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GAPS IN THE SPINLESS COLLECTIVE EXCITATIONS OF ^{3}He-B IN THE PRESENCE OF THE DIPOLE FORCES*

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Gaps in the density and transverse current excitations were investigated in the presence of the Fermi-liquid and dipole interactions. It was shown that the gap in the density excitations is always greater than the gap in the transverse current excitations, provided that dipole forces are taken into account. The dipole correction parameter properties are discussed in detail.

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The purpose of this paper is to discuss the effects which occur in the superfluid ^{3}He-B and which can be helpful for evaluation of the second Landau parameter F_2. The precise calculation of the F_2 is essential to improve the description of the response of the superfluid system. Now, it is known [1–4] that parameter F_2 is strongly connected with a gap in the spinless, i.e., density and transverse current, collective excitation spectrum. We discuss this problem in detail for the dipole interaction taken into account.

The spinless collective excitations can be defined by means of the poles having the following correlation functions: density-density, density-current and current-current, [1–5].

The collective excitations which appear in the ^{3}He-B can be classified in terms of the two-particle states [2, 6–8]. These states have the following quantum numbers: the angular momentum $L = 1$, spin $S = 1$ and the total angular momentum J, where $0 \leqslant J \leqslant 2$ and its projection M is such that $|M| \leqslant J$. Collective excitations with the gap, given by the pole of the density autocorrelation function are the excitations to the state $J = 2$, $M = 0$. On the other hand, the collective excitations given by the pole of the transverse current autocorrelation function are the excitations to the states $J = 2$, $M = \pm 1$ [1, 2, 8]. Both kinds of excitations can be observed experimentally. They are connected with the propagation and attenuation of the sound [2–4,9]. When investigating the density auto-

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correlation function, one obtains that the oscillations of the density are attenuated for the frequency equal to the gap longitudinal collective excitation ($J = 2$, $M = 0$), whereas for the frequency equal to the gap of the transverse collective excitations ($J = 2$, $M = \pm 1$), the homogeneous oscillations of the transverse current become possible. This problem was discussed by Czerwonko [1], Wölfle [3, 4], Maki and Ebisawa [10], and Maki [8, 11]. They showed that the value of the gap can be computed from the equation

$$\omega^2[1+\tfrac{3}{25} F_2F(\omega)] = \tfrac{12}{5} \Delta^2[1+\tfrac{1}{5} F_2F(\omega)], \tag{1}$$

where

$$F(\omega) = \int\limits_0^\infty d\varepsilon \frac{\operatorname{th} \dfrac{\sqrt{\varepsilon^2+\Delta^2}}{2T}}{\sqrt{\varepsilon^2+\Delta^2}} \cdot \frac{4\Delta^2}{4(\varepsilon^2+\Delta^2)-\omega^2},$$

and this frequency is identical for two kinds of the collective excitations discussed, i.e., for $J = 2$, $M = 0$ and $J = 2$, $M = \pm 1$. This is not surprising, because our system, in the absence of dipole forces, is invariant under separate rotation of spin and momentum variables and at the homogeneous limit, i.e., for $k = 0$, and $M = 0$ with $M = \pm 1$ are different components of the same representation with $J = 2$.

To solve such specified problems we used the microscopic formalism developed by Czerwonko [5]. The dipole interaction, which is a weak spin interaction, can only modify the pairing interaction, (cf. [7, 8]). Hence, the total interaction in the particle-particle channel has the form

$$V_{ij} = -3g_1\hat{p}_n\hat{p}'_n\delta_{ij}+\tfrac{4}{3} g_D[\delta_{ij}-3(\hat{p}_i-\hat{p}'_i)(\hat{p}_j-\hat{p}'_j)/|\hat{p}-\hat{p}'|^2], \tag{2}$$

where g_1 and g_D are dimensionless factors connected with the pairing and dipole interactions, respectively. Indices i and j define the parts of the interaction in the spin space. We have also $g_1 \gg g_D > 0$. This inequality allows us to restrict ourselves to the first Legendre harmonic only of this interaction. Our further calculations are analogous to those, which we have developed for the investigations of spin collective excitations of ^{3}He-B in [7]. Since the present calculations are very arduous and some of their interesting features can be obtained easily [7] we omit them and restrict ourselves only to the discussion of the imposed conditions. First of all, we emphasize that the dipole contributions to both these cases completely coincide.

