Multi-Quanta Abrikosov Vortices as the Micro Spin Batteries

Z. Bąk*

Institute of Physics, Jan Dlugosz University of Czestochowa, al. Armii Krajowej 13/15, 42-200 Częstochowa, Poland

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*e-mail: z.bak@ujd.edu.pl

The aim of our paper is to study the multi-quanta Abrikosov vortices injected into a superconductor layer by twisted light impulses. We predict that the condensate circulating around the core of a multi-quanta vortex may differ in nature from that of the bulk superconductor. Special attention is paid to the possibility of the appearance of quasi-angular momentum of Cooper pairs and its compensation via their recombination to fermion quartets. We show potential applications of multi-quanta vortices as massive memories, as well as spin, angular momentum, and energy batteries. Finally, we predict the emergence of half-quantum multi-quanta Josephson vortices under certain conditions.

topics: superconductivity, fractional spectral dimension

1. Introduction

In the type-II superconductors, beyond certain critical limit of the external magnetic field, supercurrent vortices circulating around normal cores arise [1]. The supercurrents forming the Abrikosov vortex induce a magnetic field with the total magnetic flux $\Phi_o = h/2e$ (fluxon). As a rule, spontaneously formed Abrikosov vortices are single-valued, i.e., each carries a single fluxon and forms regular (Abrikosov) lattices. Nevertheless, along with the experimental verification of early predictions of a ground state consisting of a lattice of singly quantized vortices, many reports on their nucleation have been made (see [2] and references therein). In a laminar system, vortices are strongly influenced by the sample geometry; the lateral confinement may even force them to merge into multi-quanta vortices otherwise unstable in bulk geometry [3, 4].

For technological applications, we cannot rely on spontaneously formed multi-quanta vortices. The simplest way to create them in a controlled manner is the excitation by the vortex light beams. Conventional optics involves photons that carry only linear momentum, however, light can carry both linear and orbital angular momentum (OAM). The latter has a spin component associated with polarization and an orbital part associated with the spatial structure [5]. The light beams with nonzero angular momentum are referred to as the *twisted light* or *vortex beams*. A useful feature of optical beams carrying orbital angular momentum is that they can be easily manipulated and transferred. This opens new horizons across various fields, much like the use of additional degrees of freedom in other types of excitations, such as electrons [6]. Notably, new applications range from mechanical micromanipulation [7] and imaging science [8] to communication technologies [9]. Beyond optical wavelengths, OAM now plays a major role in electron engineering [10, 11]. In biophysics and micromechanics, it has been proposed to utilize OAM-induced torque to drive molecular motors and micromachines [12]. It has also been shown that with the vortex light beams, we can excite twisted magnons, excitons, polarons, or electron wave packets.

The aim of our research is to study the interaction of vortex beams with magnetic and Abrikosov vortices in solid laminar systems. We focus our research on the theoretical study of various AOM excitations and their propagation in a multi-layered system consisting of an isolator film sandwiched between two superconducting layers. It is interesting that for a relatively thin normal interlayer, we have the Josephson junction, and under certain conditions, half-valued vortices (Josephson vortices) can be formed [13]. Adjoint half-valued Josephson vortices, each carrying the semifluxon, can merge with the Abrikosov vortex and form multi-half-valued items. In the following, attention will be paid to the formation of multi-quanta Abrikosov-like vortices generated either by the external magnetic field or by injection via OAM photons. Finally, we predict a novel form of the Hall-like effect due to the interaction of OAM beams with the multi-quanta Abrikosov vortices.

2. Vortex beams

Theoretical studies of the OAM excitations are not only of a cognitive nature. The practical benefit is that the orbital angular momentum allows OAM light/excitation to carry more bits of information per photon than spin alone, so the ability to manipulate the orbital component could open up new communication methods (e.g., in the 5G telecommunication [14] or faster data transfer). In addition, it is evident that controlling the interaction of spin and orbital angular momentum for the same quasiparticles may allow for novel kinds of logic operations in future optical devices.

The use of OAM current, i.e., a flow of twisted excitations, instead of the charge or spin current, is considered as a possible route to overcome the limitations of Moore's law. In the construction of faster and more compact computing circuits, information processing is inevitably accompanied by a certain amount of heat generation. The problems with removing redundant heat make studies of the problem turn back to the superconducting (SC) memories, which greatly reduce the dissipation power and increase the speed of operation. Especially since maintenance costs exceed the cost of refrigeration to cryogenic temperatures [15], Abrikosov vortices, representing spatially isolated single magnetic flux quanta in superconductors (SCs), can be inherent carriers of digital information in advanced microprocessors and memories [15, 16]. In some cases, magnetic-flux qubits are preferable to charge qubits, holding the potential to overcome certain limitations, such as the flux noise [17]. Until now, all concepts of Abrikosov lattice applications were limited to the mono-quanta vortices.

