

Computation-Efficient Precise Determination of Tilt Using Acceleration Measurements

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A custom instrument designed to minimize errors of tilt detection (based on acceleration measurements) due to misalignments of the employed microelectromechanical system sensors is presented. Owing to special housing, the sensors can be easily aligned and calibrated, and the instrument can be precisely mounted in the end device that utilizes tilt detection. The procedure of a standard calibration for the sensors is briefly described, and a novel method of their alignment is discussed. Thus, a new computation-efficient solution for the precise determination of dual-axis tilt using low-cost accelerometers is introduced. A related application of the instrument (monitoring of axial run-out) is mentioned.

topics: microelectromechanical system (MEMS), accelerometer, tilt, alignment

1. Introduction

When a physical vector quantity is to be measured, it is crucial to align the vector/component vectors with the sensitive axis/axes of the measuring instrument used. If this issue is neglected, not only are the measurements less accurate, but even the calibration of the measurement instrument may be affected [1].

In the case of using low-cost instruments like MEMS accelerometers or gyroscopes, their alignment is usually too complicated or too costly, thus another solution is proposed, i.e., numerical compensation for the existing misalignments. A matrix equation for the estimated misalignments values is typically used [2–4], both in the case of calibrating the sensors by means of test rigs [5] or autocalibration [6]. Besides, the application of empirical formulas [7] or affine coordinate system [8] has been proposed.

A normalized vector of acceleration $\mathbf{a}_n$ can be determined using the following relation [8]

$$\mathbf{a}_n = T S (\mathbf{a}_m - \mathbf{b}), \quad (1)$$

where T is misalignment matrix; S — sensitivity matrix; $\mathbf{a}_m$ — measured vector of acceleration; $\mathbf{b}$ — offset vector.

In general, the misalignment matrix includes 6 terms (2 component misalignments for each axis), the scale factor matrix includes 9 terms (3 main and 6 cross-axis sensitivities) and the offset vector 3 terms.

On the one hand, numerical compensation considerably reduces the influence of existing misalignments. Yet, on the other hand, it complicates the related computations, which is unacceptable in some cases (e.g., due to low computational power, limited data storage or necessity of a quick response).

An alternative solution involves a physical alignment of each sensitive axis of the accelerometer/accelerometers, which makes it possible to realize precise and simple measurements. However, physically aligning the accelerometer is a toilsome work of an experimental nature. In order to make the work easier, a special sensor and alignment procedure have been developed.

We present a novel solution for precise determination of dual-axis tilt using low-cost sensors.

2. Mechanical structure of the sensor

2.1. The housing

When aligning an integrated three-axis *microelectromechanical system* (MEMS) accelerometer, it is not possible to do it precisely in the case of all three sensitive axes, because of the existing perpendicularity errors, which have their origin in an imperfect microfabrication process. A solution to this problem may be the application of three uniaxial accelerometers with a mechanical structure that uses a method of their mounting, which allows their sensitive axis to be precisely aligned.

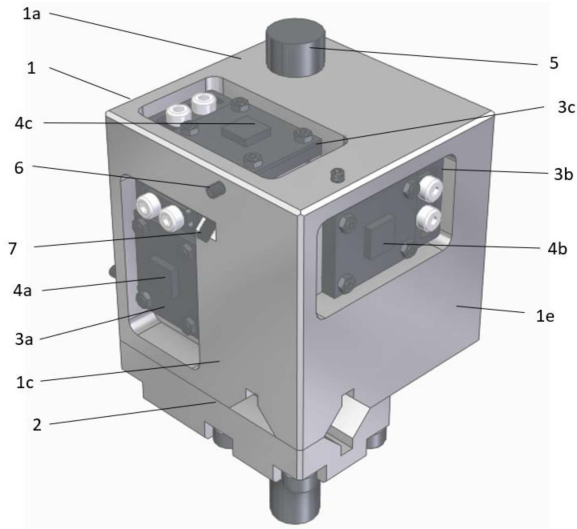


Fig. 1. The sensor mounted on the alignment shaft using the hole — top view.

