DESCRIPTION OF THE DOMAIN STRUCTURE IN THE THIN FERROMAGNETIC FILM NEAR THE PHASE TRANSITION POINT FROM A STATE OF HOMOGENEOUS MAGNETIZATION TO THE DOMAIN STRUCTURE INDUCED BY A CHANGE IN THE EXTERNAL MAGNETIC FIELD

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On the basis of the self-consistent theory of the phase transition process from the homogeneous magnetization state to the domain structure, analytic expressions for magnetization in the domain, the structure's period and the domain wall's thickness as functions of the external field and the film thickness are derived. On the basis of analytical expressions numerical calculations are made and the results are shown in the graphical form. The material data for cobalt and permalloy are given. The value of the critical field is found. The region of the external field's intensities, for which the method presented here may be applied, is established.

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1. *Introduction*

Suppose we have a thin ferromagnetic film of thickness L with surfaces placed in an (x, y) plane and the easy axis of magnetization perpendicular to the film plane. Suppose an external homogeneous magnetic field $\vec{H} = [0, H, 0]$ is present. We make the assumption that the thickness L is larger than the critical thickness L_c and the external field H

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is less than the critical field H_c. The energy F of the thin ferromagnetic film is described by the function

$$F = \int_V \{\tfrac{1}{2}\alpha(\nabla\vec{M})^2 - \tfrac{1}{2}\beta M_z^2 - HM_y - \tfrac{1}{2}\vec{H}^d\vec{M}\}dV, \tag{1}$$

where α is the isotropic exchange constant, $\beta < 4\pi$ is the uniaxial anisotropy constant, $\vec{H} = (0, H, 0)$ — the external magnetic field intensity vector, $\vec{H}^d$ — the demagnetization magnetic field vector. In order to find the demagnetization field vector $\vec{H}^d = \vec{H}^d(\vec{r})$ as a function of the magnetization distribution $\vec{M} = \vec{M}(\vec{r})$ in the film we take into account Maxwell's equations in quasi-static approximation

$$\mathrm{rot}\,(\vec{H}+\vec{H}^d) = 0, \quad \mathrm{div}\,(\vec{H}^d + 4\pi\vec{M}) = 0. \tag{2}$$

The requirement of the extremum of the energy function in the form (1), with (2) taken into account leads (see [1, 2]) to the equation

$$(1+4\pi/h)\frac{\partial^2}{\partial x^2}\left[\alpha\frac{\partial^2 m_z}{\partial x^2} + (\beta-h)m_z - \frac{\beta}{2}m_z^3\right] - 4\pi\frac{\partial^2 m_z}{\partial z^2} = 0. \tag{3}$$

In the above equation we restrict ourselves to nonlinear term proportional to m_z^3. We introduce the notation: $h = H/M_0$; $m_z = M_z/M_0$; $M_0 = M = \mathrm{const}$.

2. *The description of the domain structure in the thin film*

As it is shown in [1] for external magnetic field values near the critical value h_c we may investigate the solution of equation (3) in the form $m_z(x, z) = m(x)\cos(k_1 z)$ (where $k_1 = \pi/L$), assuming that $m(x)$ is a periodic and oscillating function, which for $h \to h_c$ behaves like $\sin(\kappa x)$.

By h_c we denote the external magnetic field intensity value for which the phase transition from the homogeneous magnetic state to domain structure occurs. In the interval of the magnetic field value $\beta - h < 4k_1[4\pi\beta\alpha/(4\pi+\beta)]^{1/2}$ (see [1]) we can restrict ourselves to taking into account the first harmonic $\cos(k_1 z)$ only. Then equation (3) takes the form

$$\frac{\partial^2}{\partial x^2}\left[\alpha\frac{\partial^2 m(x)}{\partial x^2} + (\beta-h)m(x) - \tfrac{3}{8}\beta m^3(x)\right] + k_1^2\frac{4\pi h}{4\pi+h}m(x) = 0. \tag{4}$$

In the first step of the iteration procedure of equation (4) (see Appendix) we obtain for the magnetization distribution the equation

$$\frac{\partial^2 m(x)}{\partial x^2} + Bm(x) - 2Am^3(x) = C, \tag{5}$$

where

$$A = \tfrac{3}{16}\,\frac{\beta}{\alpha}, \tag{6}$$

$$B = \alpha^{-1}(\beta - h - \alpha\kappa_0^4(h)\kappa^2(h)), \tag{7}$$

$$\kappa_0^4(h) = k_1^2\,\frac{4\pi h}{\alpha(4\pi + h)}. \tag{8}$$

Integrating equation (5) we obtain

$$\left(\frac{dm(x)}{dx}\right)^2 = Am^4(x) - Bm^2(x) + 2Cm(x) + C_1, \tag{9}$$

where C_1 is the new constant. If $m(x)$ oscillates around the value $m = 0$ then the constant C must be equal to zero. The constant C_1 will be determined later.

