Vol. A58 (1980) ACTA PHYSICA POLONICA No 3

CRITICAL POINTS OF ANISOTROPIC FERROMAGNETS WITH EXTERNAL FIELD

By G. Ritter and J. Sznajd

Institute for Low Temperature and Structure Research, Polish Academy of Sciences, Wrocław*

(*Received November 14, 1979*)

By using the conditions for the existence of the critical points in anisotropic ferromagnets with a field, derived in [1], some concrete ferromagnets are considered. The directions for the uniaxial, orthorhombic, cubic and tetragonal ferromagnets in which the directed external field does not destroy the continuous phase transitions (described either by *XY*-like or Ising-like critical exponents) are found. In some cases for the definite values of the magnetic field, depending on the interaction parameters, the multicritical points may appear.

1. *Introduction*

The Hamiltonian of an anisotropic ferromagnet with an external uniform magnetic field may be written in the form:

$$\mathscr{H} = -\tfrac{1}{2}\int_{q}\sum_{i}(r_i^0+q^2)\sigma_q^i\sigma_{-q}^i - \int_{q}\int_{q_1}\int_{q_2}\sum_{i,j} u_{ij}^0\sigma_q^i\sigma_{q_1}^i\sigma_{q_2}^j\sigma_{-q-q_1-q_2}^j + (\boldsymbol{h}, \boldsymbol{\sigma}_0), \tag{1}$$

where $\boldsymbol{\sigma}_q$ is a classical three-component spin vector of wave vector $\boldsymbol{q}$, $\boldsymbol{h}$ is the external magnetic field and $r_i^0 = a_i(T-\Theta_i)$ (Θ_i the respective critical temperature of σ^i in the free field case).

The Hamiltonian (1) has a stable fixed point if the relations between the interaction parameters r_i^0, u_{ij}^0 and the external field, which were derived in [1] are satisfied. In the present paper we will check these relations for some anisotropic ferromagnets described by the Hamiltonian (1) with definite symmetry (with some additional conditions for the interaction parameters). In other words, we will find the directions in the anisotropic ferromagnets given the symmetry in which the directed external magnetic field does not destroy the continuous phase transition.

The influence of the external field on the phase transitions in the anisotropic ferromagnets has been studied for many years (see Refs. 1–13 to [1]). It was shown that in uniaxial [2], orthorhombic [3] and tetragonal [4] (with easy axis) ferromagnets, the contin-

* Address: Instytut Niskich Temperatur i Badań Strukturalnych PAN, Pl. Katedralny 1, 50-329 Wrocław, Poland.

uous phase transition still exists if the field is directed in the plane perpendicular to the easy axis. In the cubic four-axial ferromagnet [5] the external field directed in any direction in the plane {100} does not destroy such a phase transition, whereas in the cubic three--axial ferromagnet [6] the field ought to be directed in the part of the plane formed by a diagonal of a face of the cube and a main diagonal but not along the latter diagonal and not in some finite region in its vicinity.

In this paper we will find: the ordinary critical points of the anisotropic ferromagnets mentioned above in the presence of an external field of arbitrary direction, multicritical points and some conditions for the existence of the critical points in the general case (undefined symmetry).

After the following two transformations

$$\sigma_q^i \to \sigma_q^i + M_i\delta(q), \tag{2}$$

where M_i is given by the relation

$$h_i = M_i(r_i^0 + 4\sum_j u_{ij}^0 M_j^2), \quad M_i \to 0 \quad \text{for} \quad h_i \to 0, \tag{3}$$

and

$$\sigma_q^i \to \sigma_q^i = \sum_j t_{ji} S_q^j, \tag{4}$$

where t_{ij} are the elements of the matrix T

$$T = \begin{bmatrix} \cos\varphi & \sin\theta\cos\theta & \sin\varphi\sin\theta \\ -\cos\psi\sin\varphi & \cos\psi\cos\varphi\cos\theta - \sin\theta\sin\psi & \cos\psi\cos\varphi\sin\theta + \cos\theta\sin\psi \\ \sin\psi\sin\varphi & -\sin\psi\cos\varphi\cos\theta - \cos\psi\sin\theta & -\sin\psi\cos\varphi\sin\theta + \cos\theta\cos\psi \end{bmatrix}$$