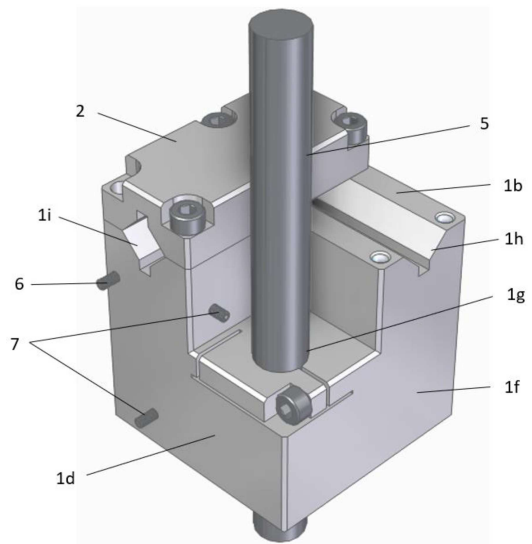


Fig. 2. The sensor mounted on the alignment shaft using the hole — bottom view.

In order to minimize the existing misalignments, which are of a random character, we implement some experimental procedures using dedicated test rigs, presented, e.g., in [9], striving for each sensitive axis of the tested accelerometer to be aligned with respect to the rotation axis of the test rig.

High accuracy of the presented sensor is achieved owing to a patented custom housing [10] illustrated in Figs. 1–4, containing three single-axis accelerometers instead of an integrated triaxial accelerometer.

The housing (no. 1) is made of hard metal (we suggest steel or hard aluminum alloy such as AW-2017A). The housing is basically cubic in shape, with a side length of 65 mm. On three (nos. 1a, 1c, 1e) adjacent walls of the cube there are located cut-outs that allow *printed circuit boards* (PCBs) with MEMS accelerometers (nos. 4a, 4b, 4c) to be installed on special aligning units (nos. 3a, 3b, 3c), which allow the accelerometer sensitivity axis to be aligned with the calibration planes of the cube. A cut-out is located on the opposite corner to ensure weight reduction and technological access to one of the aligning screws. Three other walls (nos. 1b, 1d, 1f) are also calibration planes.

Two perpendicular grooves there are created in the plane (no. 1b). Perpendicularly to the plane (no. 1a), a hole (no. 1g) with the diameter fitting the alignment shaft (no. 5) is drilled. The hole clamp with a screw was milled on the opposite side to secure the sensor on the shaft (no. 5).

The sensor can be also secured on the shaft (no. 5) by means of the clamp (no. 2) and the prismatic groove (nos. 1h or 1i), each formed by a pair of perpendicular surfaces in the sensor housing (no. 1).

The sensitivity axes of the MEMS accelerometers are perpendicular to each other since their mounting on compliant fixtures allows them to be precisely

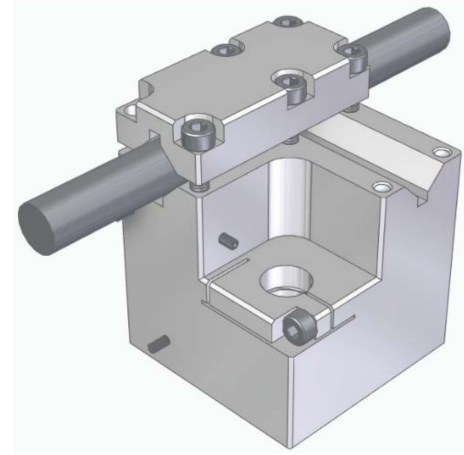


Fig. 3. The sensor mounted on the alignment shaft using the first groove and the clamp.

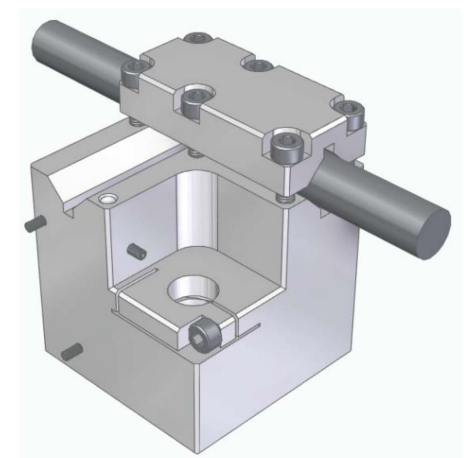


Fig. 4. The sensor mounted on the alignment shaft using the second groove and the clamp.

