

AN APPROACH TO SPIN-FLUCTUATION RESISTIVITY OF FERROMAGNETIC METALS. II. ANISOTROPIC EFFECTS IN UNIAXIAL AND CUBIC FERROMAGNETS

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The formalism within the Boltzmann-Bloch transport equation proposed in the first part of the paper for treating the spin-fluctuation resistivity, ϱ , of anisotropic magnetic systems is applied to uniaxial and cubic ferromagnetic metals. In the uniaxial case the formulae for ϱ and $d\varrho/dT$ are calculated in the first order approximation and the influence of anisotropic fluctuations, following from the generalized Landau theory appropriate for uniaxial crystals, on anisotropy in ϱ and $d\varrho/dT$ is shown and discussed. The anisotropic magnetic fluctuations are also shown to create an anisotropy of ϱ in cubic materials. In this case the anisotropic effects manifest themselves in the next order approximation within the Landau theory and the Boltzmann-Bloch equation approach.

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1. Introduction

In the first part of this paper¹ [1] an appropriate formalism within the Boltzmann-Bloch transport equation was proposed for treating the spin-fluctuation (s-f) resistivity, ϱ , of anisotropic magnetic systems, in particular, ferromagnetic metals. Such an approach was needed because of the recent interest of experimentalists in anisotropic effects which occur in the conduction electron scatterings within the temperature region encompassing the critical temperature [2, 3]. Also recently, experimentalists and theoreticians have focussed their attention on anisotropy in critical X-ray scattering (see, e.g., [4]).

Most of the previous theoretical papers concerned with the s-f resistivity have been devoted to isotropic models in the close vicinity of the critical temperature (see, e.g., references in [1]), where the magnetic and band structure anisotropy is not essential and the temperature behaviour of ϱ , or rather $d\varrho/dT$, is governed by the appropriate critical

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¹ When referring to some formulae of [1], we shall denote them (I...).

indices. However, most experiments are still carried out outside this hardly accessible region and the details of magnetic interactions and band structure become important in the interpretation of these results [5]. In order to include these details in the theoretical description, besides the solution of the Boltzmann–Bloch equation in the anisotropic case, an appropriate spin–spin correlation function is needed. In the first approximation to uniaxial systems this may be the correlation function derived in [6] on the grounds of the Landau theory supplemented with anisotropic fluctuation terms. However, in order to describe the experimental results for ϱ in a qualitatively correct manner, this function should be modified like the standard Ornstein–Zernike function [1] and, additionally, the magnetocrystalline anisotropy effects should be included. These generalizations of the correlation function derived in [6] are made in the present paper, and the function applied to calculate the s-f resistivity and its temperature derivative on the grounds of the approach proposed in [1]. The results are illustrated for reasonable values of the parameters of model materials. Some of these results has been already reported [7, 8]. In particular, this is an effect of the anisotropy in the s-f resistivity which arises from the anisotropic fluctuations [6]. In our opinion, the anisotropy in the s-f resistivity that occurs in uniaxial ferromagnetic metals and alloys [9, 2] may be, at least partly, a consequence of the anisotropic fluctuations, following, in turn, from the departure of the lattice from the cubic symmetry and the existence of the magnetic anisotropy.

As for cubic crystals there has been neither experimental nor theoretical evidence for the existence of an anisotropy in the s-f resistivity. Also an anisotropy of the magnetic fluctuations has never been studied. In the present paper we attempt to answer the basic questions as to the existence of such anisotropic effects in cubic ferromagnetic metals, too, on the basis of simple theoretical arguments.

The outline of the paper is as follows. The consequences of the approach of [1] in the application to anisotropic systems are discussed in general terms in Section 2. In Section 3 the explicit expressions for the s-f resistivity and its temperature derivative are calculated and illustrated in diagrams. Section 3 is devoted to the anisotropic corrections in cubic ferromagnets. Finally, in the last Section, the results are summarized.

2. Formulation of the problem

The problem of the s-f resistivity, studied again in [1] for isotropic systems but from a more general standpoint, is as follows: the conduction electrons described in the free-electron approximation are scattered on magnetic fluctuations of the localized magnetic moments $\mu_B S_v(\mathbf{R})$ ($v = x, y, z$) being periodically arrayed in a lattice. In the present paper, as in the considerations of [1], we assume the electron energy in the form $\varepsilon_k = (1/2)\hbar \mathcal{M}_{\mu\nu} k_\mu k_\nu$ where $\mathcal{M}_{\mu\nu}$ is $\mathbf{k}$ -independent (summation convention will be used over repeated Greek indices denoting components). The exact form of the Hamiltonian describing the interactions within the magnetic system, $\mathcal{H}_S$, is not essential at this stage of the considerations. The interaction between these two systems is specified as in [1] in the simplest form, i.e., as the *isotropic* contact exchange interaction between the localized magnetic moments and the moments of the conduction electrons. Then, the kernel of the

Boltzmann-Bloch equation — being proportional to the scattering cross-section — is given in the metallic limit ($\varepsilon_F \gg k_B T$) by

$$\Omega(\mathbf{k}, \mathbf{k}') = \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) P(\mathbf{k}, \mathbf{k}') \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_F), \quad (2.1a)$$

$$P(\mathbf{k}, \mathbf{k}') = \left(\frac{\mathcal{N}G}{2\pi} \right)^2 \frac{V_a S(S+1)}{\hbar} \sum_{\mathbf{R}} \Gamma(\mathbf{R}, T) e^{-i(\mathbf{k}-\mathbf{k}')\mathbf{R}}, \quad (2.1b)$$

where G is the interaction constant and $\mathcal{N}$ the number of the magnetic ions in the volume $V = \mathcal{N}V_a$ of the crystal. The basic quantity in Eqs. (2.1) is $\Gamma(\mathbf{R}, T)$, which — according to (I.2.2) — stands for the *sum of the diagonal components* of the tensor of the magnetic correlations

$$\Gamma_{\mu\nu}(\mathbf{R}, T) = [S(S+1)]^{-1} (\langle S_\mu(\mathbf{R}) S_\nu(\mathbf{0}) \rangle - \langle S_\mu(\mathbf{R}) \rangle \langle S_\nu(\mathbf{0}) \rangle). \quad (2.2)$$

