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Dynamics of Superconducting Correlations Induced by Hopping in Serial Double Quantum Dot System

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We study the quench dynamics of superconducting pairing correlations in the double quantum dot system coupled to superconducting and normal metallic electrodes. The quantum dots are initially isolated from each other, and the subsequent dynamics are induced by the sudden switching on hopping between them. We focus on the time-dependence of the real and imaginary parts of dots pairing potential and the role of the hopping amplitude and on-site Coulomb correlations. For relatively small hopping values, the evolution of the pairing potential is suppressed due to a strong single-occupation blockade. As the hopping amplitude increases, the pairing potential is dynamically redistributed between the dots and can eventually assume values of opposite signs. This effect is enhanced by the presence of strong on-site Coulomb interactions. The discussed numerical results are obtained by means of the time-dependent numerical renormalization group approach.

topics: quench dynamics, proximity effect, double quantum dot

1. Introduction

The dynamical and transport properties of nanostructures hybridized with superconductors are currently under extensive theoretical and experimental investigations [1–3]. Such systems can provide important applications in the area of modern nanoelectronics, quantum computing, as well as information processing and storage [4]. Research in this field is also important from a fundamental point of view and brings new ways to explore and test contemporary problems of basic science and, in particular the quantum theory of condensed matter. Advantageously, artificial systems based on quantum dots allow to study various interactions and competition between superconducting and electronic correlations in a precise and controllable manner. An important effect present in quantum dots proximitized by a superconductor is the formation of Andreev bound states (ABS) [5]. Several recent works have explored the dynamical aspect of this phenomenon [6–8] and in this article, we extend those studies focusing on the dynamics of superconducting correlations mediated by interdot hopping between two interacting quantum dots.

2. Model and method

The system (Fig. 1) consists of two quantum dots (QD1,QD2) arranged serially between the superconducting (S) and normal (N) leads. The total Hamiltonian can be expressed by

$$\hat{H} = \hat{H}_N + \hat{H}_S + \hat{H}_{mix} + \hat{H}_{DQD}, \tag{1}$$

where

$$\hat{H}_N = \sum_{\mathbf{k}} \varepsilon_{N\boldsymbol{k}\sigma}\, \hat{c}^{\dagger}_{N\boldsymbol{k}\sigma} \hat{c}_{N\boldsymbol{k}\sigma} \tag{2}$$

describes the electrons in normal metallic lead, and the superconducting electrode is given in the BCS-form

$$\hat{H}_S = \sum_{\boldsymbol{q}\sigma} \varepsilon_{S\boldsymbol{q}}\, \hat{c}^{\dagger}_{S\boldsymbol{q}\sigma} \hat{c}_{S\boldsymbol{q}\sigma} + \sum_{\boldsymbol{q}} \left[\Delta\, \hat{c}^{\dagger}_{S\boldsymbol{q}\uparrow} \hat{c}^{\dagger}_{S-\boldsymbol{q}\downarrow} + \text{h.c.}\right]. \tag{3}$$

Here, $\hat{c}_{N\boldsymbol{k}\sigma}$ ($\hat{c}_{S\boldsymbol{q}\sigma}$) stands for the annihilation operator of the electrons in normal N (superconducting S) lead, with the momentum $\boldsymbol{k}$ ($\boldsymbol{q}$), the energy $\varepsilon_{N\boldsymbol{k}\ (S\boldsymbol{q})}$, and the spin $\sigma{=}\uparrow,\downarrow$. The pairing potential Δ is assumed to be real and is considered the largest energy scale in the problem. The double quantum dot part is described by

$$\hat{H}_{DQD} = \sum_{\sigma,j=1,2} \left[\varepsilon_{j\sigma}\, \hat{c}^{\dagger}_{j\sigma} \hat{c}_{j\sigma} + U\, \hat{c}^{\dagger}_{j\uparrow} \hat{c}_{j\uparrow} \hat{c}^{\dagger}_{j\downarrow} \hat{c}_{j\downarrow}\right] + \sum_{\sigma} \left[\upsilon(t)\, \hat{c}^{\dagger}_{1\sigma} \hat{c}_{2\sigma} + \text{h.c.}\right] \tag{4}$$

