SUPPLEMENT

Supplementary Material for "Time Scales in Dynamical Sauter–Schwinger Pair Creation: Dependence on Transverse Momentum"

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1. Overview

This section is included to provide complete clarity on the numerically generated simulation data. As outlined in the result section in the main text in [1], we solved the Dirac equation for the time-dependent electric field to determine the relevant time scales for understanding the dynamics of pair production in a time-dependent electric field.

As discussed in the section on the longitudinal momentum spectra (LMS) of the created particles [1], we provide a quantitative estimation to identify the relevant time scales for the dynamics of the LMS function, which exhibits distinct behaviors and substructures. The time scales for various values of the electric field parameters are summarized in Tables SI–SIII.

Specifically, one table is provided for each time scale, i.e., the time scale $t_{\rm cp}$ is analyzed in Table SI, the time scale $t_{\rm sep}$ in Table SII, and the time scale $t_{\rm dis}$ in Table SIII.

TABLE SI

Numerical data regarding the time scale $t_{\rm cp}$ (for the central peak formation in the distribution function) obtained by evaluating the function $f(p_\parallel, p_\perp, t)$ for a homogeneous Sauter pulse, $E(t) = E_0\,\mathrm{sech}^2(\mathrm{t}/\tau)$, at different times. All quantities are expressed in units of the electron mass. The absolute values obtained are shown on the right side, as are the relative deviations from our fit $0.46 t_c^{-2.04}\tau^{1.141}E_0^{-1.273}p_\perp^{0.648}$.

τ	E_0	$p_\perp$	$t_{\rm cp}$	Rel. error	Abs. error
6	0.2	0.25	12.70	0.11642	1.47848
6	0.3	0.25	8.68	0.22823	1.98102

TABLE SI cont.

τ	E_0	$p_\perp$	$t_{\rm cp}$	Rel. error	Abs. error
6	0.4	0.25	5.85	0.20587	1.20434
10	0.2	0.25	24.52	0.18056	4.42743
10	0.3	0.25	14.53	0.17448	2.53521
10	0.4	0.25	9.34	0.10940	1.02176
14	0.2	0.25	36.09	0.18288	6.60021
14	0.3	0.25	19.64	0.10363	2.03530
14	0.4	0.25	12.40	0.01543	0.19135
18	0.2	0.25	46.34	0.15243	7.06340
18	0.3	0.25	26.42	0.11252	2.97281
18	0.4	0.25	18.02	0.09765	1.75965
22	0.2	0.25	55.40	0.10874	6.02410
22	0.3	0.25	32.60	0.09582	3.12377
22	0.4	0.25	20.20	0.01195	0.24141
6	0.2	0.5	14.05	0.25125	3.52999
6	0.3	0.5	10.33	0.01596	0.16484
6	0.4	0.5	7.16	0.01649	0.11804
10	0.2	0.5	31.22	0.00825	0.25766
10	0.3	0.5	17.80	0.05570	0.99141
10	0.4	0.5	10.67	0.22133	2.36162
14	0.2	0.5	43.67	0.05793	2.52964
14	0.3	0.5	26.67	0.03412	0.91009
14	0.4	0.5	17.22	0.11071	1.90646
18	0.2	0.5	58.40	0.05363	3.13197
18	0.3	0.5	34.00	0.08039	2.73312
18	0.4	0.5	22.25	0.14490	3.22398
22	0.2	0.5	72.00	0.07436	5.35385
22	0.3	0.5	40.00	0.15446	6.17841
22	0.4	0.5	28.50	0.12366	3.52417
6	0.2	0.75	18.50	0.23565	4.35952
6	0.3	0.75	12.80	0.06614	0.84659

TABLE SI cont.

τ	E_0	$p_\perp$	$t_{\rm cp}$	Rel. error	Abs. error
6	0.4	0.75	5.85	0.61773	3.61374
10	0.2	0.75	40.00	0.02327	0.93086
10	0.3	0.75	24.50	0.00266	0.06525
10	0.4	0.75	17.00	0.00322	0.05480
14	0.2	0.75	58.00	0.03576	2.07406
14	0.3	0.75	33.00	0.08675	2.86278
14	0.4	0.75	23.00	0.08132	1.87041
18	0.2	0.75	76.00	0.05278	4.01091
18	0.3	0.75	46.00	0.03836	1.76460
18	0.4	0.75	31.00	0.06852	2.12418
22	0.2	0.75	94.00	0.07005	6.58433
22	0.3	0.75	56.00	0.07226	4.04644
22	0.4	0.75	38.00	0.09583	3.64149
6	0.2	1.00	23.46	0.17397	4.08138
6	0.3	1.00	17.30	0.04962	0.85845
6	0.4	1.00	13.70	0.16774	2.29799
10	0.2	1.00	49.57	0.00517	0.25609
10	0.3	1.00	32.00	0.08002	2.56077
10	0.4	1.00	23.00	0.11236	2.58425
14	0.2	1.00	76.00	0.04766	3.62216
14	0.3	1.00	47.00	0.08068	3.79216
14	0.4	1.00	32.80	0.08646	2.83588
18	0.2	1.00	101.00	0.04556	4.60205
18	0.3	1.00	61.00	0.05660	3.45273
18	0.4	1.00	42.00	0.04980	2.09166
22	0.2	1.00	125.00	0.03052	3.81498
22	0.3	1.00	78.00	0.07251	5.65544
22	0.4	1.00	53.00	0.05339	2.82991

