REGGE-POLE DESCRIPTION OF THE K^+p BACKWARD SCATTERING

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Regge fits to the K^+p backward differential cross-section with the exchange of $\Lambda_\alpha(1115, \frac{1}{2}^+)$ and $\Lambda_\gamma(1520, \frac{3}{2}^-)$ trajectories are presented. Assumptions of complete and partial $\Lambda_\alpha-\Lambda_\gamma$ exchange degeneracy are tested. The results are compared with FESR predictions.

1. Introduction

In this paper we discuss the Regge-pole model of the K^+p backward scattering. Earlier discussions of this problem can be found in Refs. [1] and [6].

We start with two assumptions proposed by Barger [1]:

(*i*) The Σ trajectory contributions to the backward peak are negligible in comparison with the contribution of the Λ trajectory exchange.

(*ii*) The reaction is dominated by the exchange of $\Lambda_\alpha(1115, \frac{1}{2}^+)$ and $\Lambda_\gamma(1520, \frac{3}{2}^-)$ trajectories.

Assumption (*i*) is due to the fact that the Σ trajectory is significantly weaker coupled to the $\bar{K}N$ system than the Λ trajectory. The Λ_β (1405, $\frac{1}{2}^-$) trajectory is expected to lie about one unit below the Λ_α trajectory which justifies assumption (*ii*)[1].

It follows from duality [2] that the imaginary part of the high energy s-channel amplitude should vanish as there are no prominent KN resonances. This is equivalent to the exchange degeneracy of the Λ_α and Λ_γ Regge-poles. It can be seen from the Chew-Frautschi plot on Fig. 1 that the trajectories are really degenerate. The degeneracy of residues, however, is not yet established.

Therefore we test separately two assumptions:

(A) Both residues and trajectories are degenerated.

(B) Only trajectories are degenerated.

The differential cross-sections corresponding to assumptions (A) and (B) are fitted to experimental data. The residue parameters are obtained from the fit.

In Section 2 we discuss the form of the amplitude, Section 3 presents results of the fits and Section 4 contains the comparison of fits with the sum rules.

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[1] More detailed arguments concerning these assumptions are given in Ref. [1].

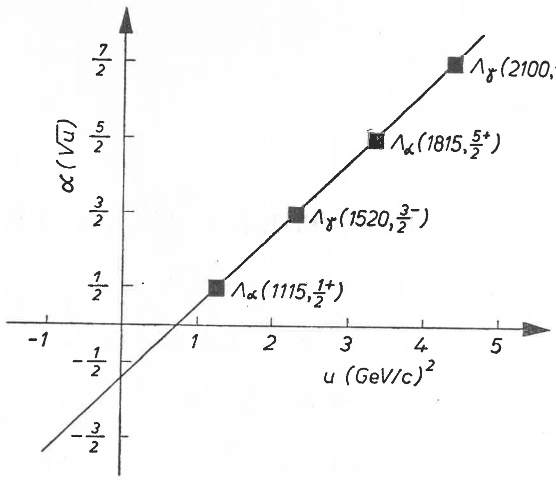

Fig. 1. The Chew-Frautschi plot for Λ_α and Λ_γ trajectories

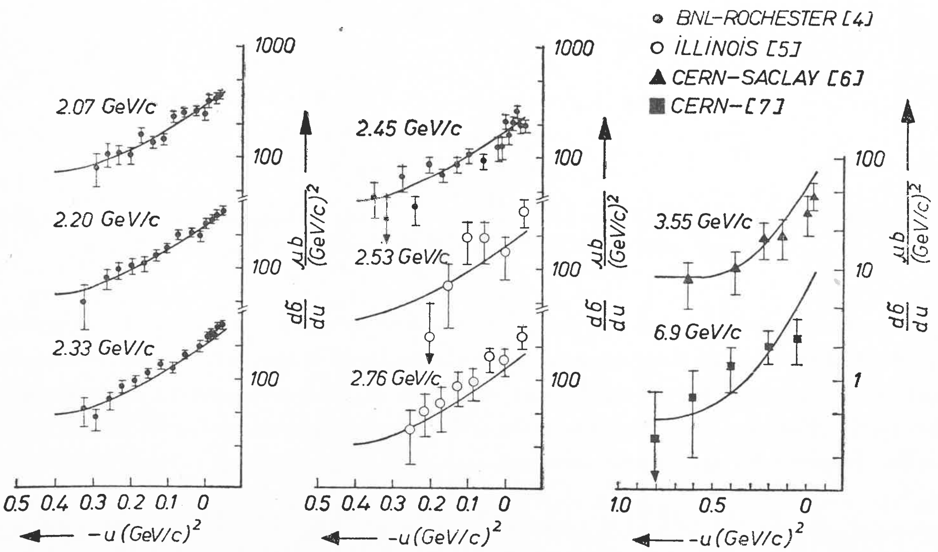

Fig. 2*a*. Predictions for the $d\sigma/du$ ($K^+p \to pK^+$) from the partially exchange degenerate solution (B) compared with the data from Refs [4]–[7]