(the angles φ and θ define the direction of the vector M) the Hamiltonian (1) takes the form (see [1])

$$\begin{aligned} \tilde{\mathscr{H}} = & -\tfrac{1}{2}\int_q \sum_{i,j} (r_{ij} + q^2\delta_{ij}) S_q^i S_{-q}^j - \int_q\int_{q_1} \sum_{i,j} w_{ij} S_q^i S_{q_1}^j S_{-q-q_1}^j - 2w \int_q\int_{q_1} S_q^1 S_{q_1}^2 S_{-q-q_1}^3 \\ & - \int_q\int_{q_1}\int_{q_2} \sum_{i,j} u_{ij} S_q^i S_{q_1}^i S_{q_2}^j S_{-q-q_1-q_2}^j - 4\int_q\int_{q_1}\int_{q_2} \sum_{i\neq j} v_{ij} S_q^i S_{q_1}^j S_{q_2}^j S_{-q-q_1-q_2}^j \\ & - \int_q\int_{q_1}\int_{q_2} \sum_{i\neq j\neq k} m_{ijk} S_q^i S_{q_1}^i S_{q_2}^j S_{-q-q_1-q_2}^k. \end{aligned} \tag{6}$$

The new interaction parameters after the transformations (2) and (3) were given in [1].

According to paper [1] the conditions for the existence of the XY-like critical point (solution of the C_2 class) have the form

I $\quad r_{33} = r_{22} < r_{11}$,

II $\quad r_{ij}^{(1)} = 0, \quad$ for $\quad i \neq j$,

III $\quad w_{ij} = 0, \quad i, j = 2, 3$,

$$\text{IV}\quad v_{ij}-ww_{ij}/2r_{11} = 0 \quad \text{for} \quad i \neq j \quad (i, j = 2, 3),$$

$$\text{V}\quad 0 < u_{23}-(2w^2-w_{12}w_{13})/2r_{11} < \Lambda \tag{7}$$

(Λ some function of $\bar{u}_{ii} = u_{ii}-w_{1i}^2/2r_{11}$).

Whereas for the Ising-like critical point (solution of the C_1 class)

$$\text{I}\quad r_{33} < r_{11}^{(1)}, r_{22}^{(1)},$$

$$\text{II}\quad r_{ij}^{(1)} = 0, \quad \text{for} \quad i \neq j$$

$$\text{III}\quad w_{33} = 0,$$

$$\text{IV}\quad u_{33}-\sum_{i=1}^{2} [w_{i3}^{(1)}]^2/2r_{ii}^{(1)} > 0, \tag{8}$$

where $r_{ij}^{(1)}$ and $w_{ij}^{(1)}$ are the interaction parameters after the additional rotation $[t_{ij}^{(1)}(\alpha)]$ which diagonalize the Hamiltonian (6) in the plane (1.2).

In the following sections we will consider the XY-like and the Ising-like critical points of the anisotropic ferromagnets in the external field directed: along [100], in any direction in the plane {100}, outside the plane {100}. It is easy to see from (3) that the vector $\boldsymbol{M}$ is directed in these directions as well, although except for the first case the vector $\boldsymbol{M}$ is not parallel to the vector $\boldsymbol{h}$.

The complete analysis will be presented only for the case when

$$u_{ii}^0 = u^0+v^0 \quad \text{and} \quad u_{ij} = u^0 \quad \text{for} \quad i \neq j, \tag{9}$$

but it is easy to see that for all the ferromagnets mentioned above the relations (9) are satisfied. However, by using the same general formalism we could find the critical points in any case (it means for a ferromagnet of an arbitrary given symmetry). In the cases of the systems for which (9) is not fulfilled we will restrict ourselfs to some general remarks.

