 10.16 mm, with an accuracy

TABLE I

Film No.	Film thickness [μm]	VSWR	Conductivity (mean value) $\left[\frac{kS}{m}\right]$
1	1.48	3.40	
2	1.48	3.48	
3	1.48	3.36	
4	1.48	3.43	3.14
5	1.65	3.95	
6	1.65	4.09	
7	1.65	4.00	
8	1.65	4.07	3.52
9	1.90	4.71	
10	1.90	4.86	3.82

of 1%. Taking into account the above-mentioned factors one can assume that the relative error of the measurement of conductivity does not exceed 10%.

The described method of conductivity measurement of thin films is simple, quick, and non-destructive, and it permits elimination of the influence of the ohmic or rectifying contacts. Other applications of the microwave technique for the study of thin semiconducting films can be found in papers [2–6].

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