In the following, we will discuss the OAM photon-induced multi-quanta Abrikosov vortices that offer new possibilities for technological applications. Knowledge about the structure and dynamics of collections of vortices is of importance both for understanding the basic physics of superconductors and for the design of devices. The use of multi-quanta Abrikosov vortices in the construction of computer memories increases their capacity. Simultaneously, superconductivity allows for the reduction of the redundant heat generated.

The interaction of twisted light with a solid is always associated with the exchange of both linear and angular momentum. Such a type of excitation allows the creation of a variety of quasiparticles that carry the angular momentum. When the electron, exciton, etc., possess orbital momentum, the Schrödinger equation written in the cylindrical coordinate system (ρ, θ, z) gives the solutions that are no longer plane waves, i.e.,

$$\Psi_p(\boldsymbol{r}, t) \propto \mathrm{e}^{\mathrm{i}\,(\boldsymbol{p}\cdot\boldsymbol{r} - E(p)\,t)/h}, \tag{1}$$

with $\boldsymbol{r}$ being the position operator, but instead are given by the monochromatic twisted (vortex) Bessel beams [5]

$$\Psi_{p_\rho, p_z} = |p, l\rangle = \alpha\, J_{|l|}\left(\frac{p_\rho\, \rho}{h}\right) \mathrm{e}^{\mathrm{i}\,(p_z\, z - E\, t)/h}\, \mathrm{e}^{\mathrm{i}\, l \theta}, \tag{2}$$

or alternatively the Laguerre–Bessel (LG) beams

$$\Psi_{l,n}^{\mathrm{LG}} \propto \left(\frac{r}{w(z)}\right)^{|l|} L_n^{|l|}\left(\frac{2r^2}{w^2(z)}\right) \times \exp\left(-\frac{r^2}{w^2(z)} + \mathrm{i}\, k \frac{r^2}{2R(z)}\right) \mathrm{e}^{\mathrm{i}\,(l\phi + kz)}. \tag{3}$$

This wave function is the product of three types of functions: a plane wave with momentum p_z in the z-direction, a radial component written as a Bessel function of the first kind $J_{|l|}$, where the p_ρ is the linear momentum in the radial direction, and finally an azimuthal component written as $\exp(\mathrm{i} l \theta)$, where l (i.e., m_z) represents the magnetic quantum number related to the angular momentum L_z in the z-direction. Twisted light interacts with matter differently than spin does. If a particle sitting off-axis in a light beam absorbs a photon with orbital angular momentum, it responds by circulating around the beam, not by spinning on its axis.

The beams with $l > 0$ show quantum vortex $\exp(\mathrm{i} l \phi)$, spiral phase structure, nonzero azimuthal probability current, and the probability density vanishing on the axis $|\Psi|^2\big|_{r=0} = 0$. Gaussian beams or wave packets are often implied in quantum models of free electrons, because they do not contain any intrinsic structures and degrees of freedom. In contrast to that, higher-order modes with $(l, n) \neq (0, 0)$ exhibit a variety of structures related to different (l, n) or Bessel beams with different l, constituting a complete set of orthogonal monoenergetic modes for the free-space Schrödinger equation.

For such beams, the wavefront has a helical spatial structure around the beam axis. This refers not only to the optical waves but also to any other solid-state excitations induced by the OAM beams. For example, twisted electron beams can be produced at scanning electron microscopes with energies up to keV and orbital momentum as high $m\hbar = 1000\hbar$ (see [18] and references therein). However, with ultrashort pulses we are interested in, there is a fundamental restriction to the topological charge of the vortex, and hence upper bound to the OAM, carried by a pulse [19]. Generally, from the monochromatic waves, the localized wave packets (with the same m_z in the bundle) can be formed to produce the required properties useful in technological applications [20]. Recently, many forms of two-dimensional, structured light beams, such as Weber beams, Mathieu beams, vector beams, and beams with arbitrary transverse shapes, have been found [21]. In the following, we limit consideration to the Bessel (see (2)) and LG (see (3)) beams.