2. *XY-like critical points*

In this section we will check conditions (7) for various directions of the external field.

A. The external field along one of the directions ⟨100⟩

As mentioned above in this case the vector $\boldsymbol{M}$ is parallel to $\boldsymbol{h}$. Without loss of generality we can assume that

$$M_1 = M_2 = 0 \quad \text{i.e.} \quad \cos\theta = \cos\varphi = 0, \tag{10}$$

and the value of M is given by the relation

$$h = M(r_3^0+4u_{33}^0M^2). \tag{11}$$

a) Condition (7.II). It is easy to see that if the relations (10) are satisfied the condition (7.II) may be written in the form

$$r_{23} = \sin\psi\cos\psi[r_2^0 - r_1^0 + 4M^2(u_{23}^0 - u_{13}^0)] = 0, \tag{12}$$

(the remaining equalities of (7.II) become identities) thus, the condition (7.II) resolves itself into

$$r_2^0 - r_1^0 + 4M^2(u_{23}^0 - u_{13}^0) = 0, \tag{13}$$

or

$$\psi = \tfrac{1}{2}k\pi, \qquad k = 0, 1, 2, \ldots. \tag{14}$$

b) Condition (7.III). In the case under consideration the equalities (7.III) are always fulfilled.

c) Condition (7.I). For $M_1 = M_2 = 0$ the effective temperature variables have the form

$$\begin{aligned} r_{11} &= r_3^0 + 12M^2 u_{33}^0, \\ r_{22} &= r_1^0\cos^2\psi + r_2^0\sin^2\psi + 4M^2(u_{13}^0\cos^2\psi + u_{23}^0\sin^2\psi), \\ r_{33} &= r_1^0\sin^2\psi + r_2^0\cos^2\psi + 4M^2(u_{13}^0\sin^2\psi + u_{23}^0\cos^2\psi). \end{aligned} \tag{15}$$

Thus, the first part of the condition (7.I) i.e., $r_{22} = r_{33}$, yield

$$(\cos^2\psi - \sin^2\psi)\,[r_2^0 - r_1^0 + 4M^2(u_{23}^0 - u_{13}^0)] = 0. \tag{16}$$

It is easy to see, comparing (16) with (13) and (14) that only (16) and (13) can be fulfilled simultaneously. It means that in our further considerations we can omit the possibility (14).

The condition (13) is satisfied for an arbitrary value of M if

$$r_1^0 = r_2^0 \quad \text{and} \quad u_{13}^0 = u_{23}^0. \tag{17}$$

The above relations resolve equation (16) into

$$r_{22} = r_{33} = r_1^0 + 4M^2 u_{23}^0 \tag{18}$$

If the relations (17) are not satisfied the condition (13) is real only for $M = M^*$, where

$$(M^*)^2 = \frac{r_1^0 - r_2^0}{4(u_{23}^0 - u_{13}^0)}. \tag{19}$$

The second part of the condition (7.I), i.e., the inequality $r_{22} < r_{11}$, when related to (13), leads to

$$r_3^0 - r_1^0 + 4M^2(3u_{33}^0 - u_{13}^0) > 0. \tag{20}$$

Before checking the remaining conditions (7.IV) and (7.V) one can note that in order to exist the XY-like critical point, the inequality (20) and one of the conditions (17) or (19) must be satisfied.

d) Condition (7.IV). It is not hard to show that

$$\tilde{v}_{23}, \tilde{v}_{32} \sim \sin\psi\cos\psi. \tag{21}$$

In the particular case of the symmetry given by the relations (9)

$$\tilde{v}_{23} = \tilde{v}_{32} = \sin\psi\cos\psi(\cos^2\psi - \sin^2\psi)v^0. \tag{22}$$

Thus, the condition (7.IV) is always fulfilled for $\psi = k\frac{\pi}{2}$. It is clear that the other possibilities exist as well, and so for the symmetry (9) if $\psi = (2k+1)\frac{\pi}{2}$ or for arbitrary ψ if $v^0 = 0$ the condition (7.IV) is fulfilled too.