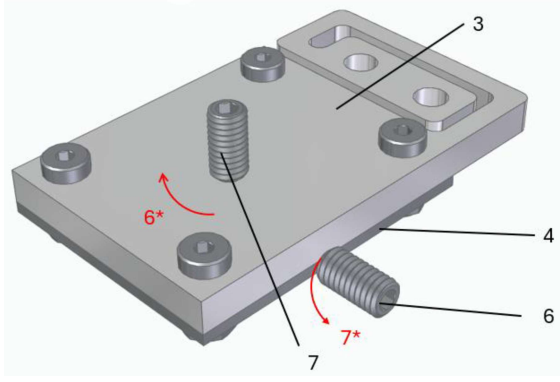


Fig. 5. The aligning unit.

aligned with the appropriate calibration surfaces of the housing by means of aligning screws (nos. 6 and 7).

2.2. The aligning unit

The aligning unit presented in Fig. 5 was also used in our other inventions, presented, e.g., in [11]. It consists of a flat compliant metal plate, secured inside the recesses in the sensor housing by means of two screws (see Fig. 1). Its deformations are used to precisely align the sensitive axis of the accelerometer with respect to the datum axes and calibration planes of the housing. The accelerometer PCB (no. 4) is screwed to the rigid part of the plate (no. 3). The deformation of the plate is obtained by applying pressure to the surfaces of the unit using the aligning screws (nos. 6 and 7), which results in a rotation according to arrows (nos. 6* or 7*, respectively).

The proposed sensor consists of three aligning units mounted inside the housing, coupled with three corresponding pairs of the aligning screws.

3. Preparation of the sensor

3.1. Alignment

Since we aim at precise tilt measurements, we must ensure not only a precise calibration of the sensor, but most importantly, appropriate alignment of its sensitive axes. This results from the fact that the related complex uncertainty of the measurements is mostly affected by these components of the gravitational acceleration that have a small magnitude. In such a case, they are very sensitive to any misalignments.

In order to align the sensitive axis of each accelerometer, the special aligning shaft (no. 5) is used. It is to be inserted first into the hole (no. 1g)

(as illustrated in Fig. 2), and then into the two prismatic grooves (no. 1h) and (no. 1i) and fixed using the clamp (no. 2) (as illustrated in Figs. 3 and 4).

The alignment consists in a parallelism of the particular sensitivity axis of the MEMS accelerometer with the axis of symmetry of the aligning shaft (no. 5). The parallelism is precisely adjusted using the alignment units (nos. 3a–3c). The alignment shaft (no. 5) with the attached housing (no. 1) is mounted in a special test rig, having the symmetry axis of the shaft oriented horizontally. Then, it is rotated around the symmetry axis.

The aligning procedure consists in minimizing, by means of the aligning screws (nos. 6 and 7), the amplitude of the recorded output signal of the sensitive axis of a respective accelerometer, generated while rotating the aligning shaft (no. 5) and the housing (no. 1), as expressed by the following equations

$$\min(A_x, A_y, A_z) = \begin{cases} A_x = U_{x_{\max}} - U_{x_{\min}}, \\ A_y = U_{y_{\max}} - U_{y_{\min}}, \\ A_z = U_{z_{\max}} - U_{z_{\min}}. \end{cases} \quad (2)$$

The procedure is illustrated in Fig. 6. While the sensitive axis is misaligned, the amplitude of the output signal is considerable. However, after few repetitions of the procedure, the desired accuracy is achieved. The remaining variations of the output signal represent the measurement noise.

The alignment procedure can be performed periodically to eliminate errors that may occur over time due to various reasons.

3.2. Calibration

In order to calibrate the accelerometer outputs, it is necessary to use an appropriate reference source. In our case, owing to the relatively small measurement range of the applied accelerometers, the reference is the gravitational acceleration. The related calibration procedure is illustrated in Fig. 7, according to [12].

The sensor housing (no. 1) is placed on a flat levelled surface, and the output signal of all the three accelerometers is read while successively supporting the housing on each of its six calibration walls (nos. 1a–1f).