Although the dipole interaction breaks the symmetry of the system, it distinguishes no direction. Nevertheless, we assume that our system distinguishes one macroscopic direction $\hat{k}$, and hence the equilibrium state is the linear combination of two-particle states with a fixed $M = 0$. This problem was discussed in detail [7] recently, and as we have shown such an approach is the most general possible. The existence of the distinguished direction causes the collective excitations with the same M to mix mutually.

For convenience we can perform the transformation which allows us to link the separate collective excitations with the quantum number J. The transformation changes also the equilibrium state into the $J = 0$ state and strongly disturbes the interaction in the particle-

-particle channel (cf. [7]). This causes splitting of the density oscillation gap and the transverse current oscillation gap, in spite of the great symmetry of such selected problem. Since we search only the collective excitation gaps we can restrict ourselves to the homogeneous limit, ($k = 0$). Taking $k = 0$ when simultaneously the direction $\hat{k}$ is held, is a quite justified procedure. It is equivalent to the one where we first calculate the correlation functions for an arbitrary $\boldsymbol{k}$, and next looking for the collective excitation gaps, we tend to zero with k in the polar parts. Roughly speaking, we assume there exists a residual inhomogeneity, $\boldsymbol{k}$, which defines the direction, $\hat{k}$, but which is so small that all the contributions from it can be neglected. Let us write down now the denominators of the correlation functions. We introduce them in the form of determinants

$$D_{\parallel} = \begin{vmatrix} \pi_{11}, & 0, & \pi_{13} \\ 0, & \pi_{22}, & 0 \\ \pi_{13}, & 0, & \pi_{33} \end{vmatrix}, \tag{3}$$

where

$$\pi_{11} = \frac{F(\omega)}{4\Delta^2}\left\{\omega^2 - 4\Delta^2\left[1 + \frac{27 g_D}{4 g_1^2 F(\omega)}\right]\right\},$$

$$\pi_{13} = \pm\tfrac{27}{20}\sqrt{5}\, g_D/g_1^2,$$

$$\pi_{22} = \frac{\omega^2 F(\omega)}{4\Delta^2[1+\frac{1}{5}F_2 F(\omega)]},$$

$$\pi_{33} = \frac{F(\omega)}{4\Delta^2[1+\frac{1}{5}F_2F(\omega)]}\left\{\omega^2[1+\tfrac{3}{25}F_2F(\omega)] - \tfrac{12}{5}\Delta^2[1+\tfrac{1}{5}F_2F(\omega)]\left[1+\frac{9g_D}{4g_1^2F(\omega)}\right]\right\},$$

and

$$D_{\perp} = \begin{vmatrix} \pi_{44}, & \pi_{45}, & 0, & \pi_{47} \\ \pi_{45}, & \pi_{55}, & \pi_{47}, & 0 \\ 0, & \pi_{47}, & \pi_{44}, & -\pi_{45} \\ \pi_{47}, & 0, & -\pi_{45}, & \pi_{55} \end{vmatrix}, \tag{4}$$

where

$$\pi_{44} = \frac{(\omega^2 - 4\Delta^2)F(\omega)}{4\Delta^2},$$

$$\pi_{45} = \mp\tfrac{9}{20}\sqrt{5}\, g_D/g_1^2,$$

$$\pi_{47} = \tfrac{9}{4}\, g_D/g_1^2,$$

$$\pi_{55} = \frac{F(\omega)}{4\Delta^2[1+\frac{1}{5}F_2F(\omega)]}\left\{\omega^2[1+\tfrac{3}{25}F_2F(\omega)] - \tfrac{12}{5}\Delta^2[1+\tfrac{1}{5}F_2F(\omega)]\left[1-\frac{3g_D}{2g_1^2F(\omega)}\right]\right\}.$$

$D_{\parallel}$ is the denominator of the density autocorrelation function, whereas $D_{\perp}$ is the denominator of the transverse current autocorrelation function. Since $g_1 < 1$ and $g_D/g_1 \ll 1$, the term g_D/g_1^2 is greater than g_D/g_1 and could be even of the order of unity.