e) Condition (7.V). Consider the left-hand side of the inequality (7.V), which in our case $\left(\text{for } \psi = k\frac{\pi}{2}\right)$ takes the form

$$u_{12}^0 - \frac{8M^2 u_{13}^0 u_{23}^0}{r_3^0 + 12M^2 u_{33}^0} > 0. \tag{23}$$

Upon inserting (17) into (23) we obtain

$$u_{12}^0 - \frac{8M^2 (u_{13}^0)^2}{r_3^0 - r_1^0 + 4M^2(3u_{33}^0 - u_{13}^0)} > 0. \tag{24}$$

When the conditions (9) are satisfied the inequality (24) reduces itself to the form

$$r_3^0 - r_1^0 + 12M^2 v^0 > 0. \tag{25}$$

For $v^0 = 0$ this inequality is satisfied only if $r_3^0 > r_1^0$ (i.e. in the uniaxial case the external field must be parallel to the hard axis). On the other hand if $r_3^0 = r_2^0 = r_1^0$ inequality (25) is satisfied only for $v^0 > 0$ (cubic four-axial ferromagnet).

The remaining possibilities of the fulfilment of the condition (7.V), for $\psi \neq k\frac{\pi}{2}$ lead to the same results as in the case considered above. Similarly, the right part of the inequality (7.V) does not lead to any new conclusions.

Let us now discuss the case when $M = M^*$ (19) (i.e. $r_1^0 \neq r_2^0$ and $u_{13}^0 \neq u_{23}^0$). In this instance, there is the interesting possibility for the existence of the *XY*-like critical point for the concrete value of the field $h = h^*$ given by the formulas (11) and (9), if the conditions (19), (20) and (23) are fulfilled simultaneously. It is easy to see that for $h \neq h^*$ the condition $r_{33} = r_{22}$ is not fulfilled and at the point $h = h^*$ the expression $(r_{33} - r_{22})$ changes sign. Thus, in this case bicritical behaviour can occur.

In summary of this part we find that if the external magnetic fields is directed along the [100] direction:

1. For the symmetry given by relations (9)

1.1 the *XY*-like critical point exists in the following ferromagnets:

1.1.1 uniaxial if the field is directed perpendicular to the easy plane,

1.1.2 cubic ($r_1^0 = r_2^0 = r_3^0$) in the four-axial case ($v^0 > 0$),

1.1.3 tetragonal ($r_i^0 > r_j^0 = r_k^0, v^0 > 0$) if the field is directed along the hard axis ("i"),
1.1.4 tetragonal ($r_i^0 < r_j^0 = r_k^0, v^0 > 0$) in the field along the "i" axis for $h > h^*$, where h^* is given by the formula (11) and $(M^*)^2 = (r_i^0 - r_j^0)/12v^0$,
1.1.5 tetragonal ($r_i^0 > r_j^0 = r_k^0, v^0 < 0$) in the field directed along the "i" axis for $h < h^*$;
1.2 there is no XY-like critical point in:
1.2.1 isotropic ferromagnets,
1.2.2 orthorhombic ferromagnets ($r_1^0 \neq r_2^0 \neq r_3^0$),
1.2.3 cubic three-axial ferromagnets ($v^0 < 0$).

It is easy to see that there is no bicritical point if the relations (9) are fulfilled.

2. For arbitrary symmetry, there is the XY-like critical point only if both axes perpendicular to the field direction ("i" direction) are equivalent ($r_j^0 = r_k^0, u_{ij}^0 = u_{ik}^0$). In the other cases the solution of the C_2 class can exist only for a given value of M which corresponds to the bicritical behavior.

B. External field parallel to the one of the planes {100}.

The analysis of the conditions (7), in this case, is much the same as that carried out above. The following results are obtained.