with the energies denoted by $\varepsilon_{j\sigma}$, the on-site Coulomb interaction by U, and $\upsilon(t)$ being a time-dependent interdot hopping. Finally, the following term describes tunneling between leads and corresponding dots

$$H_{mix} = \sum_{\boldsymbol{q}\sigma} \left[V_{S\boldsymbol{q}}\, \hat{c}^{\dagger}_{S\boldsymbol{q}\sigma} \hat{c}_{1\sigma} + \text{h.c.}\right] + \sum_{\boldsymbol{k}\sigma} \left[V_{N\boldsymbol{k}}\, \hat{c}^{\dagger}_{N\boldsymbol{k}\sigma} \hat{c}_{2\sigma} + \text{h.c.}\right], \tag{5}$$

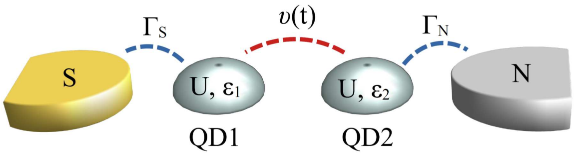

Fig. 1. Schematic of a serial double quantum dot system between superconducting (S) and normal (N) leads with couplings Γ_S and Γ_N, respectively. Considered quench is performed by abruptly switching on the hopping υ.

where $V_{S\boldsymbol{q}(N\boldsymbol{k})}$ is the tunneling amplitude between QD1(2) and lead S (N). The lead–dot couplings are then given by $\Gamma_\alpha = 2\pi \sum_{\boldsymbol{k}} |V_{\alpha\boldsymbol{k}}|^2 \delta(\omega - \varepsilon_{\alpha\boldsymbol{k}})$, where $\alpha = N, S$.

The dynamical behavior of the system was evaluated by employing the numerical renormalization group method [9–12] and its time-dependent extension (tNRG) [13–15]. The conduction band of normal lead is logarithmically discretized and then mapped to a semi-infinite 1D tight binding chain. Subsequently, diagonalization is performed in an iterative fashion. This procedure finds the complete many-body eigenbases of the initial and final Hamiltonians $\hat{H}_0$ and $\hat{H}$, respectively, $\sum_{nse} |nse\rangle_0^D \, {}_0^D\langle nse| = \hat{\mathbb{1}}$ and $\sum_{nse} |nse\rangle^D \, {}^D\langle nse| = \hat{\mathbb{1}}$, where s labels the discarded (D) eigenstates at an iteration n, and e denotes the environmental subspace [13].

Calculations of time-dependent expectation values were performed in the frequency domain. The frequency dependence is expressed as

$$O(\omega) = \sum_n^{XX' \neq KK} \sum_{n'} \sum_{ss'e} {}^X\langle ns\, e| w_{n'} \hat{\rho}_{0n'} |ns'e\rangle^{X'}$$
$$\times \, {}^{X'}\langle ns'e|\hat{\mathcal{O}}|ns\, e\rangle^X \, \delta\big(\omega + E_{ns}^X - E_{ns'}^{X'}\big). \qquad (6)$$

Here, $\hat{\rho}_{0n'}$ denotes the contribution to the initial density matrix from the n'-th iteration with weight $w_{n'}$.

In the last step of the procedure, we perform a Fourier transformation into the time domain, $O(t) = \int_{-\infty}^{\infty} \mathrm{d}\omega \, O(\omega) \mathrm{e}^{-\mathrm{i}\omega t}$ [16]. The discretization parameter in calculations was set to $\Lambda = 2$ and we kept $N_K = 2000$ states.

3. Results

We consider a quantum quench that modifies the initial Hamiltonian in a step-like fashion $\hat{H}_0 \rightarrow \hat{H}$, in which a hopping term between two initially separated quantum dots is suddenly switched on,

$$\upsilon(t) = \begin{cases} 0 & \text{for } t \leq 0, \\ \upsilon & \text{for } t > 0. \end{cases} \qquad (7)$$

We also assume the simple case of both QDs half-filled, $\varepsilon_{1/2} = -U/2$, which suppresses the charge dynamics and allows us to focus only on the time-evolution of pairing correlations. The temperature in calculations is set to $T = 0$.