Let us summarize in a passage these results of the general approach of [1] which will be useful in our further discussion. The conductivity tensor follows immediately from the expression for the electric current

$$I_\mu = - \frac{e}{4\pi^3} \int d\mathbf{k} \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) \phi(\mathbf{k}) v_\mu(\mathbf{k}), \quad (2.3)$$

where $v_\mu(\mathbf{k})$ is a component of the electron velocity. The function $\phi(\mathbf{k})$ — assumed in the form of an expansion $\phi(\mathbf{k}) = \sum_i b_i \phi_i(\mathbf{k})$ in spherical harmonics known, in turn, from the symmetry considerations — is determined through the coefficients b_i which have to satisfy the set of linear equations

$$\sum_{i,j} C_{ij} b_j + D_i = 0 \quad (2.4)$$

with $D_i = eE_\mu \int d\mathbf{k} \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) \phi_i(\mathbf{k}) v_\mu(\mathbf{k})$. The inner products $C_{ij} = (\phi_i, C\phi_j)$ with the collision operator C are expressed through the kernel $\Omega(\mathbf{k}, \mathbf{k}')$ and defined by (I.3.7). For lattices of arbitrary symmetry the first harmonic is $\phi_1(\mathbf{k}) = A_1(\varepsilon_F) k_\mu n_\mu$, where $A_1(\varepsilon_F)$ is the normalization constant and the n_μ 's stand for the components of the unit vector pointing in the direction of the applied electric field $\mathbf{E}$. The next harmonic that contributes to Eq. (2.3), e.g., for cubic lattices, is of the form

$$\phi_3(\mathbf{k}) = (1/2) A_3(\varepsilon_F) (5k_\mu^3 - 3k^2 k_\mu) n_\mu \quad (2.5)$$

($k^2 = k_\mu k_\mu$) in the coordination frame with the axes along the four-fold axes of rotation. Similarly, all the higher harmonics in the general case are $\phi_i(\mathbf{k}) = A_i(\varepsilon_F) \varphi_{i,\mu}(\mathbf{k}) n_\mu$, where $\varphi_{i,\mu}(\mathbf{k})$ is a polynomial of an odd (i -th) degree in k_μ .

As far as the considerations are restricted to the parabolic band all D_i except D_1 are equal to zero due to the orthogonality relations. For the same reason only the first harmonic *explicitly* contributes to Eq. (2.3): $b_1 \neq 0$ and $b_i = 0$ if $i \neq 1$. The remaining

harmonics contribute to the current through the value of $b_1 = -D_1 d_{11}/\det(C_{ij})$, where d_{11} is the minor of the element C_{11} . When considerations are restricted to the first harmonic, the conductivity tensor is

$$\sigma_{\mu\mu'} = \frac{e^2}{2\pi^3} \frac{\int dk \int dk' \delta(\varepsilon_k - \varepsilon_F) \delta(\varepsilon_{k'} - \varepsilon_F) \mathcal{M}_{\mu\nu} k_\nu \mathcal{M}_{\mu'\nu'} k_{\nu'} k_\lambda n_\lambda k'_\lambda n'_\lambda}{\int dk \int dk' \Omega(k, k') (k_\mu - k'_\mu) (k_\nu - k'_\nu) n_\mu n_\nu} \quad (2.6)$$

For simplicity, confine ourselves now to the spherical Fermi surface, i.e., assume $\mathcal{M}_{\mu\nu} = (\hbar/m)\delta_{\mu\nu}$. The conductivity tensor calculated up to the l -th order in spherical harmonics is

$$\sigma_{\mu\nu}^{(l)} = \frac{e^2}{4\pi^3} \left[\frac{\hbar}{mA_1(\varepsilon_F)} \right]^2 \Delta_l^{-1} \delta_{\mu\nu}, \quad (2.7)$$

where $\Delta_1 = C_{11}$, $\Delta_3 = (C_{11} - C_{13}^2 C_{33}^{-1})$, The tensor $\sigma_{\mu\nu}^{(l)}$ is diagonal in an arbitrary coordination frame since the band is spherical. However, as seen from Eq. (2.7) and the definition C_{ij} the value of Δ_l (being proportional to the resistivity measured in the n -direction) must depend on $\mathbf{n}$, i.e., on the direction of the applied electric field with respect to the crystal axes (provided that the kernel $\Omega(k, k')$ is not of spherical symmetry). This is, of course, the reason why the s-f resistivity measured in the direction parallel and perpendicular to the easy axis of a uniaxial ferromagnet is different due to an anisotropy in the magnetic fluctuations. Similar effects can also be expected in cubic ferromagnets if the applied electric field is directed in nonequivalent crystallographic directions.

3. Uniaxial ferromagnets

3A. Correlation function

To generalize the correlation function derived in [6] we must start again from the Landau free energy density appropriate for uniaxial crystals

$$f(\mathbf{R}) = AM^2(\mathbf{R}) + KM_\perp^2(\mathbf{R}) + BM^4(\mathbf{R}) + A_{\mu\nu}^{\alpha\beta} \left(\frac{\partial M_\alpha}{\partial R_\mu} \right) \left(\frac{\partial M_\beta}{\partial R_\nu} \right), \quad (3.1)$$

($\sqrt{M_\mu(\mathbf{R})M_\mu(\mathbf{R})} \equiv M(\mathbf{R})$ is the magnetization length at a point $\mathbf{R}$; $M_\perp \equiv \sqrt{M_x^2 + M_y^2}$ — similar notation for perpendicular components will be used further) and write down the tensor $A_{\mu\nu}^{\alpha\beta}$ of the magnetic fluctuations in a more general form. Obviously, the symmetry of $A_{\mu\nu}^{\alpha\beta}$ should be still such as to describe uniaxial crystals properly. However, if this tensor is to give an account of the magnetic anisotropy (K) effects besides those of the lattice symmetry, the form $A_{\mu\nu}^{\alpha\beta} = A_{\mu\nu} \delta^{\alpha\beta}$ used in [6] is not sufficient. This is obvious when one compares the free energy (3.1) in the limit $T \rightarrow 0$ to the energy corresponding to the spin Hamiltonian

$$\mathcal{H}_S = -\mathcal{J} \sum_{\langle i,j \rangle} S(\mathbf{R}_i) S(\mathbf{R}_j) + \mathcal{K} \sum_{\langle i,j \rangle} [S_x(\mathbf{R}_i) S_x(\mathbf{R}_j) + S_y(\mathbf{R}_i) S_y(\mathbf{R}_j)] \quad (3.2)$$

($\langle i, j \rangle$ — restriction to nearest neighbours) in the classical approximation. The, similar considerations as in [1], based on the standard procedure (see, e.g., [12]), lead to

$$\begin{aligned} A_{xx}^{xx} = A_{yy}^{xx} = A_{xx}^{yy} = A_{yy}^{yy} &= \frac{\mathcal{J} - \mathcal{K}}{2V_a} d_{\perp}^2 S^2, & A_{xx}^{zz} = A_{yy}^{zz} &= \frac{\mathcal{J}}{2V_a} d_{\perp}^2 S^2, \\ A_{zz}^{xx} = A_{zz}^{yy} &= \frac{\mathcal{J} - \mathcal{K}}{2V_a} d_z^2 S^2, & A_{zz}^{zz} &= \frac{\mathcal{J}}{2V_a} d_z^2 S^2, \end{aligned} \quad (3.3)$$

where

$$d_{\perp}^2 = \sum_i r_i^2, \quad d_z^2 = \sum_i z_i^2 \quad (3.4)$$

and r_i is the distance to the (i -th) magnetic ion in the first coordination sphere, corresponding to the directions in the plane xy , and z_i — corresponding to the direction z . All the remaining components of $A_{\mu\nu}^{\alpha\beta}$ should be equal to zero as a result of the symmetry requirements and the above consideration. From the comparison of Eq. (3.1) to the mean-field energy corresponding to Eq. (3.2) one can also estimate the remaining parameters of the Landau free energy (3.1), namely

$$A = \frac{\mathcal{J} S^2 w}{2V_a} t, \quad B = \frac{3S(2S^2 + 2S + 1)w\mathcal{J}}{40(S+1)^2 V_a}, \quad K = \frac{\mathcal{K} S^2 w}{2V_a}, \quad (3.5)$$

where w is the number of nearest neighbours.