1. For the symmetry given by the relations (9) there is the XY-like critical point if

$$r_i^0 < r_j^0 = r_k^0, \quad v^0 < -\frac{u^0}{6} < 0 \tag{26}$$

for $\boldsymbol{h}$ parallel to the diagonal of the face (j, k) at the point

$$h = h^* = \left|\frac{r_j^0 - r_i^0}{v^0}\right|^{1/2} \left[r_1^0 + \frac{2u^0 + v^0}{3v^0}(r_j^0 - r_i^0)\right]; \tag{27}$$

2. There is no possibility of checking conditions (7) in the general case but it is easy to see that the C_2 points can exist for certain relations between the interaction parameters and the components of the external field. It is clear that the complete analysis can be carried out for any given symmetry of the Hamiltonian.

C. External field non-parallel to the planes {100}

Examination of the conditions (7) demands that calculations similar to those in part A be performed. It is not hard to show that in this case there are no solutions of the C_2 class for the uniaxial, orthorhombic, cubic and tetragonal symmetry. For the other symmetries the XY-like critical point can exist for a certain relation between the interaction parameters and the components of the external field.

3. *Ising-like critical points*

Similarly, as in the previous section we will check the conditions (8) for the existence of the Ising-like critical point. First of all, one can note that the conditions $r_{13}^{(1)} = r_{13}^{(1)} = 0$ may be written in the form

$$A \sin\psi \cos\psi + B(\cos^2\psi - \sin^2\psi) = 0, \tag{28}$$

$$C \sin\psi + D \cos\psi = 0, \tag{29}$$

and the condition $w_{33} = 0$ amounts to

$$a \cos^3 \psi + b \cos^2 \psi \sin \psi + c \cos \psi \sin^2 \psi + d \sin^3 \psi = 0, \tag{30}$$

where A, B, C, D, a, b, c, d as functions of the interaction parameters, external field (through M) and angles φ and θ are given in the Appendix.

A. External field along one of the $\langle 100 \rangle$ directions.

Without loss of generality we can consider the conditions for the existence of the C_1 solutions for the field directed, say, along the [001] direction. In this case the coefficients in (28) nad (30), when related to (10) and (11) take the form:

$$A = r_2^0 - r_1^0 + 4M^2(u_{23}^0 - u_{13}^0), \quad B = C = D = a = b = c = d = 0. \tag{31}$$

a) Condition (8.II). It is easy to see that this condition resolves itself to the equalities

$$\sin \psi \cos \psi [r_2^0 - r_1^0 + 4M^2(u_{23}^0 - u_{13}^0)] = 0, \tag{32}$$

and

$$F \sin \alpha \cos \alpha = 0, \tag{33}$$

where

$$F = r_3^0 - r_1^0 \cos^2 \psi - r_2^0 \sin^2 \psi + 4M^2(3u_{33}^0 - u_{13}^0 \cos^2 \psi - u_{23}^0 \sin^2 \psi). \tag{34}$$

The first equation is fulfilled if

$$\psi = k\pi/2, \tag{35}$$

or

$$r_2^0 - r_1^0 + 4M^2(u_{23}^0 - u_{13}^0) = 0, \tag{36}$$

whereas the second one if

$$\alpha = k\pi/2, \tag{37}$$

or

$$F = 0. \tag{38}$$

b) Condition (8.III). In the case under consideration this condition is always fulfilled.

Thus, considering the remaining conditions (8.I) and (8.IV) we ought to take into account the four following possibilities of the fulfilment of the condition (8.II): (*i*) (35) and (37), (*ii*) (35) and (38), (*iii*) (36) and (37), (*iv*) (36) and (38).

c) Condition (8.I). Let us check this condition in the four cases mentioned above. (*i*) After setting (35) and (37) into (8.I) we obtain

$$\begin{aligned} r_1^0 + 4M^2 u_{13}^0 &< r_3^0 + 12M^2 u_{33}^0, \\ r_1^0 + 4M^2 u_{13}^0 &< r_2^0 + 4M^2 u_{23}^0, \end{aligned} \tag{39}$$

for odd k and the analogous system of the inequalities with interchanged indices $1 \leftrightarrow 2$ for even k.

