

Phenomenological Model for the Extended Cross-Float Method

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Deadweight piston gauges are vital for metrology and calibration, with their effective area traditionally determined through labour-intensive methods or precise geometric measurements. This paper introduces an extended cross-float method that simplifies effective area determination by avoiding detailed equilibrium point adjustments. The proposed method uses dynamic displacement data instead of static comparisons, potentially improving calibration efficiency. Experiments using laser sensors revealed a clear link between fall rate variations and mass discrepancies, emphasising the need for consistent reference levels. While the extended method shows promise, further research is necessary to address the impact of variables like piston temperature, rotation speed, and air density, particularly for high-pressure and gaseous systems.

topics: cross-float method, pressure balance calibration, piston fall rate, effective area calibration

1. Introduction

Deadweight piston gauges are currently the most widespread and arguably the fundamental measuring instruments used as reference standards in National Metrology Institutes and calibration laboratories worldwide [1]. These instruments are considered primary standards because they realise the unit of pressure based on units from other measurement domains. One such domain is the unit of length in relation to the dimensions of the piston and cylinder, which is represented by the parameter known as the effective area of the measurement assembly [2]. The most common method of determining this parameter is the so-called cross-float method [3]. An alternative approach involves calculating the effective area based on precise geometric measurements of the piston and cylinder [4]. However, this method has several drawbacks, including high labour intensity, the necessity of using laboratories from a different measurement domain, and, most importantly, the limitation imposed by the size of the measurement assembly — this method is typically applied to assemblies with large diameters [5]. The aim of this paper is to present a phenomenological model of the extended cross-float method, which is an approach based on determining the effective area without the need for laborious equilibrium point determination.

2. State of the art

The determination of the effective area using the cross-float method is based on the fundamental principle of identifying the equilibrium point between the standard and the calibrated deadweight piston gauges [6]. According to this principle, the fall rate of the piston in the measurement assembly of the standard gauge should be identical when operating individually (isolated from the rest of the measurement system) and when connected to the calibrated instrument. To compare this parameter, it is essential to measure the piston's fall rate [7]. Various displacement sensors or optical methods are employed for this purpose. This approach has the undeniable advantage that the accuracy of the displacement measurement (and consequently the fall rate) is not critical — rather, repeatability is essential. In this context, the sensors serve more as indicators than measuring devices, confirming that the attained state corresponds to the desired measurement point, as defined. However, achieving the equilibrium point in the cross-float method requires a labour-intensive process of adjusting masses by adding or removing additional weights from the standard weights [8]. This process is also time-consuming, further complicating its execution due to time-dependent factors affecting

the measurements, such as the piston temperature (which directly influences thermal expansion) or air density (which affects the buoyancy of the standard weights). With the continuous advancements in technology and the improved quality of measurement assemblies, the detectable differences in mass during these measurements can be as small as a single milligram [9].

An attempt to address the challenges of the standard cross-float method is the extended cross-float method. This method is based on the concept that determining the effective area does not necessarily require the laborious and time-consuming process of balancing the measurement assemblies but can instead be performed dynamically based on recorded displacement. This could be implemented as a final variant, with a full uncertainty budget and final results, or as an intermediate variant, serving to estimate the mass needed to be added or removed to achieve the equilibrium point. This method has the potential to significantly simplify the procedure, ultimately leading to increased availability of dead-weight piston gauge calibration services in calibration laboratories, particularly in accredited laboratories.

3. Methodology

To establish a phenomenological model of the behaviour of deadweight piston gauge assemblies using the extended cross-float method, a measurement setup was prepared, consisting of two measurement assemblies with similar effective areas (Fig. 1). These instruments were connected into a single measurement system, allowing for smooth pressure adjustment, separation of assemblies by a valve, and measurement of the fall rate of the standard piston [10]. For this purpose, two Micro-Epsilon ILD1420-25 laser sensors with a measurement range of 0–25 mm were used [11]. These sensors were placed beneath the first standard weight, enabling measurements at successive measurement points without the need to reposition or reconfigure the sensors. This placement did not allow for the measurement point to be positioned optimally, i.e., along the piston axis, so two sensors were positioned symmetrically relative to the piston's axis of rotation (Fig. 2). This setup significantly reduced the impact of runout and systematically smoothed out the imperfections of the standard weight [12].