The generalization (3.3) does not change the form of the correlation function, which as in [6], in the inverse space is

$$\hat{\Gamma}_{\mu\nu}(\mathbf{q}, t) = \frac{\pi V_a^{-2/3} \Gamma_0(t) \xi_{\mu\nu, \perp}^2}{1 + \xi_{\mu\nu, z}^2 q_z^2 + \xi_{\mu\nu, \perp}^2 q_{\perp}^2} \quad (3.6)$$

($\mathbf{q}$ — fluctuation wave vector, $\Gamma_0(t) = \text{const}(1+t)$, $t = (T - T_c)/T_c$) for the diagonal components and it is equal to zero if $\mu \neq \nu$. The only change is in the amplitude of the correlation lengths $\xi_{\perp, \mu}$ of $\Gamma_{\perp}(\mathbf{q}, t)$, in which the additional factor $(1 - K/A')^{1/2}$ ($A \equiv A'|t|$) arises². Thus, the correlation lengths defined in [6, 7] read

$$\begin{aligned} \xi_{zz, z} &= \xi_0 \eta_z(t), & \xi_{\perp, z} &= \xi_0 \eta_{\perp}(t), \\ \xi_{zz, \perp} &= \xi_0 \lambda \eta_z(t), & \xi_{\perp, \perp} &= \xi_0 \lambda \eta_{\perp}(t), \end{aligned} \quad (3.7)$$

where

$$\xi_0 = d_z w^{-1/2}, \quad \lambda = d_{\perp}/d_z \quad (3.8)$$

² An error has been made in [7, 8], in comparing the mean-field free-energy corresponding to (3.2) to the respective form of (3.1). The correct form of $\eta_{\perp}$ in these papers should be $\eta_{\perp} = \kappa^{-1/2}$ for $t < 0$, and $\eta_{\perp} = (t^{2\nu} + \kappa)^{-1/2}$ for $t > 0$ ($\kappa \equiv K/A'$, $\nu = 1/2$).

and the functions $\eta_\mu(t)$ are in the ferromagnetic phase ($t < 0$)

$$\eta_z(t) = 2^{-1/2}|t|^{-\nu}, \quad \eta_\perp(t) = (\kappa^{-1} - 1)^{1/2} \quad (3.9a)$$

and in the paramagnetic phase ($t > 0$)

$$\eta_z(t) = t^{-\nu}, \quad \eta_\perp(t) = [(1 - \kappa)/(t^{2\nu} + \kappa)]^{1/2}, \quad (3.9b)$$

where $\kappa \equiv K/A' = \mathcal{K}/\mathcal{J}$ and $\nu = 1/2$. As in [6] the isocorrelation surfaces ($\hat{r}(\mathbf{q}) = \text{const}$) are ellipsoids of revolution with the ratio of axes equal to λ . From a purely physical point of view it is obvious that the parameters λ and κ should be interrelated. With tetragonal lattices and pseudodipolar interactions being equivalent to (3.2), one can find such a relation (see, e.g., [13]). It simply says that κ is proportional to λ , and κ is positive if the cubic lattice is contracted in the z -direction. The latter statement is, of course, in agreement with common sense since the magnitude of the exchange interactions decreases with distance. However, even in this simplest case one can hardly estimate the value of the proportionality constant. Therefore, we shall consider λ and κ as two independent model parameters.

To include, at least partly, the large- q correlations essential for the s-f resistivity [14] we modify the temperature dependent factor in Eq. (3.6) by requiring [15] the correlation function to satisfy the exact sum rule

$$\sum_{\mathbf{q}} \hat{r}(\mathbf{q}, t) = \mathcal{N}(1 - m^2), \quad (3.10)$$

where $m(t)$ is the homogeneous part of the reduced magnetization. The details of this procedure for isotropic systems are described in [1]. In the uniaxial case considered here we integrate the function $\hat{r}(\mathbf{q}, t) = \hat{r}_{zz}(\mathbf{q}, t) + 2\hat{r}_{\perp\perp}(\mathbf{q}, t)$ over the volume in $\mathbf{q}$ -space equal to that of the Brillouin zone and bounded by an isocorrelation surface. Then, $\Gamma_0(t)$ — assumed as the normalization factor to be found from the requirement (3.10) — is given by

$$\gamma(t) = \frac{\Gamma_0(t)}{\Gamma_0(0)} = \frac{3(1 - m^2)}{3 - u_z(t) - 2u_\perp(t)}, \quad (3.11)$$

where

$$3\Gamma_0(0) = 2^{2/3}(\pi/3)^{1/3}\lambda^{-2/3}, \quad (3.12a)$$

$$u_\mu(t) = (6\pi^2\lambda^2)^{-1/3}(\eta_\mu r)^{-1} \arctan [(6\pi^2\lambda^2)^{1/3}\eta_\mu r], \quad (3.12b)$$

$$m(t) = \begin{cases} m_0|t|^\beta, & t < 0 \\ 0, & t > 0 \end{cases} \quad r = \xi_0 V_a^{-1/3} \quad (3.12c)$$

and within the Landau theory $\beta = 1/2$ and $m_0 = (A'/2B)^{1/2}$.

Further refinements of the correlation function can be obtained by requiring the correlation length of the order parameter to be described by the index ν ($\simeq 2/3$ from experiments and more accurate theories). Then, by also applying the scaling relation $\nu = 2\beta$ (valid for a 3-dimensional system), we obtain the generalized ($\nu \neq 1/2$) relations (3.9). Such a generalization should be, of course, made with caution, but there are premises to consider that it shifts the validity of our approximation closer to the critical region [16, 1].

After these refinements the diagonal components of the correlation function (3.6) can be represented as

$$\hat{F}^{(\mu)}(\mathbf{K}, t) = \frac{\hat{F}_0 \gamma(t)}{r^{-2} \eta_\mu^{-2}(t) + (K_z^2 + \lambda^2 K_\perp^2)}, \quad (3.13)$$

where $(\mu) = zz, \perp\perp$, $\hat{F}_0 = 2^{2/3}(\pi\lambda/3)^{4/3}$, $K_\mu = V_a^{1/2} q_\mu$, and r is defined in (3.12c). In the isotropic case ($\lambda = 1$, $\kappa = 0$) the sum $\hat{F}^{(z)}(\mathbf{K}, t) + 2\hat{F}^{(\perp)}(\mathbf{K}, t) \equiv \hat{F}(\mathbf{K}, t)$ corresponds³ to (I.4.10).