Then two accelerometer calibration parameters are calculated as follows

$$a_{x,y,z} = \frac{U_{x,y,z_{\max}} + U_{x,y,z_{\min}}}{2}, \quad (3)$$

$$b_{x,y,z} = \frac{U_{x,y,z_{\max}} - U_{x,y,z_{\min}}}{2}, \quad (4)$$

where a is offset of output signal; b — scale factor of output signal; $U_{x,y,z_{\max}}$, $U_{x,y,z_{\min}}$ — maximum and minimum values of output signal assigned to particular sensitive axis at six calibration positions.

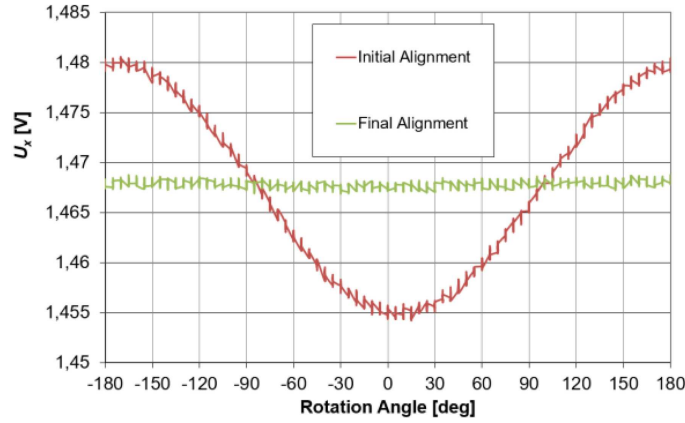


Fig. 6. Output signal of the sensitive axis during the alignment procedure (while misaligned and then aligned).

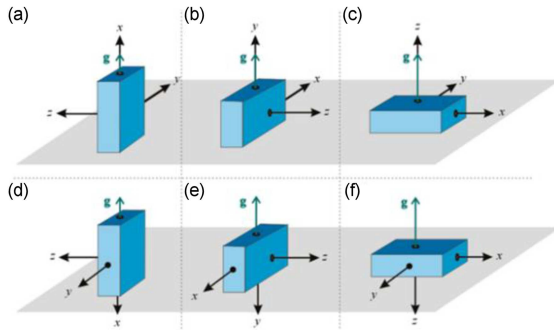


Fig. 7. Six-position calibration of the sensor.

Knowing the values of offsets and scale factors, the cartesian components g_x , g_y , g_z of the gravitational acceleration g can be determined as follows

$$g_{x,y,z} = g \frac{U_{x,y,z} - a_{x,y,z}}{b_{x,y,z}}. \quad (5)$$

The procedure can be performed several times in order to average the indications and in this way obtain higher accuracy.

4. Tilt measurements

While performing tilt measurements, the sensor housing is fixed to the monitored device using the hole (no. 1g) with compliant clamp or alternatively one of the prismatic grooves (nos. 1h or 1i) and the clamp (no. 2).

Owing to precise alignment of the sensitive axes of the accelerometers, the components of a dual-axis tilt can be determined on the basis of their output signals according to the following simple formulas [6]

$$\alpha = \arctan \left(g_x / \sqrt{g_y^2 + g_z^2} \right), \quad (6)$$

$$\gamma = \arctan \left(\frac{g_y}{g_z} \right), \quad (7)$$

where α is pitch, and γ is roll.

In the case of determining single-axis tilt [13] or axial tilt, only a simplified form of (5) is used, namely

$$\alpha = \arctan \left(\frac{g_x}{g_z} \right). \quad (8)$$

Application of formulas based on the arctangent function to determine tilt (both pitch, roll and axial tilt) ensures the lowest uncertainty of the measurements [14].

5. Conclusions

A novel tilt sensor is proposed. Owing to application of a special housing, it is possible to precisely align all three sensitive axes of the MEMS accelerometers used. Then, it is possible to use simple formulas to determine the pitch and roll without compromising accuracy and ensuring at the same time computation efficiency. Simplicity of computations is often striven for, when a quick response of a control system is required, as, e.g., in [15] or in the case of a limited data storage or low computational power, as, e.g., in [16].