The fact that OAM beams can transfer both spin and angular momentum to the vortex opens the question about the nature of superconductivity that forms circulating supercurrents. One would expect the appearance of unconventional superconductivity associated with the multi-quanta vortices. As we know, the s-state Cooper pair states easily develop in an isotropic medium, however, in laminar systems, when the layer thickness is less than the coherence length, s-pairing is not favored. The bulk condensate forms the Cooper pairs that are not coherent with the vortex supercurrents, which means that both condensates, bulk and vortex, are completely decoupled. The latter forms a droplet that rotates with certain angular velocity ω, thus both condensates can be of different natures. Moreover, some experiments suggest spatial segregation of pairs formed of different electronic states [22, 23]. A stronger magnetic field within multi-quanta vortices, as well as a laminar structure, should favor non-s superconductivity. This can be, e.g., the triplet pairs or even OAM-endowed electron pairs. Additionally, the hallmark of triplet superconductivity is the host spontaneous currents at the sample surface [24], and Abrikosov vortices perfectly meet this expectation. Available experimental techniques allow for observation of vortices in nonconventional superconductors [24].

3. Quasi-angular momentum

In theoretical research, the focus is on intentionally generated excitations carrying the OAM. However, spontaneous formation of quasiparticles having nonzero angular or quasi-angular momentum can arise. The most known example of such an item is the electron/hole Cooper pair. As we have pointed out above, in laminar systems, when the layer thickness is less than the coherence length, s-pairing is not favored, as the motion of electrons is squeezed by the adjacent layer potentials. This is especially interesting for superconducting systems being the focus of our study. The Cooper pair with electron quasi-momenta $\hbar\boldsymbol{k}$ and $-\hbar\boldsymbol{k}$ and separated at a distance f the order of coherence length ξ forced to in-plane motion should exhibit quasi-angular momentum (quasi-AM) of the order of $\boldsymbol{L} \propto \boldsymbol{\xi} \times \hbar k$. As only the electrons close to the Fermi surface are active in the Cooper pair formation, we can estimate L as $l = \hbar\xi k_{\mathrm{F}}$. The opposite helicities produce opposite quasi-AM and respective magnetic moments. Quasi-momenta of Cooper pairs are quantized, and at a temperature of condensation, all pairs should exhibit the same quasi-momentum quantum number. At the onset of superconductivity, the quasi-momenta of Cooper (hole) pairs in the bulk systems are compensated [25, 26]. However, when there is a single Cooper pair tunneling across a magnetic barrier, it can sense the magnetization M of the spacer and, as a result, exerts action of the potential

$$V = V_c(r) + V_s(r)\, \boldsymbol{M} \cdot \boldsymbol{L}, \qquad (4)$$

with $\boldsymbol{M}$ and $\boldsymbol{L}$ being the barrier magnetization and Cooper pair quasi-angular momentum, respectively. In (4), V_c is the spin-independent potential, while $V_s(r)$ represents the quasi-momentum interaction with the magnetization of the spacer. This means that, e.g., Cooper pairs tunneling across a magnetic barrier should sense different heights, and the ferromagnetic layer should filter the Cooper pairs of different chirality. We should point out here that the skew scattering can also arise, represented by an asymmetric part of the transition probability [27]. For us, it is important that within the context of scattering experiments, the quasi-angular momentum and the true angular momentum play identical roles and cannot be distinguished [28].

At the interfaces, the mechanisms of compensation of the quasi-AM valid in the bulk [25] are not effective, and another mechanism can be switched on to lower the barrier given by (3). The Cooper pairs with opposite orientations of the quasi-AM can undergo a second pairing when two distinct pairs recombine into a four-electron/hole aggregate (quasiparticle) — a quartet [29]. This effect can be experimentally observed by the measurements of the Josephson current. In some systems, the persistent current across the Josephson junction oscillates in time with a basic period of $4eV/\hbar$ instead of $2eV/\hbar$, which can be interpreted as the effect of electron tunneling in quartets [29].

Such a scenario can be easily illustrated in the layered copper oxide superconductors. The high in-layer mobility of electrons causes the quasi-AM of the copper pairs to be oriented perpendicularly to the $\mathrm{CuO_2}$ plane. Suppose that there is another Cooper pair in the adjacent $\mathrm{CuO_2}$ layer with the opposite orientation of the quasi-AM, then their dipolar momenta compensate each other. The theory of such four electron binding into quartets that involves other mechanisms of coupling is presented in [30]. We should point out here that additional interactions can change the spectrum of quartets, and thus, the nature of the transition to the SC state can be modified [31].