On the grounds of Eqs. (3.4) and (3.8) one can estimate the value of λ and r for a given crystal structure. For the simple tetragonal lattice and the hcp structure these values along with the corresponding ratio ξ_0/b (b is the lattice constant in the basal plane, c — in the z -direction) are given in Table I. As with the cubic lattices [1] the values of ξ_0 are of the order of the lattice spacings, which is in accordance with experimental estimations (see, e.g., [17]).

TABLE I

The values of the parameters r , λ and ξ_0/a following from the comparison of the Landau free energy to the microscopic one (see text)

Lattice	Quantity	ξ_0/a	r	λ
simple tetragonal		$3^{-1/2}$	$3^{-1/2} \simeq 0.58$	a/c
hcp		$3^{-1/2}$	$2^{1/6} 3^{-1/2} \simeq 0.65$	$3^{1/2}/2 \simeq 0.87$

3B. Spin-fluctuation resistivity

Consistent with the symmetry of the correlation function (3.6), (3.13) we assume

$$\mathcal{M}_{\mu\nu} = (\hbar/m_z)\delta_{z\mu}\delta_{z\nu} + (\hbar/m_\perp)(\delta_{x\mu}\delta_{x\nu} + \delta_{y\mu}\delta_{y\nu}), \quad (3.14)$$

which according to Eq. (2.6) yields the following expression for the resistivity in the easy and hard direction ($\mu = z, \perp$)

$$\varrho^{(\mu)} = \frac{2\pi^3}{e^2} \frac{\int d\mathbf{k} \int d\mathbf{k}' \Omega(\mathbf{k}, \mathbf{k}') [v_\mu(\mathbf{k}) - v_\mu(\mathbf{k}')]^2}{[\int d\mathbf{k} \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) v_\mu^2(\mathbf{k})]^2}, \quad (3.15)$$

where $v_\mu(\mathbf{k})$ is the electron velocity in the respective direction. The above expression corresponds to that used in [18] to calculate the effects of the band structure anisotropy on the

³ As seen from Eqs. (3.7) and (3.9) the correspondence to the isotropic case is obvious in the paramagnetic phase. It is also obvious that for the isotropic ferromagnet the correlation length cannot depend on direction in the crystal. Indeed, in this case we have $A_{\mu\nu}^{ab} = A\delta^{ab}\delta_{\mu\nu}$ and on the grounds of the definition of the correlation length that is used here [6] we have $\eta_z(t) = \eta_\perp(t) = 2^{-1/2}|t|^{-1/2}$ for the ferromagnetic phase.

s-f resistivity and obtained by a different method. If the kernel $\Omega(\mathbf{k}, \mathbf{k}')$ and the band are of spherical symmetry, from Eq. (3.15) one can easily obtain the formula (I.3.19) corresponding, in turn, to the result of the relaxation time approach.

It is easy to note that Eq. (3.15) can be represented by

$$\varrho^{(\mu)}(t) = \varrho_0 \sum_{\mathbf{R}} F^{(\mu)}(\mathbf{R}) \Gamma(\mathbf{R}, T), \quad (3.16)$$

where

$$\varrho_0 = \frac{3\pi S(S+1)m_{\perp}(\mathcal{N}G)^2 V_a}{2e^2 \hbar \varepsilon_F}, \quad (3.17)$$

$$F^{(\mu)}(\mathbf{R}) = \frac{\varepsilon_F}{3m_{\perp}} \frac{\int d\mathbf{k} \int d\mathbf{k}' \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_F) e^{-i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{R}} [v_{\mu}(\mathbf{k}) - v_{\mu}(\mathbf{k}')]^2}{[\int d\mathbf{k} \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) v_{\mu}^2(\mathbf{k})]^2} \quad (3.18)$$

and $F^{(\mu)}(0) = m_{\mu}/m_{\perp}$. If $\mathbf{R}$ is considered as a continuous variable as in Eq. (3.1), Eq. (3.16) in the inverse space is

$$\varrho^{(\mu)}(t) = \frac{\varrho_0 V_a}{(2\pi)^3} \int d\mathbf{q} \hat{F}^{(\mu)}(\mathbf{q}) \hat{\Gamma}(\mathbf{q}, t), \quad (3.19)$$

where the Fourier transforms are defined $f(\mathbf{q}) = V_a^{-1} \int d\mathbf{R} f(\mathbf{R}) \exp(-i\mathbf{q}\cdot\mathbf{R})$. The transform of Eq. (3.17) is

$$\hat{F}^{(\mu)}(\mathbf{q}) = \frac{3\pi^2}{2V_a} \frac{q_{\mu}^2(m_{\mu}/m_{\perp})}{k_{F,z}^2 k_{F,\perp}^2 q'} \Theta(2k_{F,z} - q'), \quad (3.20)$$

where $\Theta(x)$ is the step function ($\Theta(x) = 1$ if $x > 0$ and 0 if $x < 0$), $q' = (q_z^2 + \omega^2 q_{\perp}^2)^{1/2}$ with $\omega^2 = m_z/m_{\perp}$, $2k_{F,\mu}$ is the Fermi surface caliper in the μ -direction and the summation convention with respect to μ is not used.

The application of Eqs. (3.13) and (3.20) to Eq. (3.19) yields the expression for the reduced s-f resistivity

$$\mathcal{R}^{(\mu)}(t) = \alpha^{(\mu)} \gamma(t) \{3\mathcal{J}_0^{(\mu)} - \mathcal{J}^{(\mu)}(\eta_z) - 2\mathcal{J}^{(\mu)}(\eta_{\perp})\}, \quad (3.21)$$

where

$$\mathcal{R}^{(\mu)}(t) = \frac{\varrho^{(\mu)}(t) p^2}{6\varrho_0 \hat{\Gamma}_0}, \quad (3.22a)$$

$$p = 2k_{F,z} V_a^{1/3}, \quad \alpha^{(z)} = \omega^2, \quad \alpha^{(\perp)} = \omega^{-2}, \quad (3.22b)$$

the values of $\mathcal{J}_0^{(\mu)}$ are given in Table II, and the integrals $\mathcal{J}^{(\mu)}$ are defined as

$$\mathcal{J}^{(\mu)}(\eta) = \int_0^1 dx g^{(\mu)}(x) \frac{\ln[1 + (p\eta)^2 a^2(x)]}{(p\eta)^2 a^4(x)}, \quad (3.23)$$

TABLE II

The values of $\mathcal{J}_0^{(\mu)}$ in the formula (3.21)