For this measurement setup, measurement points were evenly distributed across the measurement range of the deadweight piston gauges used: 5, 10, 15, 20, and 25 kg, corresponding approximately to pressures of 5, 10, 15, 20, and 25 bar. At each of these points, the balancing of the measurement assemblies was experimentally conducted. Adding or removing trim masses eventually led to finding the equilibrium point according to the standard

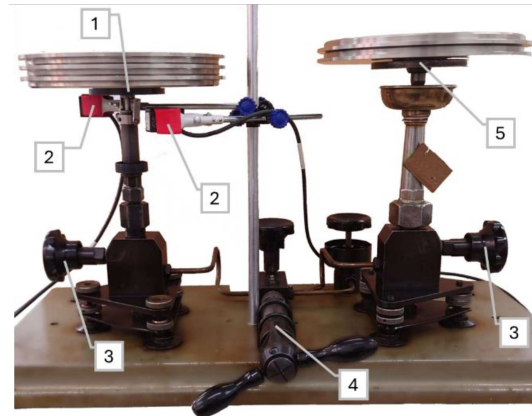


Fig. 1. Measurement setup; 1 — pressure balance standard, 2 — laser sensors, 3 — shut-off valves, 4 — pressure pump, 5 — pressure balance under test.

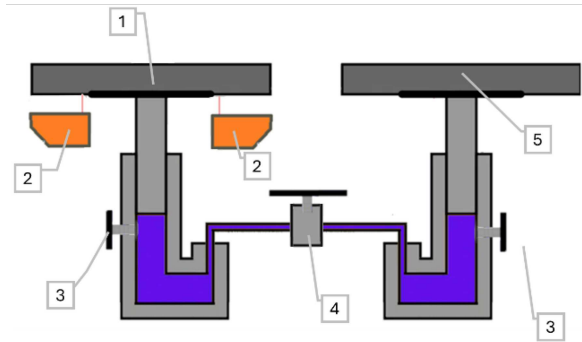


Fig. 2. Schematic diagram of the test stand; 1 — pressure balance standard, 2 — laser sensors, 3 — shut-off valves, 4 — pressure pump, 5 — pressure balance under test.

cross-float method. Subsequently, the measuring instruments were separated using a valve, after which the mass on the standard piston was modified by the following values: $-30, -20, -10, -5, -2, 0, +2, +5, +10, +20, +30$ g. The valve was then opened, causing the system to lose equilibrium on one side. This change was recorded by measuring the displacement of the standard piston using laser sensors.

4. Results

The collected data required additional processing to enable direct comparison. The fall rate of a piston operating individually, without connection to the entire system, depends primarily on the load placed on it. This means that the recorded passive fall curve will have a different slope for a measurement point with a 5 kg load than for one with a 30 kg load. Therefore, it was necessary to adjust the data individually, relative to the trend line determined each

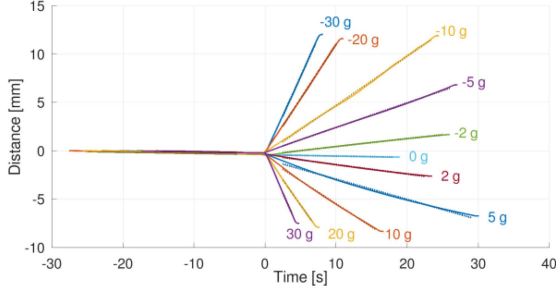


Fig. 3. Distance [mm] vs time [s] for total mass of 25 kg with separated measurements series.

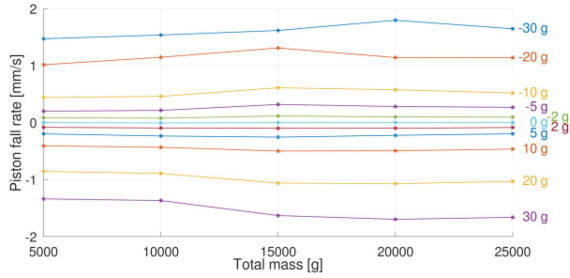


Fig. 4. Fall rate of the piston [mm/s] vs total mass [g] with separated measurements series.

time for each data set. This approach allowed for the introduction of corrections, eliminating differences resulting from the sum of the standard weights in different measurement points [13].

The nature of data collection necessitated that preliminary computation also includes standardisation of the approach concerning the moment of cut-off valve opening. This critical event is distinctly observable on the graph as a sudden transition from the nearly flat line representing the passive fall rate segment to the sloped lines reflecting the change in the piston's movement speed (Fig. 3).