An alternative mechanism to explain the compensation of quasi-AM of Cooper pairs at T_c is a two-fluid model. Cooper pairs of different chirality form separate condensates with the same gap and the same spatial density. In this scenario, opposite quasi-momenta are compensated similarly as the momenta of “spin up” and “spin down” electrons in the conduction bands of simple metals.

4. Interaction of twisted beams with vortices

We focus our interest on a laminar system, namely a SC/no-SC/SC sandwich, where SC denotes superconducting material. For an extremely thin central spacer (of the order of a few nanometers), this is just the Josephson junction, but we assume a rather mesoscopic dimension with larger thickness and lateral extension.

In the bulk type-II superconductor, under the action of the magnetic field perpendicular to the sample's surface, the formation of mono-quanta (Abrikosov) vortices arises. This occurs above a certain critical field strength H_{c1}. The vortex density increases with increasing field strength. In thin films, when type-II superconductors are fabricated, even small magnetic fields $B_c \approx 1\ \mu$T result in the creation of Abrikosov vortices, i.e., regions of supercurrent that circulate a non-superconducting core. In laminar systems, the formation of the Abrikosov vortices is not limited to the type-II superconductors. It is well known that superconducting films made of a type-I material can demonstrate a type-II magnetic response, developing stable vortex configurations in a perpendicular magnetic field [32]. We should point out here that vortex formation (orbital effect) strongly destabilizes the superconductivity, leading to a small upper critical field Hc_2. By contrast, when the field is applied parallel to the layers, most of the flux lines can penetrate between the SC layers, and then the orbital effect is strongly quenched. This is the main reason why Hc2 in the parallel direction is much higher than that in the perpendicular direction.

The way a superfluid acquires angular momentum is perhaps one of its most interesting properties. The creation of the Abrikosov lattice under the action of the external magnetic field is not the only way. With the use of OAM photons, we can inject multi-quanta vortices into the mesoscopic SC layer. Coherent pumping, injecting OAM particles directly into the condensate at an energy $\hbar\omega$, is described by a source term [33]

$$\hbar\nabla_t\psi = F\,\mathrm{e}^{\mathrm{i}\omega t}\mathrm{e}^{\mathrm{i}l\theta}. \tag{5}$$

OAM phase twist, defined in terms of the rotation of the isophase surface of the wave function, was shown to have physical, measurable effects in terms of the production of new defects [34]. The focused incident OAM light impulse of sufficient intensity can transfer part of its energy and angular momentum to the superfluid, forming a local phase defect. The orbital angular momentum may be removed from the beam mode and converted into currents within a superfluid. As we are focused on the laminar systems, the injected vortices can be the multi-quanta ones. This way, an alternative method for Abrikosov vortices creation in one component superfluid arises. Recently, such a direct transfer of light's OAM onto a non-resonantly excited polariton superfluid has been observed [35]. It is possible that the impact of a focused OAM photon can change the topological charge of the preexisting vortex. As the multi-quantized vortex in Bose–Einstein condensate (BEC) can carry any integer number of circulation quanta, the storage efficiency of cryogenic memory in this dense-coding of information is possible. This resembles the properties of the OAM signals, which also enable infinite communication channels for both classical and quantum communications. As a result, the application of SC active layers as the base of cryogenic memory opens the possibility of writing all of the information carried by a multi-quantum OAM wave packet in one multi-quantum Abrikosov vortex.

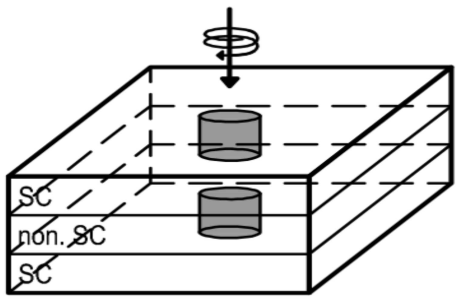

Fig. 1. Creation of twin vortices by the OAM photon impact.

Suppose we apply a magnetic field perpendicular to the surface, above H_{c1} a net of multi-quanta Abrikosov vortices is produced. If in the upper layer an Abrikosov vortex is generated, its magnetic field penetrates the second layer to a usual depth λ. The screening current crosses the (normal) interlayer and induces the superfluid of the other one. Provided that the thickness of the second layer is less than λ, the twin vortex in the lower SC layer is created. In this way, the geometry causes the formation of coaxial twin vortices in both SC layers (Fig. 1).