	$\mathcal{J}_0^{(z)}$	$\mathcal{J}_0^{(\perp)}$
$0 < \lambda/\omega < 1$	$Y(1 - X^{-1} \arctan X)$	$2^{-1} \left(Y^{1/2} \left \frac{\lambda}{\omega} \right ^{-1} \arctan X - \mathcal{J}_0^{(z)} \right)$
$\lambda/\omega = 1$	$1/3$	
$\lambda/\omega > 1$	$ Y (X ^{-1} \operatorname{arctanh} X - 1)$	$2^{-1} \left(Y ^{1/2} \left(\frac{\lambda}{\omega} \right)^{-1} \operatorname{arctanh} X - \mathcal{J}_0^{(z)} \right)$
$X \equiv \frac{1 - (\lambda/\omega)^2}{(\lambda/\omega)^2} \quad Y \equiv \frac{1}{1 - (\lambda/\omega)^2}$		

in which

$$g^{(\mu)}(x) = \begin{cases} x^2, & \mu = z \\ (1-x^2)/2, & \mu = \perp \end{cases} \quad a^2(x) = [1 - (\lambda/\omega)^2]x^2 + (\lambda/\omega)^2. \quad (3.24)$$

For the isotropic ferromagnet the above expression corresponds to (I.5.8) (see also footnote 3). The temperature derivative of $\mathcal{R}^{(\mu)}$ is

$$\begin{aligned} \frac{d\mathcal{R}^{(\mu)}}{dt} = & \frac{\operatorname{sgn}(t)\gamma(t)v}{3|t|} \left\{ 6\alpha^{(\mu)}[(W^{(\mu)}(\eta_z) - \mathcal{J}^{(\mu)}(\eta_z)) + 2\Theta(t)(1 + \kappa t^{-2\nu})^{-1}(W^{(\mu)}(\eta_\perp) - \mathcal{J}^{(\mu)}(\eta_\perp))] \right. \\ & \left. - \frac{\mathcal{R}^{(\mu)}(t)}{1 - m_0^2|t|^\nu} \left[\left(\frac{1}{1 + (d\eta_z)^2} - u_z \right) + 2\Theta(t)(1 + \kappa t^{-2\nu})^{-1} \left(\frac{1}{1 + (d\eta_\perp)^2} - u_\perp \right) \right] \right\} \\ & + \mathcal{R}^{(\mu)}(t) |t|^{\nu-1} \frac{\nu m_0^2}{1 - m_0^2|t|}, \end{aligned} \quad (3.25)$$

where the functions $W^{(\mu)}(\eta)$ are defined in Table III and $d \equiv (6\pi^2\lambda^2)^{1/3}$.

Fig. 1 (dashed curves) illustrates the dependence of ϱ on the reduced temperature in the case $\omega = 1$ (spherical band). The anisotropy of the s-f resistivity, caused solely by the lattice (hcp structure, $\lambda < 1$, see Table I) and the magnetic anisotropy is evident. The band structure anisotropy increases the splitting between the s-f resistivity values corresponding to z - and $\perp$ direction (solid curves). The same dependence is shown in Fig. 2 but for $\omega = \lambda = 1.01$ (which may roughly correspond to tetragonal TbZn [3]) and the same value of κ as in Fig. 1. Note that now, contrary to Fig. 1, $\mathcal{R}^{(z)} > \mathcal{R}^{(\perp)}$. The next two figures (3,4) show the influence of the value of p and κ on $d\mathcal{R}^{(\mu)}/dt$. It is seen that even considerable change in the effective number of the current carriers (p) have little influence on the behaviour of $d\mathcal{R}^{(\mu)}/dt$ in the ferromagnetic phase. Also the influence of the magnetic

TABLE III

The function $W^{(\mu)}(\eta)$ of the formula (3.25)

	$W^{(z)}(\eta)$	$W^{(\perp)}(\eta)$
$0 < \lambda/\omega < 1$	$Y(Z^{-1/2} \arctan Z^{1/2} - X^{-1/2} \arctan X^{1/2})$	$2^{-1}Y[(X^{-1/2} + X^{1/2}) \arctan X^{1/2} - (Z^{-1/2} + Z^{1/2}) \arctan Z^{1/2}]$
$\lambda/\omega = 1$	$3^{-1}(1 + p^2 r^2 \eta^2)^{-1}$	
$\lambda/\omega > 1$	$ Y (X ^{-1/2} \arctan h X ^{1/2} - Z ^{-1/2} \arctan h Z ^{1/2})$	$2^{-1} Y [(Z ^{-1/2} - Z ^{1/2}) \arctan h Z ^{1/2} - (X ^{-1/2} - X ^{1/2}) \arctan h X ^{1/2}]$

$$Z = Z(\eta) = \left(1 + \frac{p^2 r^2 \lambda^2}{\omega^2} \eta^2\right)^{-1} \left(1 - \frac{\lambda^2}{\omega^2}\right) (pr\eta)^2$$

X and Y defined in Table II.

anisotropy constant κ on the anisotropy of $d\mathcal{R}^{(\mu)}/dt$ is small in comparison to the combined effect of the lattice ($\lambda \neq 1$) and band structure ($\omega \neq 1$) anisotropy.

The influence of the value of ν (correlation length index) and m_0 (amplitude of magnetization) on the behaviour of dQ/dT as a function of T is much the same as in the isotropic case (see, [1]) with the magnitude of dQ/dT split for the z - and $\perp$ -direction.

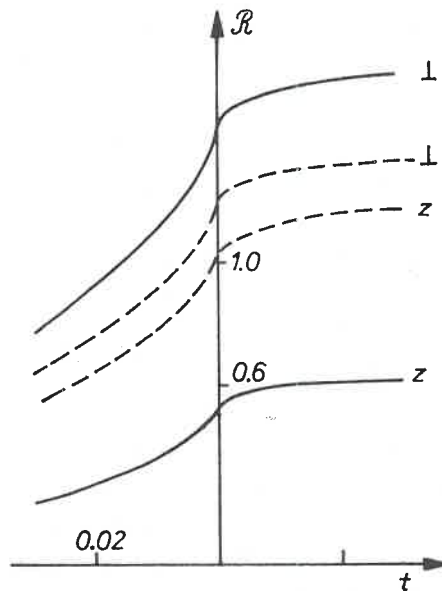


Fig. 1. The dependence of the (reduced) s-f resistivity on $t(\equiv (T - T_c)/T_c)$ for the hcp structure (r, λ according to Table I), $\omega = 1$ (dashed curves) and $\omega = 0.8$ (solid curves). In both the cases $p = 12$, $\kappa = 0.1$ and $S = 1$

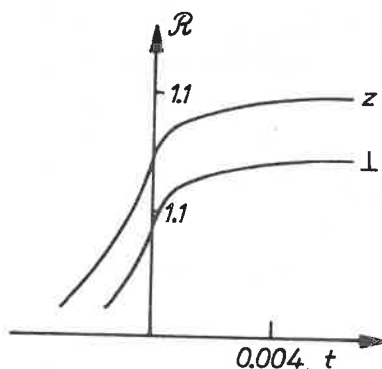


Fig. 2. The dependence of the (reduced) s-f resistivity on t for $\omega = \lambda = 1.01$ (approximately TbZn) and the values of the remaining parameters as in Fig. 1. Pay attention that now, contrary to Fig. 1, $\varrho^{(z)} > \varrho^{(l)}$