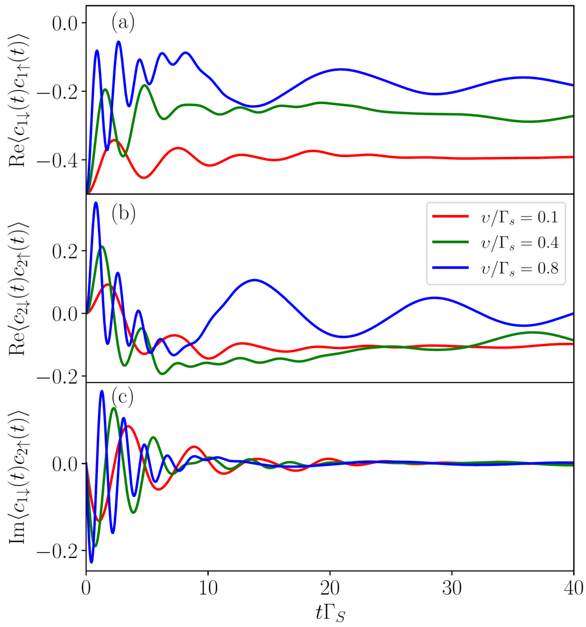

Fig. 2. (a, b) The real part of the individual on-dot pairings $\langle c_{j\downarrow}(t) c_{j\uparrow}(t)\rangle$ and (c) the imaginary part of the interdot pairing $\langle c_{1\downarrow}(t) c_{2\uparrow}(t)\rangle$ as a function of time. The results are obtained for indicated values of υ/Γ_S, while $U/\Gamma_S = 0.2$, $\varepsilon = -U/2$ and $\Gamma_N/\Gamma_S = 0.1$.

Figure 2 shows the time evolution of the real and imaginary parts of the pairing correlations following the proposed quench protocol. Generally, the characteristic features of these functions have a relation to differential conductance and can indicate excitation energies and in-gap bound states. A detailed analysis of this correspondence was performed in the earlier work, to which reference is made [8]. The curves are evaluated for different υ values in the final Hamiltonian. For all cases presented, initially only QD1 is in contact with the superconducting lead. At time $t \leq 0$, this is manifested as the maximum absolute value of pairing potential, $\mathrm{Re}[\langle c_{1\downarrow} c_{1\uparrow}\rangle] = -0.5$, while QD2 is not affected by superconducting correlations, $\mathrm{Re}\langle c_{2\downarrow} c_{2\uparrow}\rangle = 0$. At $t = 0$, the finite value of hopping coupling υ (as indicated in the legend in Fig. 2) is switched on and we observe the subsequent time evolution of pairing potentials. At short times, all three functions are dominated by oscillations, strongly dependent on the strength of the coupling to superconductor Γ_S. The redistribution of the real part of the pairing potential between the dots is revealed. As the higher values of the υ hopping are evaluated, we observe then that in the long time limit, the pairing potential of both dots is evening up. However, when $\upsilon/\Gamma_S > 0.5$, the dynamics expose additional long-time oscillations that are only found in the real parts. Moreover, in this regime, the real part of the paring potential on QD2 also assumes positive values.