2. *Parametrisation of amplitudes*

We use the standard technique of Reggeisation of the backward scattering amplitudes for the $0^-\frac{1}{2}^+ \to 0^-\frac{1}{2}^+$ processes [3].

The Regge-pole contribution to the u-channel spin amplitude f_1 is written in the form

$$f_1(\sqrt{u}, s) = \frac{E_u+M}{\sqrt{u}}\, \gamma(\sqrt{u})\, \frac{1+i\tau \exp\left[-i\pi\alpha(\sqrt{u})\right]}{\cos \pi\alpha(\sqrt{u})}\, s^{\alpha(\sqrt{u})-\frac{1}{2}}. \tag{1}$$

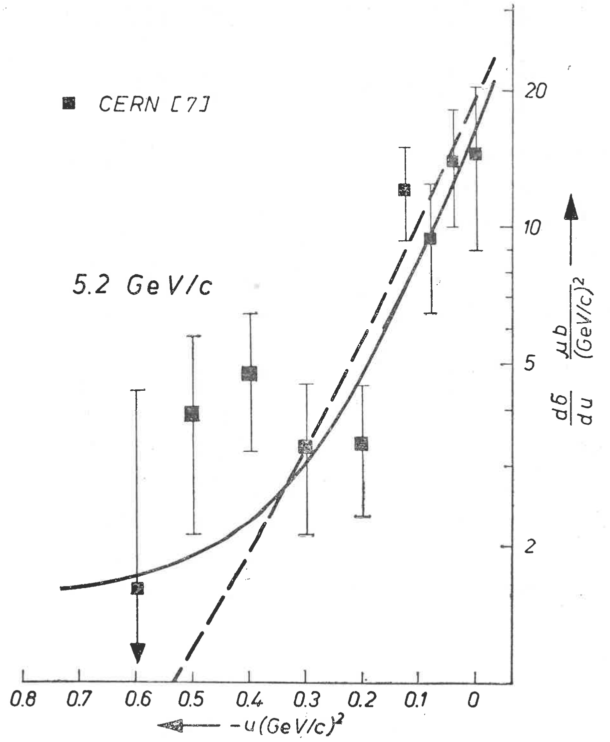

Fig. 2*b*. Predictions for the $d\sigma/du$ ($K^+p \to pK^+$) from the completely exchange degenerate solution (*A*) (dotted line) and solution (*B*) (full line) compared with data from Ref. [7]

Here u is the squared momentum transfer from the incoming proton to the outgoing K^+, E_u is the CMS energy of the proton in the u-channel, M is the mass of the proton; $\alpha(\sqrt{u})$, $\gamma(\sqrt{u})$ and τ are trajectory, residue and signature of the exchanged fermion Regge-pole, respectively. Further, s denotes the total CMS energy squared in GeV^2.

The formula for the $u \leftrightarrow s$ crossing

$$f_1(\sqrt{s}, u) = \frac{E_s+M}{2\sqrt{s}}\left[\frac{\sqrt{u}-\sqrt{s}+2M}{E_u+M} f_1(\sqrt{u}, s) + \frac{\sqrt{u}+\sqrt{s}-2M}{E_u-M} f_1(-\sqrt{u}, s)\right] \tag{2}$$

yields the following expression for the f_1 amplitude in the s-channel

$$f_1(\sqrt{s}, u) = \frac{E_s+M}{2\sqrt{s}}\left\{[\gamma(\sqrt{u})+\gamma(-\sqrt{u})] - \frac{\sqrt{s}-2M}{\sqrt{u}}[\gamma(\sqrt{u})-\gamma(-\sqrt{u})]\right\} R(u, s).$$

$$R(u, s) \equiv \frac{1+i\tau \exp[-i\pi\alpha(\sqrt{u})]}{\cos \pi\alpha(\sqrt{u})}\, s^{\alpha(\sqrt{u})-\frac{1}{2}}. \tag{3}$$

The amplitude f_2 is obtained from the relation

$$f_2(\sqrt{s}, u) = -f_1(-\sqrt{s}, u). \tag{4}$$

The normalisation of amplitudes is such that the differential cross-section is

$$\frac{d\sigma}{du} = \frac{\pi}{q_s^2}\,[|f_1+f_2\cos\Theta|^2+|f_2|^2\sin^2\Theta], \tag{5}$$

where q_s is the CMS momentum and Θ is the CMS scattering angle.