TABLE SII

Numerical data regarding the time scale $t_{\rm sep}$ (for the peak separation). At this time, the central peak becomes dominant or the time after the two peaks become distinctly separated. The data were obtained by evaluating the function $f(p_\parallel, t)$ for a homogeneous Sauter pulse, $E(t) = E_0\,\mathrm{sech}^2(\mathrm{t}/\tau)$, at different times. The absolute values obtained are shown on the right side, as are the relative deviations from our fit $0.7808\, t_c^{-1.71} \tau^{1.125}\, E_0^{-1.048}\, p_\perp^{0.537}$.

τ	E_0	$p_\perp$	$t_{\rm sep}$	Rel. error	Abs. error
6	0.2	0.25	15.37	0.02315	0.35585
6	0.3	0.25	11.07	0.11312	1.25220
6	0.4	0.25	8.52	0.14753	1.25695
10	0.2	0.25	30.39	0.12259	3.72539
10	0.3	0.25	20.32	0.14193	2.88393
10	0.4	0.25	15.09	0.14520	2.19109
14	0.2	0.25	44.59	0.12704	5.66457
14	0.3	0.25	29.20	0.12831	3.74655
14	0.4	0.25	21.45	0.12214	2.61995
18	0.2	0.25	58.42	0.11613	6.78439

TABLE SII cont.

τ	E_0	$p_\perp$	$t_{\rm sep}$	Rel. error	Abs. error
18	0.3	0.25	38.01	0.11169	4.24532
18	0.4	0.25	27.78	0.10084	2.80144
22	0.2	0.25	72.12	0.10282	7.41525
22	0.3	0.25	46.60	0.09205	4.28937
22	0.4	0.25	34.01	0.07966	2.70929
6	0.2	0.5	18.13	0.20146	3.65242
6	0.3	0.5	13.13	0.08481	1.11359
6	0.4	0.5	10.09	0.04432	0.44717
10	0.2	0.5	36.02	0.07398	2.66482
10	0.3	0.5	24.20	0.04529	1.09612
10	0.4	0.5	18.00	0.03965	0.71365
14	0.2	0.5	53.00	0.06552	3.47273
14	0.3	0.5	34.85	0.05962	2.07769
14	0.4	0.5	25.65	0.06505	1.66850
18	0.2	0.5	69.66	0.07540	5.25258
18	0.3	0.5	45.25	0.08255	3.73556
18	0.4	0.5	33.12	0.09416	3.11871
22	0.2	0.5	86.00	0.09155	7.87319
22	0.3	0.5	55.60	0.10403	5.78396
22	0.4	0.5	40.67	0.11657	4.74084
6	0.2	0.75	22.34	0.21215	4.73942
6	0.3	0.75	16.50	0.07317	1.20731
6	0.4	0.75	12.80	0.02340	0.29958
10	0.2	0.75	45.20	0.06398	2.89209
10	0.3	0.75	30.70	0.02435	0.74757
10	0.4	0.75	22.90	0.01591	0.36439
14	0.2	0.75	67.10	0.04628	3.10563
14	0.3	0.75	44.30	0.03629	1.60767
14	0.4	0.75	32.70	0.03859	1.26174
18	0.2	0.75	88.50	0.05231	4.62963
18	0.3	0.75	57.78	0.05396	3.11775
18	0.4	0.75	42.40	0.06253	2.65115
22	0.2	0.75	109.60	0.06479	7.10103
22	0.3	0.75	71.00	0.07480	5.31115
22	0.4	0.75	52.00	0.08565	4.45373
6	0.2	1.00	28.18	0.12143	3.42178
6	0.3	1.00	21.07	0.01925	0.40550
6	0.4	1.00	16.48	0.07238	1.19274
10	0.2	1.00	58.00	0.03235	1.87634
10	0.3	1.00	39.87	0.07952	3.17057
10	0.4	1.00	29.87	0.09107	2.72037
14	0.2	1.00	87.00	0.05827	5.06976
14	0.3	1.00	57.80	0.07311	4.22557
14	0.4	1.00	42.70	0.07182	3.06652
18	0.2	1.00	115.20	0.05657	6.51736
18	0.3	1.00	75.50	0.05870	4.43209
18	0.4	1.00	55.40	0.05100	2.82514
22	0.2	1.00	143.00	0.04762	6.80944
22	0.3	1.00	93.00	0.04241	3.94458
22	0.4	1.00	68.00	0.03115	2.11827