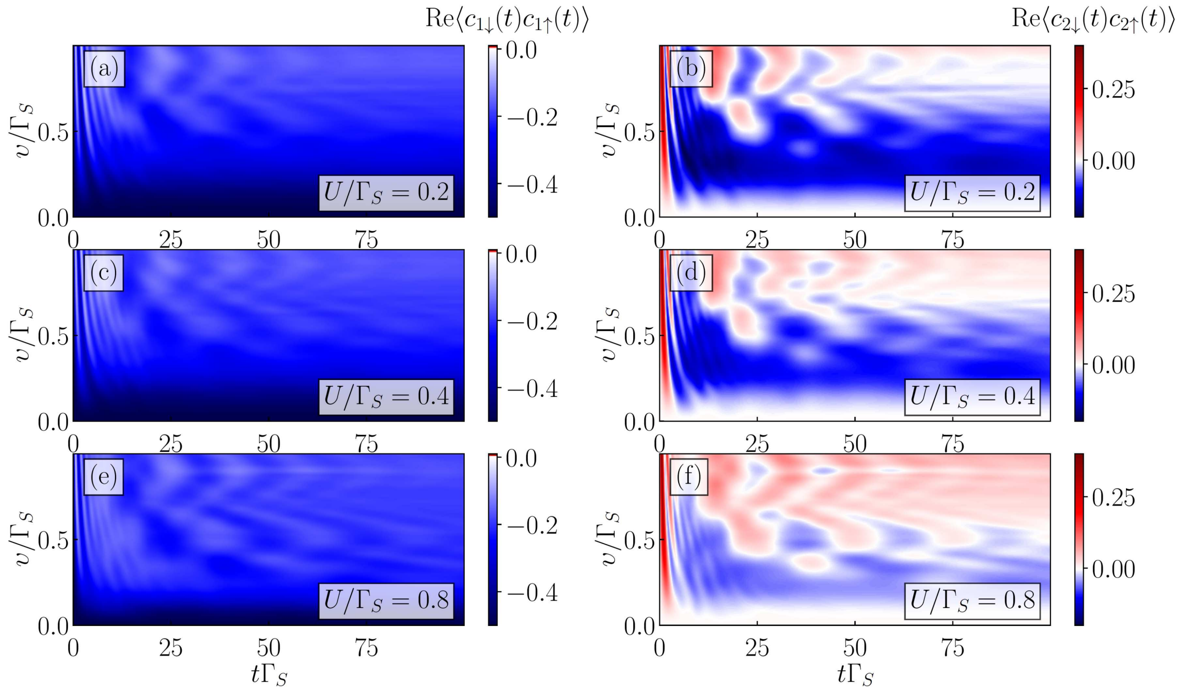

Fig. 3. The real part of the individual on-dot pairings $\langle c_{j\downarrow}(t)c_{j\uparrow}(t)\rangle$ as a function of time and hopping coupling υ. The results are obtained for indicated values of U/Γ_S (a–f), while $\varepsilon = -U/2$ and $\Gamma_N/\Gamma_S = 0.1$.

To inspect this effect in more details, we plot the relevant time dependencies on a longer time scale and for a wide range of hopping υ. Additionally, we consider the role of the intradot Coulomb interactions U. Figure 3 presents the results obtained for three different values of U. Here we note that the influence of the Coulomb interaction is only revealed in the real part of pairing potential, while the imaginary part of interdot potential remains unaffected and therefore we do not show it.

In all considered sets of parameters, the pairing potential on QD1 remains negative and, depending on the amplitude of υ, is reduced after the quench. In turn, the pairing potential on QD2 is evolving to a new finite value. For relatively small values of hopping $\upsilon/\Gamma_S < 0.1$, the pairing potential remains intact on QD1 and does not develop on QD2 due to the presence of a strong blockade in the single occupation regime. In the limit of $0.1 < \upsilon/\Gamma_S < 0.5$, the pairing potential tends to evolve toward an even distribution between both dots. Furthermore, for substantial values of the hopping amplitude $\upsilon/\Gamma_S > 0.5$, we predict that the pairing potential of QD2 can acquire a positive sign. For the case $U/\Gamma_S = 0.2$, in a long time limit, a positive value of the pairing potential on QD2 is only predicted when $\upsilon \approx \Gamma_S$. However, as U is enhanced, this tendency is predicted for a wide range of hoppings values where $\upsilon/\Gamma_S > 0.5$. This dynamical behavior reveals another interesting and non-trivial competition between Coulomb and superconducting correlations and the corresponding dynamics in strongly interacting nanoscopic systems.

4. Conclusions

In summary, we analyzed the dynamics of superconducting correlations in a hybrid double quantum dot system after sudden switching on of the interdot hopping coupling. The presented results can be of great importance for understanding how in-gap bound states are formed and modified in complex hybrid nanostructures induced by sudden perturbation of the system. We have specifically focused on switching on the hopping interaction between two quantum dots and shown that in the regime of small amplitudes, the dynamics for superconducting correlations can be blocked. For the intermediate regime, the time evolution tends toward an even distribution of the pairing potential between the dots. However, when the hopping parameter is considerable, the time-dependence reveals additional oscillations and the system needs more time to achieve the long-time limit, while the pairing potentials of both dots may assume opposite signs. Finally, we show that Coulomb interactions enhance this effect.

Acknowledgments

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