Proceedings of "Applications of Physics in Mechanical and Material Engineering" (APMME 2024)

Effect of Voltage Amplitude on Behavior of RLD System

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The aim of the present paper was to study the chaotic behavior in a resistor–inductor–diode circuit induced by modulation of voltage amplitude. Time evolutions of the voltage or current signal revealed an extremely chaotic response of the simulated system. These dependences were taken into account in the construction of the phase space. Moreover, based on a bifurcation diagram, the Feigenbaum constant was calculated and verified with reliable and noticeable accuracy.

topics: deterministic chaos, Feigenbaum's constant, phase space

1. Introduction

Some nonlinear equations or systems of nonlinear equations are extremely sensitive to initial conditions. This could result in the generation of chaotic behavior called as deterministic chaos [1]. The iteration of a parabolic function for specific initial conditions is one of the simplest example of a chaotic system. Chaos is observed in all systems where period doubling has been detected [2, 3]. Period doubling induces bifurcations that change one or more parameters of a system. In the example for a parabolic function, the period is initially equal to 1 and after several iteration takes the following values: 2, 4, 8, and so on and so forth. The changes of period are observed after a specific, constant step called Feigenbaum's constant δ [4–6]. An analysis of the system behavior allows one to produce plot known as Feigenbaum's diagram, which has a fractal structure and shows its self-similarity of each part. Similar fractal forms built by copper aggregation and tuning of RLD circuit were studied in our previous work [6–8]. The RLD series circuit is a relatively simple system showing chaotic behavior. A scheme of this circuit (where R is a resistor, L — an inductor; and D is a diode) is presented in Fig. 1. The element generating period doubling is a diode that is a nonlinear part of the electrical circuit. Such a system is very sensitive to changes in frequency, as it was shown in our previous paper [7].

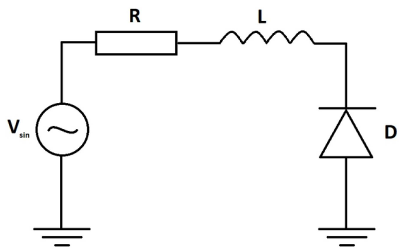

Fig. 1. Scheme of the RLD circuit.

The objective of the present paper was to study the behavior of the RLD circuit, to construct Feigenbaum's diagram, and to calculate Feigenbaum's constant, taking into account the simulation of voltage variations.

2. Procedure of simulation

The modeling of the RLD circuit was carried out in simulations using a PC computer (Processor Intel i7 10Gen, 32GB RAM) equipped with Wolfram Mathematica software. In order to simulate chaotic behavior of the system, the following equations were used

$$I(t) = \frac{\mathrm{d}q(t)}{\mathrm{d}t} \qquad (1)$$

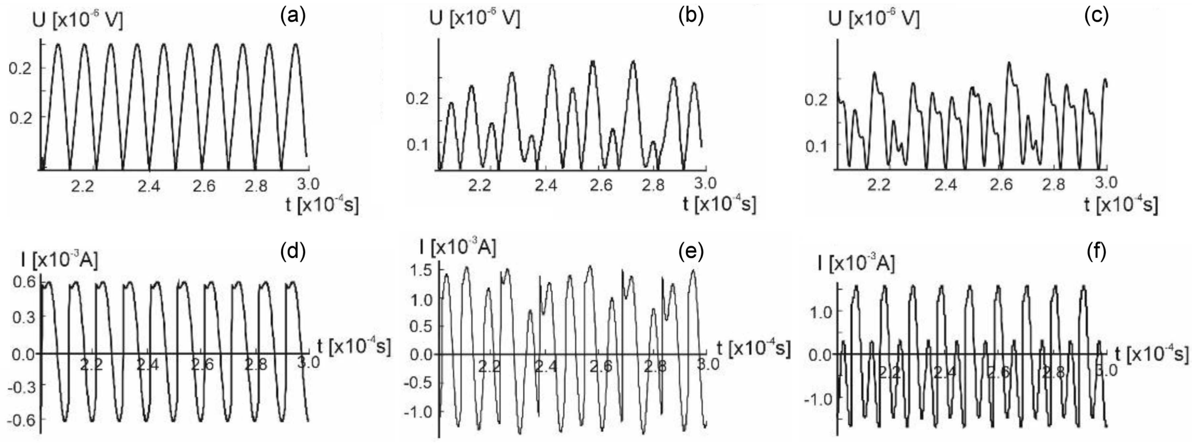

Fig. 2. Time dependences of voltage/current for the analyzed RLD circuit. Simulations were made for different values of the generator voltage (a)–(f).

and

$$LI'(t) + RI(t) + \frac{C_2 - C_1}{2C_1C_2}\left|q(t)\right| + \frac{C_1 + C_2}{2C_1C_2}\,q(t)$$
$$+U_a = U_d \sin(2\pi f t), \qquad (2)$$

where I denotes current, q denotes electric charge, t denotes time, L denotes induction of solenoid, R denotes resistance of resistor, C_1 and C_2 denote capacities of diode, U_a denotes voltage, U_d denotes maximum supply voltage, and f is a frequency of generator.

Equation (2) was implemented in Mathematica software and solved numerically using the Runge–Kutta algorithm.

The values of each parameters are: $L = 0.3$ mH, $R = 100\ \Omega$, $C_1 = 0.2\ \mu$F, $C_2 = 90$ pF, $U_a = 1.4$ V and $f = 150$ kHz. The diode parameters were taken for the real diode IN1233.

The simulation was done in a specific range of voltage.