TABLE SIII

Numerical data regarding the time scale $t_{\rm dis}$ (for the disappearance of oscillation). This indicates the time when the oscillations within the central peak fade away or the time after the primary peak (or the left-side peak) ceases to exist. The data were obtained by evaluating the function $f(p_\parallel, t)$ for a homogeneous Sauter pulse, $E(t) = E_0\,\mathrm{sech}^2(\mathrm{t}/\tau)$, at different times. The field strengths E_0 are given in the first column in terms of the critical field strength E_c and the pulse lengths τ along the first column in units of $[m^{-1}]$. The absolute values obtained are shown on the right side, as are the relative deviations from our fit $2.929\,t_c^{-1.294}\tau^{0.854}\,E_0^{-0.830}\,p_\perp^{0.318}$.

τ	E_0	$p_\perp$	$t_{\rm dis}$	Rel. error	Abs. error
6	0.2	0.25	33.37	0.04094	1.36618
6	0.3	0.25	29.37	0.16558	4.86315
6	0.4	0.25	24.52	0.21967	5.38624
10	0.2	0.25	56.39	0.04729	2.66648
10	0.3	0.25	47.53	0.20262	9.63133
10	0.4	0.25	36.09	0.18003	6.49743
14	0.2	0.25	76.76	0.06723	5.16093
14	0.3	0.25	57.67	0.12408	7.15588
14	0.4	0.25	43.34	0.09001	3.90102
18	0.2	0.25	95.42	0.07009	6.68823
18	0.3	0.25	68.56	0.08691	5.95852
18	0.4	0.25	52.68	0.07221	3.80380
22	0.2	0.25	106.00	0.00650	0.68860
22	0.3	0.25	72.60	0.02340	1.69863
22	0.4	0.25	58.60	0.01009	0.59123
6	0.2	0.5	37.06	0.16870	6.25186
6	0.3	0.5	28.32	0.07899	2.23710
6	0.4	0.5	23.70	0.00665	0.15751
10	0.2	0.5	65.20	0.02740	1.78679
10	0.3	0.5	47.00	0.00553	0.26009
10	0.4	0.5	36.00	0.02496	0.89839
14	0.2	0.5	85.00	0.05030	4.27547
14	0.3	0.5	60.69	0.03782	2.29506
14	0.4	0.5	51.00	0.03577	1.82432
18	0.2	0.5	111.00	0.00326	0.36211
18	0.3	0.5	79.25	0.01506	1.19346
18	0.4	0.5	56.20	0.08439	4.74277
22	0.2	0.5	127.00	0.03394	4.31069
22	0.3	0.5	81.00	0.14372	11.64148
22	0.4	0.5	66.00	0.09591	6.32999
6	0.2	0.75	42.00	0.17331	7.27886
6	0.3	0.75	34.00	0.02256	0.76691
6	0.4	0.75	28.00	0.03056	0.85568
10	0.2	0.75	70.00	0.08879	6.21545
10	0.3	0.75	52.00	0.03406	1.77104
10	0.4	0.75	41.00	0.02395	0.98182
14	0.2	0.75	101.00	0.00569	0.57481
14	0.3	0.75	68.00	0.05386	3.66241
14	0.4	0.75	55.00	0.01728	0.95054
18	0.2	0.75	117.00	0.07590	8.88030
18	0.3	0.75	79.50	0.11711	9.31027
18	0.4	0.75	63.00	0.10062	6.33876
22	0.2	0.75	141.00	0.05958	8.40116
22	0.3	0.75	97.00	0.08664	8.40455
22	0.4	0.75	78.00	0.05506	4.29477
6	0.2	1.00	48.50	0.11350	5.50469
6	0.3	1.00	39.00	0.02305	0.89895
6	0.4	1.00	31.00	0.04040	1.25255
10	0.2	1.00	83.00	0.00632	0.52450
10	0.3	1.00	64.00	0.07926	5.07233
10	0.4	1.00	52.00	0.11523	5.99213
14	0.2	1.00	116.00	0.04038	4.68419
14	0.3	1.00	83.00	0.05380	4.46518
14	0.4	1.00	63.00	0.02673	1.68381
18	0.2	1.00	147.00	0.06155	9.04781
18	0.3	1.00	104.00	0.06416	6.67285
18	0.4	1.00	79.00	0.03812	3.01167
22	0.2	1.00	176.00	0.06972	12.27130
22	0.3	1.00	123.00	0.06087	7.48717
22	0.4	1.00	93.00	0.03025	2.81317

TABLE SIII cont.

References

[1] D. Sah, M.P. Singh, *Acta Phys. Pol. A* **148**, 149 (2025).