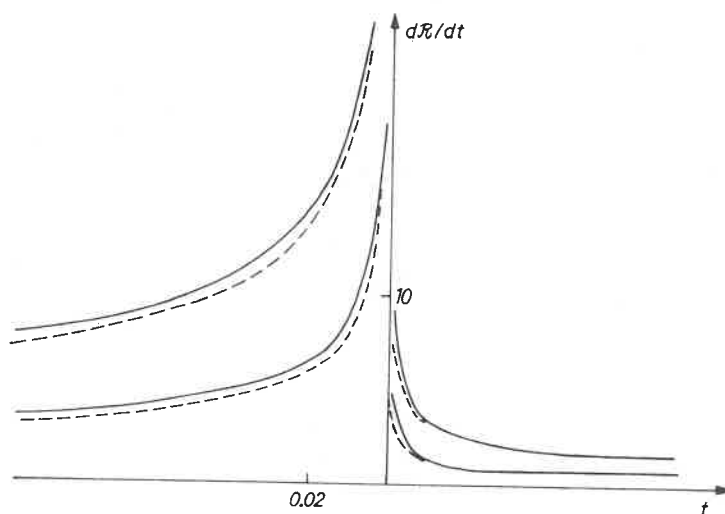


Fig. 3. The influence of the value of p on $d\mathcal{R}/dt$. The solid curves correspond to $p = 12$, dashed ones to $p = 2$; in both the cases the upper one to the perpendicular direction and $\kappa = 0.1$

4. Cubic ferromagnets

It is known that anisotropic effects in cubic ferromagnets must be described by at least fourth order terms in the magnetization components or spin operators. Thus, an approximate Landau free energy should include such anisotropic terms of the proper symmetry and, in order to describe fluctuations, also the corresponding fourth order terms in $\partial M_\alpha / \partial R_\mu$. However, to answer the basic questions it is quite sufficient to restrict the considerations to the variation of the free energy due to the fluctuations (see, e.g., [6]), which for cubic crystals should read [20]

$$\Delta f \equiv \int d\mathbf{R} [f(\mathbf{R}) - f_0] = \sum_{\mathbf{q}, \nu} L(\mathbf{q}) |\hat{M}_\nu(\mathbf{q})|^2 + 4\text{-th order terms in } \hat{M}_\nu(\mathbf{q}). \quad (4.1)$$

f_0 denotes here the free energy of a uniformly magnetized crystal and $\hat{M}_r(\mathbf{q})$ stand for the Fourier transforms of $\Delta M_v(\mathbf{R}) = M_v(\mathbf{R}) - m_r$ i.e., the departures of the magnetization components from their average values. The cubic symmetry implies the following form of the coefficients:

$$L(\mathbf{q}) = L_0 + L_2 q^2 + L_4 q^4 + L'_4(q_x^4 + q_y^4 + q_z^4) + \dots \quad (4.2)$$

The temperature dependence of Δf included in L_i is not essential for our present considerations. The last two terms of the right hand side of Eq. (4.2), which are invariant under the

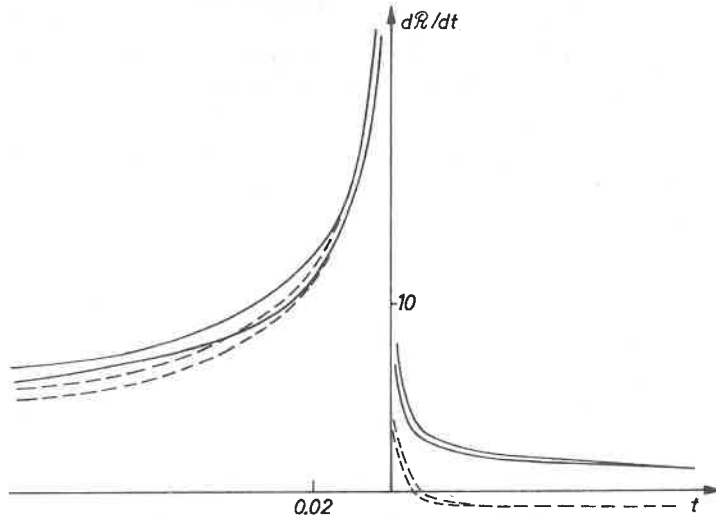


Fig. 4. The dependence of dQ/dT on t , corresponding to the solid curves of Fig. 1. The dashed curves illustrate the same dependence with the change κ for the value 0.2 (instead of 0.1)

symmetry operations of cubic lattices, arise obviously from the fourth order terms in $\partial M_\alpha / \partial R_\mu$ of the Landau free energy. The diagonal components $\langle |\hat{M}_r(\mathbf{q})|^2 \rangle$ of the tensor correlation function, being the only nonvanishing ones and equal to each other, are simply proportional to the inverse of $L(\mathbf{q})$ [19], if the fourth order terms in $\hat{M}_r(\mathbf{q})$ are omitted in the probability density $\exp(-\Delta f/k_B T)$ of the fluctuations. Thus, in the lowest order approximation we can assume

$$\hat{F}(\mathbf{q}, t) = 1/L(\mathbf{q}, t). \quad (4.3)$$

It is seen that the isocorrelation surfaces cease to be spheres, like those in the case of the Ornstein-Zernike correlation function, by simply considering higher order terms in the free energy expansion (see, Fig. 5).

We shall show that the anisotropic correlations described in this way suffice to produce an anisotropy in the s-f resistivity in cubic crystals. For this purpose we assume the spherical

band and calculate the resistivity up the third order approximation in spherical harmonics, i.e., we use formula (2.7) for $l = 3$ in which the inner products C_{ij} are calculated with the use of the harmonic (2.5). They can be represented in the form (3.19), namely

$$C_{ij} = \int d\mathbf{q} \hat{F}(\mathbf{q}, T) \hat{F}_{ij}(\mathbf{q}), \quad (4.4)$$

where, to within an accuracy of a constant factor, $F_{ij}(\mathbf{q})$ are given by

$$\hat{F}_{ij}(\mathbf{q}) = \int d\mathbf{q} \delta(\varepsilon_{\mathbf{k}} - \varepsilon_F) \delta(\varepsilon_{\mathbf{k}-\mathbf{q}} - \varepsilon_F) [\phi_i(\mathbf{k}) - \phi_i(\mathbf{k}-\mathbf{q})] [\phi_j(\mathbf{k}) - \phi_j(\mathbf{k}-\mathbf{q})]. \quad (4.5)$$

The straightforward calculations yield

$$\hat{F}_{11}(\mathbf{q}) \propto q_\mu q_\nu n_\mu n_\nu \Theta(2k_F - q), \quad (4.6)$$