The investigation of the periodic and oscillating solution satisfying the initial condition $m(x = 0) = 0$, leads to the result

$$m(x) = m_0\,\mathrm{sn}\,(\kappa x; k), \tag{10}$$

with

$$\kappa^2 = \tfrac{1}{2}\,(B - \sqrt{B^2 - 4AC_1}), \tag{11}$$

$$k^2 = (B - \sqrt{B^2 - 4AC_1})\,(B + \sqrt{B^2 - 4AC_1})^{-1}, \tag{12}$$

$$m_0^2 = A^{-1}\kappa^2 k^2. \tag{13}$$

In order to obtain the $\kappa = \kappa(h)$ dependence we have to solve equation (11) with regard to κ. The single-valued $\kappa = \kappa(h)$ and periodic $m(x)$ solution may be found if we choose

$$C_1 = A^{-1}\left[\left(\frac{\beta - h}{2\alpha}\right)^2 - \kappa_0^4(h)\right]. \tag{14}$$

Then the functions $\kappa(h)$, the modulus $k(h)$ of the elliptic integral and the amplitude $m_0(h)$ of the magnetization are of the form

$$\kappa^2(h) = \frac{\beta - h}{2\alpha} = \kappa_0^2(\beta) + \frac{h_c - h}{2\alpha}, \tag{15}$$

$$k^2(h) = 1-\kappa_0^4(h)\kappa^{-4}(h) \cong 1-\kappa_0^4(\beta)\kappa^{-4}(h), \tag{16}$$

$$m_0^2(h) = \tfrac{16}{3}\, \alpha\beta^{-1}[\kappa^2(h)-\kappa_0^4(h)\kappa^{-2}(h)] \cong \frac{8(h_c-h)\,(\beta-h+2\alpha\kappa_0^2(\beta))}{3\beta(\beta-h)}. \tag{17}$$

The value of the critical field h_c may be determined from the condition

$$\kappa_0^4(h_c) = \kappa^4(h_c), \tag{18}$$

which is equivalent to $k = 0$ or $m_0(h_c) = 0$. In the first approximation with regard to the small parameter $\alpha\kappa_0^2(\beta) \ll \beta$ the solution of equation (18) has the form

$$h_c = \beta-2\alpha\kappa_0^2(\beta). \tag{19}$$

So the process of solving the nonlinear equation (3) is finished. The approximate solution describes the magnetization distribution in the x-direction. The investigation of the magnetization heterogeneity in the perpendicular to the film plane direction has been done in [2]. The magnetization heterogeneity distribution in the x-axis direction, computed by us, shows the existence of domain structure with the period

$$\lambda = 4K(k)\left(\kappa_0^2(\beta)+\frac{h_c-h}{2\alpha}\right)^{-1/2}, \tag{20}$$

where $K(k)$ is the complete elliptic integral with modulus k defined by equation (16).

Taking into account (9), (14), (17) the thickness of the domain wall $\delta = 2m_0(dm(x)/dx)^{-1}_{x=0}$ may be rewritten as

$$\delta = 2\left[\kappa_0^2(\beta)+\frac{h_c-h}{2\alpha}\right]^{-1/2}. \tag{21}$$

In order to illustrate the dependences magnitudes m_0, λ, δ on $(h_c-h)/\beta$ we introduce parameter L_1 in the following form:

$$L_1 = \frac{2\alpha}{\beta}\kappa_0^2(\beta)L. \tag{22}$$

With the above notation expressions (15), (16) may be expressed as follows:

$$\kappa^2(h) = \frac{\beta}{2\alpha}\left(\frac{L_1}{L}+\frac{h_c-h}{\beta}\right), \tag{23}$$

$$k^2(h) = 1-\left[1+\frac{L}{L_1}\,\frac{h_c-h}{\beta}\right]^{-2}. \tag{24}$$

Choosing the thickness $L = 20L_1$ for the dependences (17), (20), (21) and the ratio δ/λ, we obtain the following plots for them

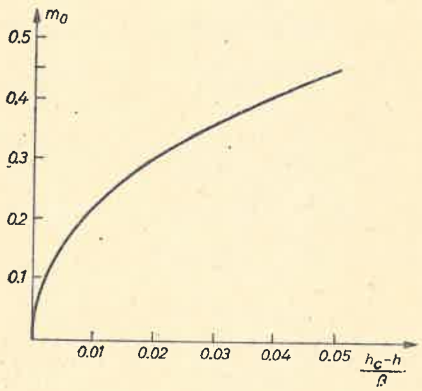

Fig. 1

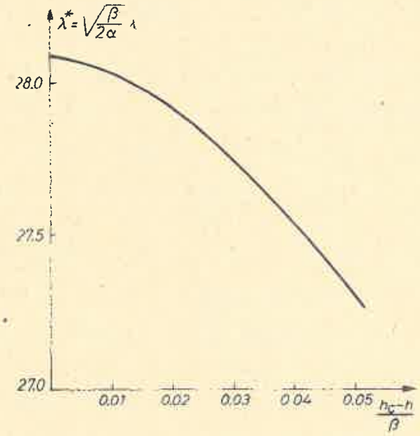

Fig. 2

Fig. 1. The plot of magnetization amplitude changes as function of the external magnetic field
Fig. 2. The plot of structure period changes as function of the external magnetic field