In this way, in systems composed of a few layers, a pancake of vortices can be formed. It is interesting that for a relatively thin normal layer, we have a Josephson junction, and, under certain conditions, a half-valued twin vortices (Josephson vortices) can be formed [13]. Adjoint half-valued vortices, each carrying a semi-fluxon, can merge with the Abrikosov vortices and form multi-half-valued items.

In equilibrium, the cores of Abrikosov vortices tend to become oriented perpendicular to the plane of the interface. Within the non-SC layer, a cylindrical region with a relatively strong magnetic field arises, which reproduces the Abrikosov lattice generated within adjacent SC layers. Thus, the interlayer mimics a magnetic lattice with localized magnetic momenta.

Now, let us consider the case when OAM wave packets travel laterally within such a non-SC layer with strongly non-uniform magnetization. The magnetic moment of the OAM photon (OAM excitation) interacts with the magnetic field that penetrates the non-SC layer. As a result, for the traveling OAM bullets, the Abrikosov lattice provides an ordered net of scattering centers. The magnetic field

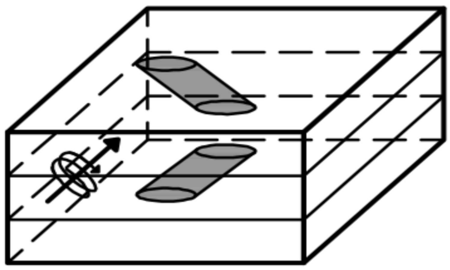

Fig. 2. Vortex tilt (not in scale) and OAM photon asymmetric deflection due to their mutual interaction.

associated with the Abrikosov influences the motion of OAM wave packets due to the magnetic interactions. The OAM bullet carries the orbital momentum $\boldsymbol{L}_b$ with the "z" component, $L_b^z = l_b \hbar$, while the Abrikosov vortex is associated with the angular momentum $\boldsymbol{L}_v$ equal to $L_v^z = l_v \hbar$; l_v and l_b represent the topological charges of the OAM bullet and Abrikosov vortex, respectively. With these abbreviations, the effective interaction can be written as $H \propto \boldsymbol{L}_b \times \boldsymbol{L}_v$. This asymmetric interaction creates a novel type of the Hall effect, the *OAM Hall effect*, which deflects asymmetrically the OAM beams and produces orbital momentum excess at one of the surfaces. Classical spin Hall effect, which is a counterpart of the *OAM Hall effect*, is composed of three components: skew scattering, side jump, and Berry phase contributions. Studies of the OAM light reflection at the interface show, along with the classical reflection, a side shift effect. This indicates that the light *OAM Hall effect* should comprise all three components.

The ballistic transport of the OAM current $\boldsymbol{J}_o$ classically is given as the expectation values of the velocity $\boldsymbol{v} = \mathrm{i}\,[H, \boldsymbol{r}]_a$ and momentum operators [36]

$$\boldsymbol{J}_o = \frac{1}{2}\,[\boldsymbol{v}, L_z]_a\,, \tag{6}$$

where $[\ ,\ ]_a$ denotes the anticommutator. When we have a strong coupling between the magnetic momenta of the AOM packet and those of the underlying medium, angular momentum currents are not conserved. The effective currents follow the formula $J_\alpha^{eff} = J_\alpha + T_\alpha$, where α denotes the OAM current component, and T_α is the dipole torque density that varies the initial currents. The continuity equation for the effective OAM current requires

$$\frac{\partial L_z}{\partial t} + \nabla \cdot \boldsymbol{J}_o = T_z^o, \tag{7}$$

where T_z^o is the torque density; with this, the effective OAM current J_o^{eff} in z direction reads as

$$J_o^{eff} = J_o + x\,\tau_o, \tag{8}$$

where $x\,\tau_0$ gives the correction due to the OAM torque.

The Abrikosov vortices are not rigid items, thus, during the scattering process, some portion of the momentum and energy of the traveling OAM wave packet can be transferred to the vortex and induce its motion. We should remind here that although vortices are commonly assumed to exit the superconductor perpendicular to the surface there were found tilted vortex lattices in $NbSe_2$ under the action of tilted fields [37]. As we are considering a SC/no-SC/SC sandwich, we should note that under action (perpendicular to the surface) of magnetic field or OAM photon, twin vortices in both SC layers are created. Now, suppose that in our system with a certain concentration of vortices within the non-SC layer, an OAM photon or beam travels. In such a situation, due to the mutual interaction of the OAM bullet and Abrikosov vortex, asymmetric Hall deflection of the OAM and the vortex shift and/or tilt appear (see Fig. 2).