We check two hypotheses:

(A) Complete $\Lambda_\alpha - \Lambda_\gamma$ exchange degeneracy *i.e.*

$$\alpha(\Lambda_\alpha) = \alpha(\Lambda_\gamma);\ \gamma(\Lambda_\alpha) = \gamma(\Lambda_\gamma).$$

The residue of each trajectory is assumed to have the form suggested by Barger [1]

$$\gamma(\sqrt{u}) = \left(\alpha+\frac{1}{2}\right)\left(\alpha+\frac{3}{2}\right)\left(1-\frac{\sqrt{u}}{M_\alpha}\right)\left(1-\frac{\sqrt{u}}{M_\gamma}\right) a \exp(bu). \tag{6}$$

The term $(\alpha+\frac{1}{2})(\alpha+\frac{3}{2})$ kills here the ghost states and the factor[2] $(1-\sqrt{u}/M_\alpha)$ $(1-\sqrt{u}/M_\gamma)$ is introduced in order to extinguish the McDowell reflections of $\Lambda_\alpha(1115, \frac{1}{2}^+)$ and $\Lambda_\gamma(1520, \frac{3}{2}^-)$. Parameters M_α and M_γ are the masses of the unobserved states $\Lambda_\beta(1115, \frac{1}{2}^-)$ and $\Lambda_\delta(1520, \frac{3}{2}^+)$. Under these assumptions the amplitudes are purely real.

(B) Partial $\Lambda_\alpha - \Lambda_\gamma$ exchange degeneracy, *i.e.* common trajectory for Λ_α and Λ_γ with different residues.

We parametrize the residues as follows

$$\gamma(\Lambda_x) = \left(\alpha+\frac{1}{2}\right)\left(\alpha+\frac{3}{2}\right)\left(1-\frac{\sqrt{u}}{M_x}\right) a_x \exp(b_x u). \tag{7}$$

The purpose of introducing the factor $(1-\sqrt{u}/M_x)$ is similar as in (A).

In both hypothesis (A) and (B) we assume the trajectories to be linear in u, *i.e.*

$$\alpha(\sqrt{u}) = \alpha(0)+\alpha' u. \tag{8}$$

The particular form (6) and (7) of our residues (especially their dependence on $\sqrt{u}$) is a strong assumption. However, it works satisfactorily in the case of πN backward scattering[3].

3. *Results and discussion*

We determine two or four parameters (depending on whether we test (A) or (B) (by fitting the backward elastic K^+p cross-section in the momentum range from 2.07 to 6.9 GeV/c; $-0.8\ (\mathrm{GeV/c})^2 \leqslant u \leqslant +0.05\ (\mathrm{GeV/c})^2$ [4]–[7].

In both cases (A) and (B) we have taken from the Chew-Frautschi plot[4] the following common $\Lambda_\alpha - \Lambda_\gamma$ trajectory

$$\alpha(\sqrt{u}) = -0.70+0.96\,u. \tag{9}$$

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[2] This choice of the sign of $\sqrt{u}/M_x$ is connected with the crossing convention applied here.

[3] see *e. g.*: V. Barger, *University of Wisconsin preprint* COO–881–216 (1968).

[4] We have also tried to fit simultaneously the parameters of the residue and trajectory, but the improvement of the fit is rather small. Moreover, the intercept of the trajectory remains constant within ± 0.01.

The overall sign of the amplitude can not be determined by fitting only the cross-section. However, the sum rules demand this sign to be negative. The fitted parameters of the residues are presented in Table I.

TABLE I

Values of fitted residues. Number of data points is 96. The overall sign of the a parameter follows from the CMSR consistency condition

Solution	Trajectory	Residue	a [GeV]^2	b [GeV]^2	χ^2	Confidence level
(A)	$\Lambda_\alpha - \Lambda_\gamma$	Eq. (6)	−6.41	0.86	118	4%
		Eq. (10)	−5.97	1.07	121	3%
(B)	Λ_α	Eq. (7)	−3.74	−0.87	103	20%
	Λ_γ	Eq. (7)	−5.73	0.57		

It can be seen that the fit with hypothesis (A) gives a lower confidence level (of 4 per cent) than the fit with hypothesis (B) (which gives 20 per cent). The difference arises mainly from the large $|u|$ region (Fig. 2b). Hypothesis (B) describes this region better and also gives a better fit to the 6.9 GeV/c data.