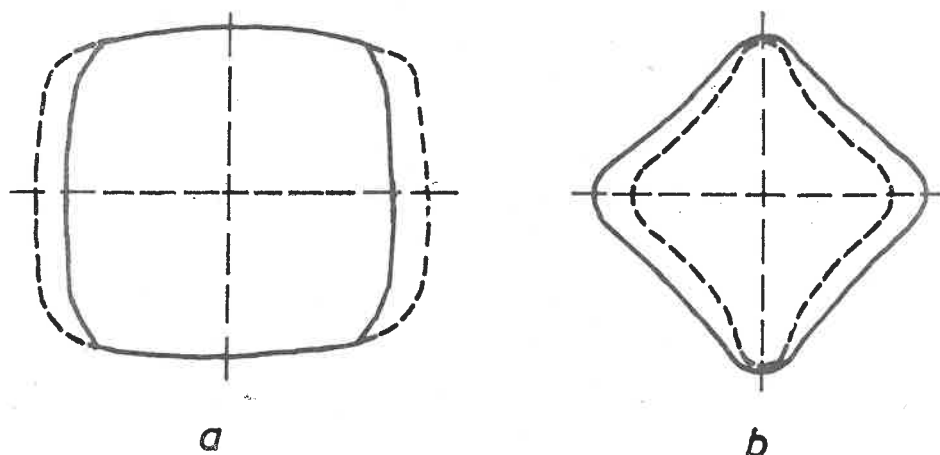


Fig. 5. The schematic isocorrelation surfaces ($\hat{F}(\mathbf{q}, t) = \text{const}$) of cubic lattices in q -space, following as in [20] from the approximation (4.3). In the spherical coordinates: $q_x = q \sin \vartheta \cos \varphi$, $q_y = q \sin \vartheta \sin \varphi$, $q_z = q \cos \vartheta$ with the polar axis parallel to the directions of the type [001], the solid curves correspond to the projection $\varphi = 0$ and the dashed (thick) ones to $\varphi = \pi/4$ in the case a) $L'_4 > 0$, b) $L'_4 < 0$

which corresponds to Eq. (3.20) specified for the spherical band, when the electric field is along the main directions, i.e., those of the type [100] — being the reference frame of the harmonics $\phi_i(\mathbf{k})$. One can easily note that the resistivity calculated up to the first order

$$\varrho^{(1)} \propto \int \hat{F}(\mathbf{q}, t) \hat{F}_{11}(\mathbf{q}) d\mathbf{q} \quad (4.7)$$

does not depend on the direction of $\mathbf{n}$, since only the terms q_x^2 , q_y^2 , q_z^2 contribute, and all equally, to the integral (4.6). However, to be consistent with the order of the approximation that we used to calculate $\hat{F}(\mathbf{q})$, we have to take into account the next, third harmonic and determine C_{13} and C_{33} .

The integrals (4.5) can be easily solved in spherical coordinates with the polar axis parallel to the direction of $\mathbf{q}$, and by then applying the identity $\int_{-1}^{+1} d(\cos \vartheta) \delta(\cos \vartheta - t)$

$= \Theta(1-t)$, ($t > 0$), where ϑ is the polar angle. Thus, we rotate the coordination system out of the main directions by means of one of the matrices ($\mu, \nu = x, y, z$)

$$(D_{\mu\nu}^A) = \begin{pmatrix} \frac{q_x}{q} & \frac{q_y}{q} & \frac{q_z}{q} \\ 0 & -\frac{q_z}{q_A} & \frac{q_y}{q_A} \\ \frac{q_A}{q} & -\frac{q_x q_y}{q_A q} & -\frac{q_z q_x}{q_A q} \end{pmatrix}, \quad q_A \equiv (q_y^2 + q_z^2)^{1/2} \quad (4.8a)$$

$$(D_{\mu\nu}^B) = \begin{pmatrix} -\frac{q_z}{q_B} & 0 & \frac{q_x}{q_B} \\ \frac{q_x}{q} & \frac{q_y}{q} & \frac{q_z}{q} \\ -\frac{q_x q_y}{q_B q} & \frac{q_B}{q} & -\frac{q_y q_z}{q_B q} \end{pmatrix}, \quad q_B \equiv (q_z^2 + q_x^2)^{1/2} \quad (4.8b)$$

$$(D_{\mu\nu}^C) = \begin{pmatrix} -\frac{q_y}{q_C} & \frac{q_x}{q_C} & 0 \\ -\frac{q_z q_x}{q_C q} & -\frac{q_y q_z}{q_C q} & \frac{q_C}{q} \\ \frac{q_x}{q} & \frac{q_y}{q} & \frac{q_z}{q} \end{pmatrix}, \quad q_C \equiv (q_x^2 + q_y^2)^{1/2}. \quad (4.8c)$$

In the new coordination system the components $\tilde{q}_\mu = D_{\mu\nu} q_\nu$ are $(q, 0, 0)$, $(0, q, 0)$, $(0, 0, q) = \sqrt{q_\mu q_\mu}$ for the transformation A, B, C , respectively.

The final expressions for $\hat{F}_{13}(\mathbf{q})$, $\hat{F}_{33}(\mathbf{q})$ are polynomials in q of the fourth and sixth order, respectively, with the coefficients expressed by the elements of the D -matrices and the direction cosines of the applied electric field. For instance, the terms of $\hat{F}_{13}(\mathbf{q})$ which contribute to (4.4) can be represented by means of $D_{\mu\nu}^C \equiv D_{\mu\nu}^C$ (transpose of (4.8c)) as

$$\begin{aligned} \sum_{\mu, \nu} \{ (\frac{1}{2}) (D_{\mu x}^2 D_{\mu z} D_{\nu z} + D_{\mu y}^2 D_{\mu z} D_{\nu z}) q^2 + [(\frac{1}{2}) (D_{\mu x}^2 D_{\mu z} D_{\nu z} + D_{\mu x}^2 D_{\mu z} D_{\nu z}) \\ + 15 D_{\mu z}^3 D_{\nu z} + 6 D_{\mu z} D_{\nu z}] (q^4 / 4 k_F^2) \\ + 5 (D_{\mu z}^3 D_{\nu z} - 3 D_{\mu z} D_{\nu z}) q^4 \} n_\mu n_\nu \Theta(2k_F - q). \end{aligned} \quad (4.9)$$

By the same arguments as in the case $\hat{F}_{11}(\mathbf{q})$ one can find out that none of the terms in the brace bracket contributes to (4.4) if $\mu \neq \nu$. This is sufficient for isotropy of C_{13} ,

since its values corresponding to the main directions should be the same. The list of the terms which contribute to C_{33} is, of course, considerably more numerous. It suffices to say that among them there are such terms of the second and fourth order in q , e.g.,

$$\sum_{\alpha, \beta, \mu, \nu} \{ (D_{\alpha\alpha} D_{\alpha\alpha} D_{\mu\mu}^2 D_{\mu\mu} D_{\nu\nu} + D_{\alpha\gamma} D_{\alpha\gamma} D_{\mu\mu}^2 D_{\mu\mu} D_{\nu\nu}) q^2 + (D_{\alpha\alpha} D_{\beta\beta} D_{\alpha\alpha} D_{\beta\beta} D_{\mu\mu} D_{\nu\nu} + D_{\alpha\gamma} D_{\beta\gamma} D_{\alpha\gamma} D_{\beta\gamma} D_{\mu\mu} D_{\nu\nu}) q^4 \} n_\mu n_\nu \Theta(2k_F - q), \quad (4.10)$$

which contribute to the dependence of C_{33} on the field direction. Thus, on the basis of Eq. (2.7) it is obvious that the magnitude of the resistivity is directionally dependent. Due to the symmetry requirements used to build the higher harmonics the s-f resistivity should be of the same value for the electric field applied to crystallographically equivalent directions. The number of the contributing terms is considerably smaller if the field is directed in one of the main directions, namely, if $\mathbf{n} = (1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$. In this case the equivalence of these directions can be shown in the most efficient way by applying respectively the transformations (4.8).