3. Results and discussion

A time evolution of the current and voltage for different values of the generator voltage amplitude were shown in Fig. 2. Low values of the generator voltage result in stability of the analyzed system (Fig. 2a and d). However, with the increase in the voltage amplitude, unstable behavior of the voltage/current waveforms is induced (Fig. 2b and e). The multiplication of states is clearly visible for voltage higher than 0.5 V (Fig. 2c and f) and the chaotic response of these parameters is clearly seen.

In order to prove these observations, the phase space construction was done (Fig. 3). For stable states, the phase space has an ovoid shape (Fig. 3a). An increase in voltage amplitude induces the appearance of a characteristics deformation of an oval (Fig. 3b), and a further growth of the generator voltage results in characteristic loops or another deformation of the oval (Fig. 3c). Such behavior is characteristic for a chaotic state [7].

The best tool for analysis chaotic behavior and observations of doubling of period is the Feigenbaum diagram (Fig. 4). The analysis of the Feigenbaum plot revealed that for voltages lower than 0.125 V, the circuit responds with a stable signal. The double period was detected for the range 0.235–0.25 V, and another doubling of period was observed up to the value of 0.255 V, for which completely chaotic behavior was seen.

In order to calculate δ (Feigenbaum's constant), the construction of Feigenbaum diagram was used, taking into account following relation [4]

$$\delta = \frac{x_{n+1} - x_n}{x_{n+2} - x_{n+1}}, \qquad (3)$$

where x_n denotes the parameter changing with one state, and x_{n+1} or x_{n+2} are the parameters corresponding to the next doubling of the period.

For the investigated case, (3) should be given in the following correct form

$$\delta = \frac{U_{n+1} - U_n}{U_{n+2} - U_{n+1}}, \qquad (4)$$

where U_n is the changing voltage until the first bifurcation, and U_{n+1} and U_{n+2} are the voltages to the next two bifurcations.

Calculations of the Feigenbaum constant δ revealed its value of 4.54 ± 0.11. Compared to the theoretical predictions, for which this constant equals 4.6692, the result of our calculations corresponds very well to the theoretical predictions. Similar values were delivered by another groups [9, 10]. Taking the above into account, our value is reliable and reasonable.

The dynamic properties of the RLD circuit and its specific characteristics could be applied as one-time pads in secret communication. Changing the

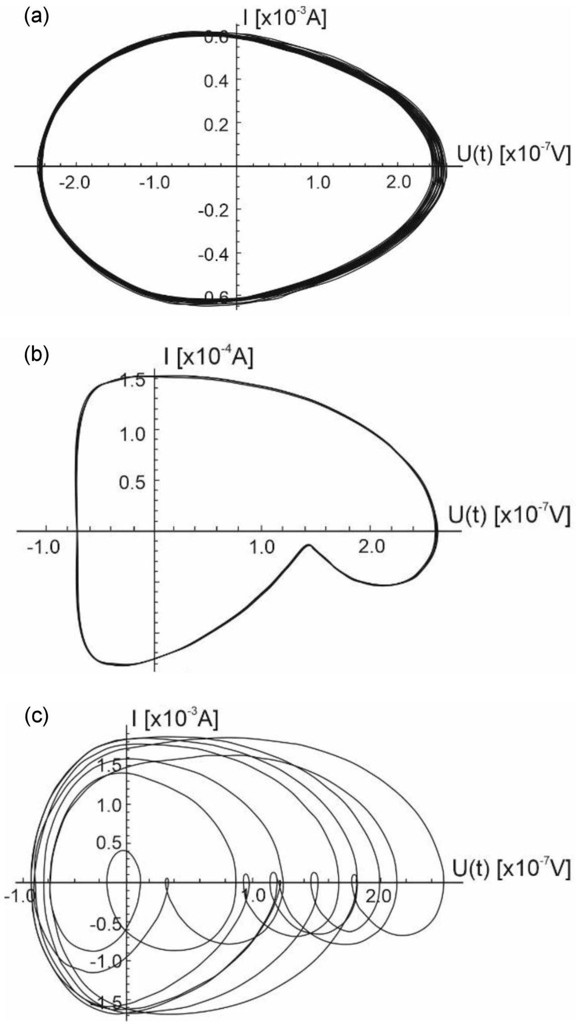

Fig. 3. Phase spaces for the analyzed RLD circuit. Simulations were made for different values of the generator voltage (a)–(c).

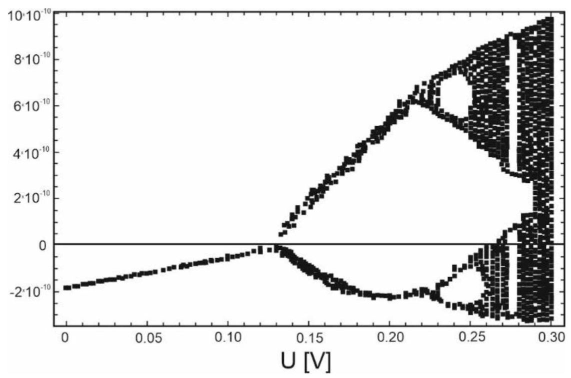

Fig. 4. The Feigenbaum's diagram constructed for the analyzed RLD circuit.

parameters will give many possibilities of results in different responses as output. Such property is extremely important during encoding and decoding.

4. Conclusions

In the present paper, the simulation of chaotic behavior of the RLD circuit was conducted. As it was presented, even such a simple system is very sensitive to the changes in voltage amplitudes. The time dependences of voltage or current showed chaotic behavior with the increase in the voltage of the generator. The chaos in the RLD circuit was confirmed by the construction of the phase space for different values of voltage. Multiplying of period was shown in the Feigenbaum diagram with the increase in voltage. The Feigenbaum diagram allowed to reveal the parameter δ (Feigenbaum's constant), which was 4.54 ± 0.11 and corresponded well to the theoretical predictions.

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