We have tried to use with hypothesis (A) another form of ghost-killing factor taking the residue

$$\gamma(\sqrt{u}) = \frac{1}{\Gamma(\alpha+0.5)}\left(1-\frac{\sqrt{u}}{M_\alpha}\right)\left(1-\frac{\sqrt{u}}{M_\gamma}\right) a \exp(bu) \tag{10}$$

instead of Eq. (6). This does not change significantly the confidence level (by about 1 per cent).

The relatively low energy data contribute to our fit with a larger statistical weight than the high energy data. Moreover, there is some evidence for the existence of an exotic Z^* resonance in the momentum region around 1.5 GeV/c [8]. This would spoil the Regge behaviour in the region of lowest momenta used here and change the values of the fitted parameters.

We would like to mention that all our fits describe also quite well the new bubble chamber data by Danysz *et al.* [9].

4. *Comparison with sum rules*

We compare our fits with the fixed u continuous moment sum rules (CMSR) derived by Gołąb-Meyer and Kwieciński [10].

If the residue of each trajectory is decomposed as follows

$$\gamma(\sqrt{u}) = -c_x(u) + \sqrt{u}\, d_x(u), \tag{11}$$

it is possible to write the sum rules for the part of the amplitude which is even in $\sqrt{u}$

$$\frac{1}{4\pi^2}\int\limits_{2s_0-\bar{s}}^{\bar{s}} \mathrm{Im}\,\{(s_0-s)^\lambda\,[A(s,u)+MB(s,u)]\}\,ds$$

$$=\frac{2}{\pi}\sum_x \frac{c_x(u)}{\alpha_x+\lambda+\dfrac{1}{2}}\left[1+\frac{\sin\pi\lambda}{\cos\pi\alpha_x}-\tau_x\frac{\cos\pi(\lambda+\alpha_x)}{\cos\pi\alpha_x}\right](\bar{s}-s_0)^{\lambda+\alpha_x+\frac{1}{2}} \tag{12a}$$

and separately for the part odd in $\sqrt{u}$

$$\frac{1}{4\pi^2}\int\limits_{2s_0-\bar{s}}^{\bar{s}} \mathrm{Im}\,\{(s_0-s)^\lambda B(s,u)\}\,ds$$

$$=\frac{2}{\pi}\sum_x \frac{d_x(u)}{\alpha_x+\lambda+\dfrac{1}{2}}\left[1+\frac{\sin\pi\lambda}{\cos\pi\alpha_x}-\tau_x\frac{\cos\pi(\lambda+\alpha_x)}{\cos\pi\alpha_x}\right](\bar{s}-s_0)^{\lambda+\alpha_x+\frac{1}{2}}. \tag{12b}$$

Here $A(s, u)$ and $B(s, u)$ denote s-channel invariant amplitudes; λ is the parameter of the sum rule. The left-hand sides (LHS) of Eq. (12) are integrated over low energy s- and t-channel amplitudes. The parameters s_0 and $\bar{s}$ are chosen as in Ref. [10] *i. e.* $s_0 = 2.49$ GeV^2 and $\bar{s} = 5.07$ GeV^2. The right-hand sides (RHS) of Eqs (12) contain the contributions from the exchanged fermion Regge-poles.

The comparison of the LHS integrals to the RHS terms of Eq. (12) with Regge-pole parameters of the (A) solution has been performed in Ref. [10].

We have investigated our solution (B) using the same numerical values of the LHS integrals. The results are presented in Fig. 3.

The agreement is reasonably good for both solutions. In particular, the chosen $\sqrt{u}$-dependence of the residues gives the correct relative sign between $\sqrt{u}$-dependent and $\sqrt{u}$-independent parts of the amplitude.

If $\lambda = 0$ then only the Λ_γ trajectory contributes to the sum rules, whereas if $\lambda = 1$ then the only contribution comes from the Λ_α exchange. It can be seen from Fig. 3 that for our solution (B) the CMSR consistency condition for Λ_α is better satisfied than the consistency condition for the Λ_γ trajectory. This is particularly striking for the $\sqrt{u}$-dependent part of the amplitude. The reason is perhaps that in our parametrisation the whole $\sqrt{u}$-dependence is contained in the factor killing the McDowell reflections.