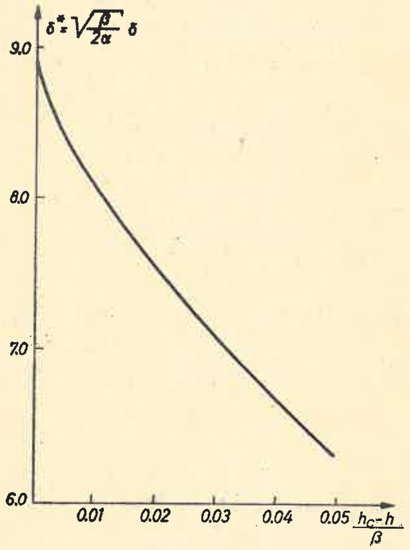

Fig. 3

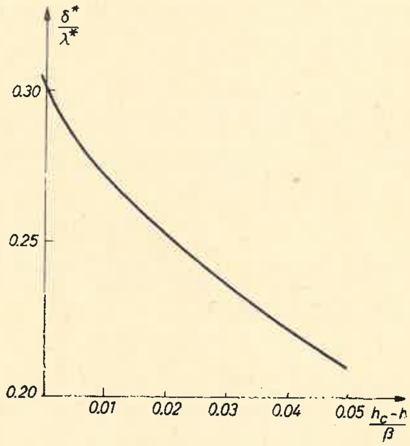

Fig. 4

Fig. 3. The plot of domain wall thickness changes as function of the external magnetic field
Fig. 4. The plot of ratio $\delta^*/\lambda^* = 1/2\, K(k)$ changes as function of the external magnetic field

3. Concluding remarks

The dependences illustrated by Figs. 1–4 describe the dimensionless magnitude, the shapes of which are independent of the kind of the ferromagnetic material forming the thin film. The choosing of the interval $0 \leqslant (h_c - h)/\beta \leqslant 0.05$ arises from the omitting, in analytical calculations, the terms involving the higher harmonics in the solution of equation (3) (see [1, 3]). To obtain real values of the magnetization in the domains $M_z = M_0 m_0$, the structure period $\lambda = \sqrt{2\alpha/\beta}\lambda^*$ and the domain wall thickness $\delta = \sqrt{2\alpha/\beta}\delta^*$, we must know the material constants which for instance, for permalloy [4] $\alpha = 2.7 \cdot 10^{-12}\text{cm}^2$, $\beta = 2.6$, $M_0 = 860$ Gs. The critical value of the external field $H_c = M_0 h_c$ for $L = 20L_1 = 116 \cdot 10^{-6}$ cm is $H_c = 2124$ Oe. The external magnetic field range of changes $2018 \text{ Oe} \leqslant H \leqslant 2124$ Oe and $\sqrt{2\alpha/\beta} = 1.44 \cdot 10^{-6}$ cm. In this range of changes H the magnetization M_z changes from 0 to 387 Gs, the structure period $39.3 \cdot 10^{-6} \leqslant \lambda \leqslant 40.5 \cdot 10^{-6}$ cm and domain wall thickness $9.1 \cdot 10^{-6} \text{ cm} \leqslant \delta \leqslant 12.9 \cdot 10^{-6}$ cm. For cobalt with $\alpha = 1.4 \cdot 10^{-12}\text{ cm}^2$ and $\beta = 2.5$, $M_0 = 1400$ Gs (see [4]) we obtained the ranges of changes $3159 \text{ Oe} \leqslant H \leqslant 3325$ Oe, $25.8 \cdot 10^{-6} \text{ cm} \leqslant \lambda \leqslant 26.6 \cdot 10^{-6}$ cm, $6 \cdot 10^{-6} \text{ cm} \leqslant \delta \leqslant 8.5 \cdot 10^{-6}$ cm.

APPENDIX

The differential equation (4) may be described as the integral-differential equation

$$\frac{\partial^2 m(x)}{\partial x^2} + \frac{\beta - h}{\alpha} m(x) - \frac{3\beta}{8\alpha} m^3(x) + k_1^2 \frac{4\pi h}{\alpha(4\pi + h)} \iint m(x) d^2x = 0. \tag{A1}$$

In order to solve this equation we employ the successive approximation method with the formula

$$\frac{\partial^2 m_{(k+1)}(x)}{\partial x^2} + \frac{\beta - h}{\alpha} m_{(k+1)}(x) - \frac{3\beta}{8\alpha} m^3_{(k+1)}(x) + k_1^2 \frac{4\pi h}{\alpha(4\pi + h)} \iint m_{(k)}(x) d^2x = 0, \tag{A2}$$

in which as the zero approximation $m_{(0)}(x)$ we choose the solution of the linear equation $m_{(0)}(x) \sim \sin \kappa x$. Finding the solution in the first step of the approximation procedure we make the following approximation:

$$\iint m_{(0)}(x) d^2x = -\kappa^2 m_{(0)}(x) \approx -\kappa^2 m_{(1)}(x) \tag{A3}$$

which enables us to reduce equation (A1) for $m_{(1)}(x)$ to a second order nonlinear equation in the form (5).

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