With external fields, this can even induce a kind of Larmour precession of the vortices. Thus, it is of interest to account for the tilt and possibly induced precession of vortices due to the interaction between the SC vortex and OAM beam since this affects the inter-vortex interactions and the Hall effect [38]. The phase of the precession in a row of vortices differs due to the delay in the time of excitation. In this way, a picture arises in which the precession of vortices mimics the semiclassical spin wave model. However, contrary to the spin waves, the precession of vortices is damped, and the initial orientation of the vortex cores is restored. As we have discussed above, the Abrikosov vortices can be created in two ways, namely by the application of external fields or by OAM photon irradiation. In the field-induced vortices, the external magnetic field forces tend to restore initial orientation. In both types of Abrikosov vortices, dissipation of the energy occurs due to the normal core resistivity. It is well-known that the motion of vortices induces resistance, and as a consequence, the effective current $\boldsymbol{J}$ swirling the normal core is given by

$$\boldsymbol{J} = \left(\frac{1}{\kappa}\nabla\Phi - \boldsymbol{A}\right)|\psi|^2 - \sigma\left(\frac{\partial \boldsymbol{A}}{\partial t} - \nabla\Phi\right), \tag{9}$$

where $\boldsymbol{A}$ and Φ are vector and scalar potentials, while σ in the second term denotes the classical conductivity [39]. The classical component is responsible for the dumping of the vortex precession.

When thinking about the Abrikosov vortex, we automatically assume the currents swirling around its normal core to have cylindrical symmetry and their monotonic decay with the separation from the center of the vortex. Evidently, for the vortices created by the external magnetic field, this is the ground state of phase defect. However, one may expect that the vortices created by the OAM photons may inherit their symmetry. Thus, another form of the Abrikosov vortex with currents reproducing spatial distribution of the LG beams (3) is worth considering, and we can have many excited forms of the vortex. The relaxation to the ground states occurs due to the energy loss within the normal core (see (9)).

5. Conclusions

Single OAM photons/excitations are of paramount importance for modern quantum information science [40], since the OAM can be used as an additional degree of freedom with a view to achieving high capacity and high spectral efficiency in communication and computational systems. For example, OAM wave packets offer efficient control of charge flow by acting on their orbital moment [41]. Recent progress in microfabrication techniques has brought electronic devices of the order of or smaller than the OAM diffusion length, which provides possibilities for a variety of applications [42]. Both achievements can be applied to improving existing or generating new technologies and computing concepts, widening the potential of solid-state orbitronics. This was the main motivation for our research.

We have focused on a study of the sandwich composed of a normal material and two superconducting overlayers. The crucial effect is that the vortex light beams/photons can transfer their orbital angular momentum to the internal dynamics of the solid-state system. In this way, in the laminar systems, the multi-quanta Abrikosov vortices can be created. This opens the possibility of application in computer memories based on Abrikosov lattices. The information can be encoded by multiplying the number of distinguishable states, because an AOM photon can carry an arbitrarily large amount of information distributed over its spin and orbital quantum states and transfer it to the multi-quanta Abrikosov vortices.

In our study, we have proved that twisted light beams can create multi-valued Abrikosov vortices, carrying many fluxons each. It has also been shown that multi-valued Abrikosov vortices can serve as an effective medium to store information, spin, angular momentum, and energy. Moreover, in the case of the Josephson junction, we postulate that half-valued vortices merge to the multi-valued ones. The hypothesis that at Josephson junctions, multi- and semi-valued vortices can be formed needs experimental verification. The same applies to the possibility of collective Larmor precession of the vortices under the action of the OAM beams. The respective experimental techniques are in common use. Thus, the verification of the hypothesis formed above should be easy.

Another concept is based on the fact that the Abrikosov vortex is immersed in a bulk condensate, and its supercurrents condensate is not coherent with it. As we know, magnetic fields within the vortex favor the non-s state. Thus, electron pairs forming vortex supercurrents can be in a different electronic state than this of bulk condensate. Considerations above show that from the basic physics point of view, there are still some very important aspects of superconductivity that need to be understood.

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