The form of the parameters g_1 and g_D can be found from the zero-temperature weak-coupling Green's functions formalism. Czerwonko [5] reckoned g_1 in the dependence on ε_p and Δ with the assumption that the dipole interaction should be neglected. The dipole interaction modified this dependence and the appropriate equation has the form [7]

$$g_1 + \tfrac{1}{2} g_D = \left[\ln \frac{\varepsilon_p^2 + \sqrt{\varepsilon_p^2 + \Delta^2}}{\Delta}\right]^{-1}. \tag{5}$$

Considering the cut-off parameter ε_p, we usually assume that $\Delta \ll \varepsilon_p \ll \varepsilon_F$ (ε_F — Fermi energy). Leggett [12] and Wheatley [9] estimated this for 0.7 K in the temperature scale, $\Delta = 1.75\, T_c$. We see from Eq. (5) that the inclusion of the dipole interaction leads to an increase of Δ ($\Delta(g_D) = \Delta(0)\exp(g_D/2g_1^2)$). The g_D value appearing in our formalism is different from those given by Wheatley [9] since the dipole interaction has been renormalized together with the pairing interaction, by the introduction of the cut-off parameter ε_p. Such renormalizing transformation was described in detail by Czerwonko [5]. The simple calculations allow us to prove that g_D/g_1^2 is an invariant under the renormalizing transformation, i.e. g_D/g_1^2 does not depend on ε_p (cf. Appendix). The expression g_D/g_1^2 is the new, universal parameter of the developed formalism and final results depend only on it. We call it further by

$$g = g_D/g_1^2. \tag{6}$$

Now using Eqs. (5) and (6) we can express g_1 and g_D on the dependence on ε_p, in the form:

$$g_1 = \left[\left(1 + 2g\Big/\ln\frac{\varepsilon_p + \sqrt{\varepsilon_p^2 + \Delta^2}}{\Delta}\right)^{1/2} - 1\right] g^{-1},$$

$$g_D = \left[\left(1 + 2g\Big/\ln\frac{\varepsilon_p + \sqrt{\varepsilon_p^2 + \Delta^2}}{\Delta}\right)^{1/2} - 1\right]^2 g^{-1}. \tag{7}$$

Furthering our calculations, we assume that all terms, g^n, for $n \geqslant 2$ are much smaller than unity and are not significant. In such a case we obtain

$$D_{\parallel} \sim \pi_{33} \quad \text{and} \quad D_{\perp} \sim \pi_{55}. \tag{8}$$

Hence, for gaps of the collective excitation spectrum, we obtain two following equations:

$$\omega^2[1 + \tfrac{3}{25} F_2 F(\omega)] = \tfrac{12}{5} \Delta^2 [1 + \tfrac{1}{5} F_2 F(\omega)]\, [1 + 9g/4F(\omega)], \tag{9}$$

$$\omega^2[1 + \tfrac{3}{25} F_2 F(\omega)] = \tfrac{12}{5} \Delta^2 [1 + \tfrac{1}{5} F_2 F(\omega)]\, [1 - 3g/2F(\omega)]. \tag{10}$$

Let us indicate the solution of these equations by $\omega_{\parallel}$ and $\omega_{\perp}$ and by ω_0 the solution of Eq. (1). The accuracy of our calculations allows us to assume that $\omega_{\parallel}$ and $\omega_{\perp}$ are modified, in comparison with ω_0, by quantities λ and μ, of the order of g, and can be taken in the form:

$$\omega_{\parallel} = \omega_0(1+\lambda) \quad \text{and} \quad \omega_{\perp} = \omega_0(1+\mu). \tag{11}$$