5. Final remarks

The results of the present paper may be summarized shortly by saying that both in uniaxial and cubic ferromagnets an anisotropy in the magnetic fluctuations leads to anisotropic effects in the electron scattering and electrical resistivity. Remarkable is the fact that these effects are caused solely by the spatial (in $\mathbf{R}$ - or $\mathbf{q}$ -space) anisotropy of the magnetic fluctuations, following in turn from the anisotropic medium in which the fluctuations propagate and from the existence of a magnetocrystalline anisotropy. No anisotropy in the magnetic interaction between the scattering system (localized magnetic moments) and the current carriers (free electrons) was assumed. The existence of such an anisotropy should yield additional anisotropic contribution to the s-f resistivity.

The anisotropy in our system was assumed at the level of the interactions within the scattering system. In the uniaxial case an anisotropy of the electronic dispersion law, and consequently that of the Fermi surface, was also assumed. As seen from Eq. (2.6) these two factors have quite a different influence on the s-f resistivity. While the latter determines the tensor properties of the conductivity (or resistivity), the first depends only functionally on the direction of the applied electric field due to the spatial anisotropy of the magnetic fluctuations. As a result, for the spherical Fermi surface when the resistivity tensor is diagonal, and — as one should expect — with the diagonal components are equal to each other, their values depend on the direction of the applied electric field.

In the uniaxial case we calculated the values of the resistivity in the (magnetic) easy and hard directions in which the resistivity had been experimentally measured [9, 2] and for which the anisotropy is most pronounced. The results of Section 3 show that the effective anisotropy of the s-f resistivity is simply related both to the Fermi surface anisotropy and that following from the magnetic fluctuations. The effect seems to be of the same order as an apparent anisotropic influence of the phonon scattering background. Therefore,

while interpreting the experimental data for such a material as GdTb [2], both scattering sources should be taken into account.

At the present stage of the investigations one can hardly describe the anisotropy of the fluctuations in a better approximation than that used in the present paper. However, this approximation seems quite sufficient, provided that the comparison of the experimental data to the theoretical results is performed in a proper temperature interval. The incorporation of a more realistic band structure into the theory is rather more important. Within the formalism that we use it is not difficult to include the band structure in the same approximate way as in [21, 18]. Without knowing of the details of the band structure of uniaxial ferromagnetic metals a better approximation is not needed. Nevertheless, even this oversimplified approximation requires numerical analysis from the very beginning. Thus, the present free electron approximation calculations were made simply to point out the existence of such anisotropic effects and to estimate its magnitude.

In the case of cubic ferromagnets one can just about to assess the magnitude of the anisotropy in the s-f resistivity. As we showed in Section 4 such effects should exist, but — as far as we know — there is no experimental evidence for their occurrence. From a theoretical point of view the difficulties in studying this effect are twofold. In the first place, it is a question of finding a proper, anisotropic correlation function. Secondly, the effect can be revealed only in a higher order approximation than that used usually in treating the transport phenomena within the Boltzmann–Bloch equation approach. In comparison to the uniaxial ferromagnets the effect is of the higher order, and, therefore, one may expect that the anisotropy of the s-f resistivity of cubic ferromagnetic metals can be even considerably smaller. At present it is difficult to estimate whether this anisotropy could be experimentally revealed amongst other anisotropic factors which apparently influence the resistivity of a metal.

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REFERENCES

- [1] K. Durczewski, *Acta Phys. Pol.* **A59**, 167 (1981).
- [2] J. B. Sousa, M. M. Amado, M. E. Braga, R. P. Pinto, J. M. Moreira, D. Hunkin, *Commun. Phys.* **2**, 95 (1977).
- [3] J. B. Sousa, M. M. Amado, R. P. Pinto, J. M. Moreira, M. E. Braga, M. Ausloos, J. P. Leburton, J. C. van Hay, P. Clippe, J. P. Vigneron, P. Morin, *J. Phys. F* **10**, 933 (1980).
- [4] J. Kociński, L. Wojtczak, *Critical Scattering Theory*, An Introduction, p. 212; PWN (Polish Scientific Publishers), Warszawa, Elsevier Publishing Company, Amsterdam–Oxford–New York 1978.
- [5] M. Ausloos, K. Durczewski, *Phys. Rev.* **B22**, 2439 (1980).
- [6] J. Klamut, J. Sznajd, *Phys. Status Solidi (b)* **60**, 795 (1973).
- [7] K. Durczewski, *Solid State Commun.* **31**, 427 (1979).
- [8] K. Durczewski, M. Ausloos, *J. Magn. Magn. Mater.* **15–18**, 927 (1980), Proc. ICM'79 (Munich).
- [9] H. E. Nigh, S. Legvold, F. G. Spedding, *Phys. Rev.* **132**, 1062 (1963).
- [10] T. Kasuya, *Prog. Theor. Phys.* **16**; 45 (1956); **16**, 58 (1956).
- [11] P. G. de Gennes, J. Friedel, *J. Phys. Chem. Solids* **4**, 71 (1958).
- [12] A. H. Morish, *The Physical Principles of Magnetism*, § 6.3, J. Wiley and Sons, Inc., New York 1965.

- [13] J. Klamut, *Acta Phys. Pol.* **25**, 711 (1964); H. Pfeiffer, J. Ulner, *Acta Phys. Pol.* **A39**, 703 (1971).
- [14] M. E. Fisher, J. S. Langer, *Phys. Rev. Lett.* **20**, 665 (1968).
- [15] S. von Molnar, T. Kasuya, *Phys. Rev. Lett.* **21**, 1757 (1968); S. von Molnar, M. W. Shafer, *J. Appl. Phys.* **41**, 1093 (1970).
- [16] G. Malström, D. J. W. Geldart, *Phys. Rev.* **B21**, 1130 (1980).
- [17] P. P. Craig, W. I. Goldburg, T. A. Kitchens, J. I. Budnick, *Phys. Rev. Lett.* **19**, 1334 (1967); L. Passel, K. Blinowski, T. Brun, *Phys. Rev.* **139A**, 1866 (1965).
- [18] D. J. W. Geldart, T. G. Richard, *Phys. Rev.* **B12**, 5175 (1975).
- [19] L. D. Landau, E. M. Lifshitz, *Statistical Physics*, Nauka, Moscow 1964, § 113.
- [20] J. Klamut, J. Sznajd, ICM '79, *Program and Abstracts*, International Conference on Magnetism, Munich 1979, page 108 (4W10).
- [21] T. G. Richard, D. J. W. Geldart, *Phys. Rev. Lett.* **30**, 290 (1973).