Since various new applications of MEMS accelerometers have been constantly proposed, such as described by the authors in [17, 18], the presented instrument provides an opportunity to develop existing or even creation of new devices whose operation requires performance of precise tilt measurements at low cost. The application of low-cost MEMS accelerometers makes it possible to obtain this goal. However, a certain challenge regarding the cost is the fabrication of the housing. Once the housing requires machining, the related costs are high. However, new techniques, such as additive manufacturing, can be applied, as in the case of other precise mechanical members, e.g., sliding bearings [19].

Since the sensor allows for periodical recalibration and readjustment of the accelerometers, it provides a possibility of eliminating errors resulting from the instability of low-cost devices [20], such as aging effects or long-term drifts [21].

Moreover, we succeeded in designing two similar sensors presented in [11], which make it possible to monitor axial run-out of a rotating element.

References

- [1] Z. Syed, P. Aggarwal, C. Goodall, X. Niu, N. El-Sheimy, *Meas. Sci. Technol.* **18**, 1897 (2007).
- [2] A. Olivares, G. Ruiz-Garcia, G. Olivares, J.M. Górriz, J. Ramirez, *Sensors* **13**, 11797 (2013).
- [3] S.P. Won, F. Golnaraghi, *IEEE Trans. Instrum. Meas.* **59**, 2144 (2010).
- [4] W.T. Ang, P.K. Khosla, C.N. Riviere, *IEEE Sens. J.* **7**, 81 (2007).
- [5] W. Yang, B. Fang, Y.Y. Tang, J. Qian, X. Qin, W. Yao, *IEEE Sensors J.* **13**, 2313 (2013).
- [7] I. Frosio, F. Pedersini, N.A. Borghese, *IEEE Sens. J.* **12**, 2100 (2012).
- [8] J. Qian, B. Fang, W. Yang, X. Luan, H. Nan, *IEEE Sens. J.* **11**, 2301 (2011).
- [9] M. Šipoš, P. Pačes, J. Rohac; P. Novacek, *IEEE Sens. J.* **12**, 1157 (2012).
- [10] M. Zams, S. Łuczak, in: *Inter-Academia 2023. Lecture Notes in Networks and Systems*, Vol. 939, Eds. Y. Ono, J. Kondoh, 2024 p. 202.
- [11] S. Łuczak, M. Zams, Polish Patent Office, patent pending; P.449531, 2024 (in Polish).
- [12] M. Zams, P. Pieńczuk, S. Łuczak, *Acta Phys. Pol. A* **146**, 419 (2024).
- [13] S. Stančin, S. Tomažič, *Sensors* **14**, 14885 (2014).
- [14] S. Łuczak, *Mechatronics. Recent Technological and Scientific Advances*, Springer-Verlag, Berlin Heidelberg 2012, p. 705.
- [15] S. Łuczak, *Recent Advances in Mechatronics*, Springer-Verlag, Berlin Heidelberg 2007, p. 511.
- [16] S. Lian, W. Meng, K. Shao, J. Zheng, S. Zhu, H. Li, *IEEE Trans. Ind. Electron.* **70**, 3975 (2023).
- [17] D. Farago, B. Maczak, Z. Gingl, *IEEE Access* **12**, 38102 (2024).
- [18] S. Łuczak, in: *Recent Global Research and Education: Technological Challenges*, Eds. R. Jabłoński, R. Szewczyk, Springer International Publishing, Switzerland 2017, p. 287.
- [19] S. Łuczak, M. Zams, *Adv. Intell. Syst. Comput.* **1044**, 274 (2020).
- [20] Z. Kusznerewicz, M. Michałowski, S. Łuczak, K. Bagiński, B. Kabziński, M. Zams, *IEEE Access* **8**, 78622 (2020).
- [21] S. Nadig, A. Lal, in: *2018 IEEE Int. Reliability Physics Symp. (IRPS), Burlingame (CA)*, 2018, p. 3C.8-1.
- [22] S. Łuczak, M. Zams, B. Bąbrowski, Z. Kusznerewicz, *Sensors* **22**, 1504, (2022).