Substituting the first formula of (11) into (9) and the latter into (10), confining ourselves to the terms up to the order of g and eliminating F_2 from the first order terms via equation (1), one finds that [1]:

$$\lambda = 9g/8E_{3/5}(\omega_0), \qquad \mu = -3g/4E_{3/5}(\omega_0), \tag{12}$$

where

$$E_s(\omega) = F(\omega)+G(\omega)\left[s-\left(\frac{\omega}{2\Delta}\right)^2\right]\left[1-\left(\frac{\omega}{2\Delta}\right)^2\right]\Big/(1-s),$$

and

$$G(\omega) = 4\Delta^2\,\frac{dF(\omega)}{d(\omega^2)} = \int_0^\infty d\varepsilon\,\frac{\operatorname{th}(\sqrt{\varepsilon^2+\Delta^2}/2T)}{\sqrt{\varepsilon^2+\Delta^2}}\cdot\frac{16\Delta^2}{[4(\varepsilon^2+\Delta^2)-\omega^2]^2}$$

and hence,

$$\frac{\omega_{\parallel}-\omega_{\perp}}{\omega_0} = 15g/8E_{3/5}(\omega_0), \tag{13}$$

and

$$\omega_0 = \tfrac{1}{5}(2\omega_{\parallel}+3\omega_{\perp}). \tag{14}$$

Let us consider now the function $E_s(\omega)$. This can be rewritten in the form:

$$E_s(\omega) = 16\Delta^2\int_0^\infty \frac{d\varepsilon}{\sqrt{\varepsilon^2+\Delta^2}}\,\frac{\operatorname{th}(\sqrt{\varepsilon^2+\Delta^2}/2T)}{[4(\varepsilon^2+\Delta^2)-\omega^2]^2}\left\{\varepsilon^2+\frac{\Delta^2}{1-s}\left[1-\left(\frac{\omega}{2\Delta}\right)^2\right]^2\right\}. \tag{15}$$

This function is, of course, always positive. Hence, from Eqs. (11)–(15) we obtain the following inequalities:

$$\omega_{\perp} < \omega_0 < \omega_{\parallel}. \tag{16}$$

Investigating the function $E_s(\omega)$, we find that its minimum value is reached for $\omega_0 = 2\sqrt{s}\Delta$, i.e. for $F_2 \to 0$. Czerwonko [1] discussed in detail the solutions of Eq. (1) as a function of F_2 and temperature. According to this calculation, ω_0 tends to 2Δ and the function $E_s(\omega_0)$ tends to infinity if F_2 tends to infinity. For F_2 tending to minus five at $T = 0$, i.e., to the border of stability of the system ω_0 tends to zero and, from Eqs. (13)–(15), $\omega_{\parallel}$ and $\omega_{\perp}$ also tend to zero. According to Eq. (13), the difference $\omega_{\parallel}-\omega_{\perp}$ is proportional to the function $\omega_0/E_{3/5}(\omega_0)$. This function has a sharp maximum in the vicinity of $\omega_0 = 2\sqrt{\tfrac{3}{5}}\Delta$. This causes the dipole interaction to be the most important if F_2 is close to zero.

The sole characteristic quantity modifying the results is the renormalization invariant, g. This value should determine the magnitude of corrections connected with the dipole interaction and, hence, decide whether the dipole interaction can be neglected or not. The experimental results carried out so far do not permit an estimation of the parameter, g. Eqs. (9)–(14) should also allow us to determine F_2 more precisely [1, 9, 13]. From the formalism developed one can find that the density autocorrelation function discussed vanishes in the homogeneous limit ($k = 0$) even without the extra assumption on g^n.