5. Discussion

The obtained results clearly indicate an observable relationship between the change in the fall rate of the piston in the deadweight piston gauge assembly and the surplus or deficit of mass in the standard weights. The experimental data collection process also highlighted that a critical aspect of the measurements is establishing a constant and unchanging reference level in successive measurement points for both the standard and the calibrated instrument. Improper positioning of one of the pistons' operating positions could be problematic, especially with a liquid medium transmitting pressure [14]. The results of the measurements also suggest that the extended cross-float method has clear limitations regarding trim mass — the more

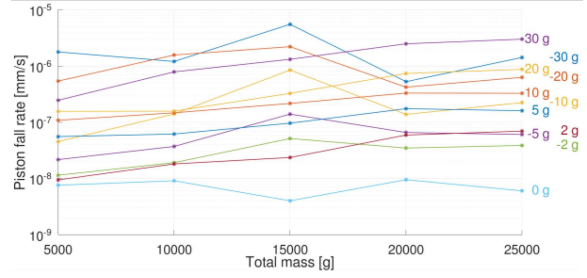


Fig. 5. Standard deviation of piston fall rate.

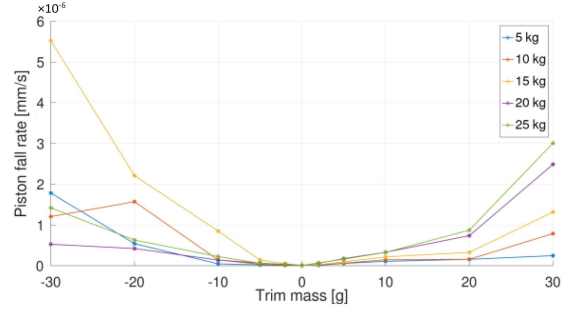


Fig. 6. Standard deviation of piston fall rate [mm/s] vs trim mass [g] with separated total mass series.

significant the mass deficit/surplus, the more difficult it becomes to accurately determine the piston's fall rate due to the limited range of available displacement. Moreover, additional studies will be required for measurement systems characterised by a gaseous reference medium, as well as for high-pressure measurement systems, where the influence of λ coefficient is significant.

Interesting correlations can be deduced from the analysis of the graph depicting the relationship between the fall rate and the total mass of the weights (Fig. 4).

Hypothetically, the symmetry should be expected in the measured fall rate of the piston values for pairs of opposite points (e.g., -30 g and 30 g). In practice, this trend is clearly observable; however, the deviations suggest the possibility of a significant influence of factors that were not recorded during the measurements, such as the temperature of the piston-cylinder unit, the rotation speed of the piston, and the air density.

This is also reflected in the standard deviation calculated for the piston fall rate, based on linear regression for displacement vs time, for each of the measurement series (Fig. 5) [15].

A clear trend is noticeable, namely the results deteriorate as the excess or deficit of mass relative to the zero point increases. The values of these deviations indicate that, for the collected data, the ± 5 g range represents the limit of data utility, beyond which reliable interpretation of the data would be difficult.

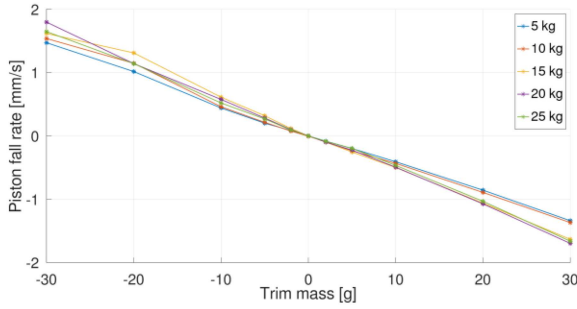


Fig. 7. Fall rate of the piston [mm/s] vs trim mass [g] with separated total mass series.

Moreover, the fall rate vs. trim mass graph (Fig. 6) shows that disturbances are more pronounced for mass deficits than for mass excesses. This may indicate that an influence, such as one related to a non-ideal design of the separating valve (a valve with a non-constant volume), has not been accounted for, which will require further investigation in the future.

This is also clearly visible in the fall rate vs trim mass graph (Fig. 7), which illustrates a relatively large variability at the edges of the graph and a relatively high consistency of data in its centre.

6. Conclusions

The gathered results have preliminarily outlined the applicability limits of the extended cross-float method, which for the selected piston-cylinder assemblies are at the level of approximately ± 5 g. They also revealed incomplete symmetry in the observed behaviour of the measurement system, which will require closer examination. It also seems that the linear approximations adopted for computations need to be validated. To broaden the scope of the method, measurement systems with different characteristics, particularly those with gaseous pressure transmitting medium and high-pressure systems, will require further investigation. It will also be worthwhile to consider the impact of the specific design of the measurement assembly on the applicability of the method, as well as the influence of the cut-off valve design.

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