5. *Conclusions*

Both our fits describe quite well the experimental data and are at the same time in reasonable agreement with the sum rules.

However, the comparison between the two fits does not answer the question whether

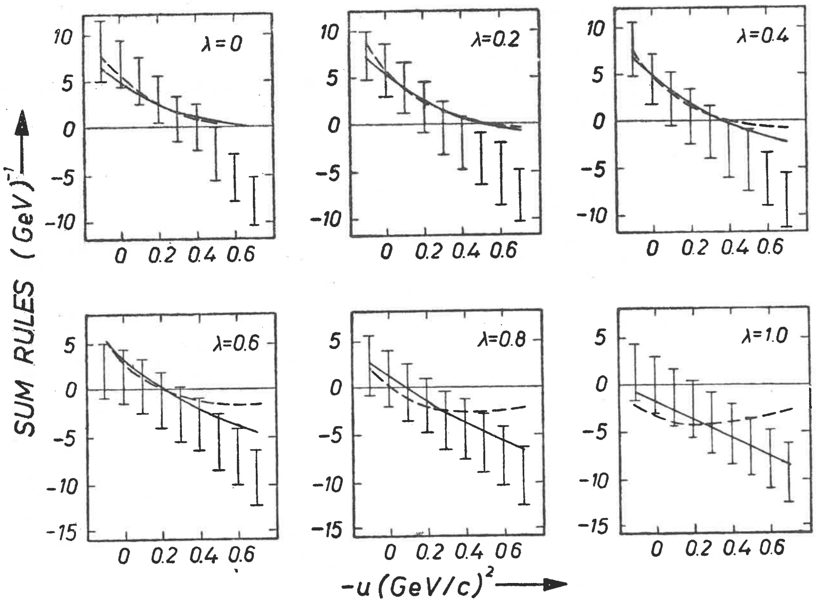

Fig. 3*a*. Comparison with CMSR for the $\sqrt{u}$-independent part of the amplitude. Vertical intervals give the values of the LHS of Eq. (12) for $g^2_{\Lambda KN}/4\pi$ varying from 5.7 (upper points) to 13.5 (lower points). These values are taken from Ref. [10]. The curves denote the RHS of Eq. (12) computed from our solutions (*A*) (dotted line) and (*B*) (full line)

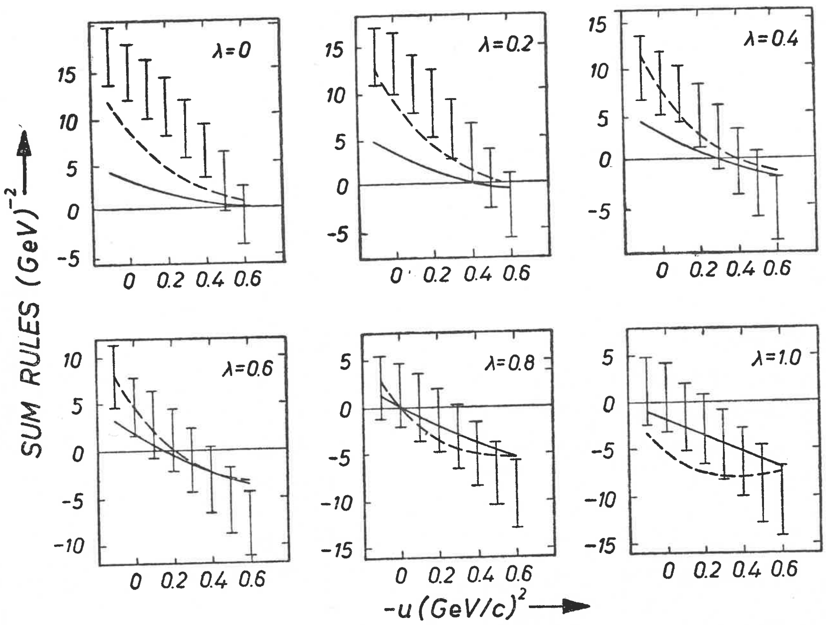

Fig. 3*b*. Comparison with CMSR for the $\sqrt{u}$-dependent part of the amplitude

the Schmid hypothesis of the complete exchange degeneracy works well in the case of K^+p backward scattering.

The measurement of the polarization in the high-energy region of the discussed reaction would provide a decisive solution of this problem (note that the completely exchange degenerate amplitude is purely real).

Experiments at higher energies would also show whether the contribution from the Σ trajectory exchange can be neglected.

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