From Eq. (3) we find also that the inclusion of the dipole interaction does not change the zero sound spectrum. These two facts confirm the corectness of the formalism developed. The factors π_{11} and π_{44} can never equal zero, since ω must be less than 2Δ in order not to destroy the superfluidity.

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APPENDIX

The formalism developed contains an artificial parameter, so-called the cut-off parameter, ε_p, and connected with it the parameter, g_1, ([5] and Eq. (12)). Since the final results contain contributions proportional to the parameter, g, Eq. (6), one can ask if the final results depend on ε_p. We show below that the parameter, g, does not depend on ε_p. Deriving the basic equations of the formalism, we apply so-called the renormalizing transformation given by the equation: [5]

$$\hat{V} = (\hat{\mathbf{1}} + \hat{V}\mathscr{A})\hat{\Gamma}. \tag{A1}$$

We use the matrix notation, where $\hat{V}$ and $\hat{\Gamma}$ are the renormalized and unrenormalized interactions in the particle-particle channel, respectively. $\mathscr{A}$ is the cut-off kernel and if $\Delta \ll \varepsilon_p \ll \varepsilon_F$ it has the form [5]

$$\mathscr{A} = G^{-}G\Theta(|\varepsilon| - \varepsilon_p). \tag{A2}$$

We assume that the renormalization transformation does not change the interaction structure, i.e., $\hat{V}$ and $\hat{\Gamma}$ have the same structure. As we showed in [7], the interaction, $\hat{V}$, in the presence of the dipole forces has the form:

$$\hat{V} = -g_1(1+\alpha)\,[\hat{\mathbf{1}} + \tfrac{3}{5}\alpha\hat{U}], \tag{A3}$$

then $\hat{\Gamma}$ has to have the form:

$$\hat{\Gamma} = -h(1+\beta)\,[\hat{\mathbf{1}} + \gamma\hat{U}]. \tag{A4}$$

The factors, $\alpha(= g_1/g_D)$, β and γ are small quantities in comparison with unity, and we hold them only in the first order in all equations. If we substitute Eqs. (A3) and (A4) into Eq. (A1) and compare the appropriate factors standing near the operator, $\hat{\mathbf{1}}$, or, $\hat{U}$, on both sides of the equation, we obtain two independent equations. Now, we can compare the quantities of the same order in the first equation, and hence, we obtain the two following equations:

$$g_1 = (1-g_1\overline{\mathscr{A}})h, \tag{A5}$$

$$\alpha g_1 = -\alpha g_1\overline{\mathscr{A}}h+\beta(1-g_1\overline{\mathscr{A}})h. \tag{A6}$$

Inserting Eq. (A5) into Eq (A6) and after further calculations we obtain

$$\frac{\alpha}{g_1} = \frac{\beta}{h}. \tag{A7}$$

Both renormalized parameters g_1 and g_D are the functions of the cut-off parameter ε_p. However, according to Eq. (A7) the choice of the cut-off parameter ε_p is quite arbitrary, thus, the parameter $g(=\alpha/g_1)$ does not depend on ε_p and is the invariant of the theory. The parameter g is always equal to β/h and independent of ε_p, although:

$$g_1 = g_1(\varepsilon_p) \quad \text{and} \quad g_D = g_D(\varepsilon_p). \tag{A8}$$

The expression

$$g = \frac{g_D(\varepsilon_p)}{g_1^2(\varepsilon_p)} \tag{A9}$$

must be independent of ε_p and is the invariant of the theory. The second one from the equations previously obtained has the from:

$$\tfrac{3}{5}\alpha g_1 = -\tfrac{3}{5}\alpha g_1\overline{\mathscr{A}}h+\gamma(1-g_1\overline{\mathscr{A}})h. \tag{A10}$$

This equation is analogous to Eq. (A6), hence

$$\gamma = \tfrac{3}{5}\beta. \tag{A11}$$

Since the dipole contribution parameter g is independent of the artificial, cut-off parameter ε_p, it shows its physical nature. We do not have enough information to estimate the parameter g, correctly. That is why we will think of g as a phenomenological parameter.

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