(*ii*) In this case (38) takes the form

$$r_3^0+12M^2u_{33}^0 = r_2^0+4M^2u_{23}^0 \tag{40}$$

for odd k ($1 \leftrightarrow 2$ for even k). Comparing (40) to (39) one can note that in this case both inequalities (39) are equivalent.

It is easy to see that the two remaining possibilities (*iii*) and (*iv*) are inconsistent with the inequalities (8.I). Thus, we conclude that in order to take into account all cases for which the conditions (8.I)–(8.III) are fulfilled one only needs to consider the conditions (35), (37) and (39).

For the symmetry given by (9) the condition (39) reduces itself into

$$r_3^0-r_1^0+4M^2(2u^0+3v^0) > 0, \quad r_1^0 < r_2^0, \tag{41a}$$

for odd k, and into

$$r_3^0-r_2^0+4M^2(2u^0+3v^0) > 0, \quad r_1^0 > r_2^0 \tag{41b}$$

for even k. In the first case (odd k) σ^1 is the critical variable whereas in the second case (even k) σ^2.

d) Condition (8.IV). If $\psi = k\pi/2$ and $\alpha = m\pi/2$ the condition (8.IV) reduces itself to

$$u_{ii}^0-\frac{8M^2(u_{i3}^0)^2}{r_3^0+12M^2u_{33}^0} > 0, \tag{42}$$

where $i = 1, 2$ for odd and even k, respectively. It is clear that without loss of generality we can consider only one of these cases, say odd k.

Since in the vicinity of the critical point $r_{33}^c \approx 0$ the condition (42) may be written in the form

$$u_{11}^0-\frac{8M^2(u_{13}^0)^2}{r_3^0+r_1^0+4M^2(3u_{33}^0-u_{13}^0)} > 0, \tag{43}$$

and for the symmetry given by (9)

$$u^0+v^0-\frac{8M^2(u^0)^2}{r_3^0-r_1^0+4M^2(2u^0+3v^0)} > 0. \tag{44}$$

We now consider some special cases:

(*i*) $v^0 = 0$, as an immediate consequence of (41) and (44) we have, in this case (for $u^0 > 0$)

$$r_1^1 < r_2^0, \quad r_1^0 < r_3^0, \tag{45a}$$

or

$$r_2^0 < r_1^0, \quad r_2^0 < r_3^0. \tag{45b}$$

This corresponds to the cases: uniaxial in the field perpendicular to the easy axis and orthorhombic in the field directed along one of the non-easy axes.

(*ii*) $r_1^0 = r_2^0 = r_3^0$, $v^0 \neq 0$, cubic ferromagnet. It is easy to see that the second inequality of (41) is not fulfilled in this case. This means that there is no solution of the C_1 class in a cubic ferromagnet in the field along an edge of a cube. In the previous section the solution of the C_2 class was found for such symmetry if $v^0 > 0$ (four-axial ferromagnet), whereas for $v^0 < 0$ (three-axial) there is no critical point.

(*iii*) $r_i^0 = r_j^0 \neq r_k^0$, $v^0 \neq 0$, tetragonal ferromagnet. There are two solutions for such symmetry; (41a) and (41b). As was mentioned above we can choose one of them, say, (41a) i.e.,

$$r_1^0 < r_2^0.$$

There are still two possibilities of realizing the tetragonal symmetry. First if

$$r_3^0 = r_1^0 < r_2^0, \quad 2u^0+3v^0 > 0, \tag{46}$$

and then from (44) we obtain

$$u^0+v^0-\frac{2(u^0)^2}{2u^0+3v^0} > 0. \tag{47}$$

Thus, the second of the inequalities (46) resolves itself into

$$v^0(5u^0+3v^0) > 0. \tag{48}$$

Since $u^0 > 0$, we get from (46) that $(5u^0+3v^0) > 0$, which gives on the basis (48) that

$$v^0 > 0. \tag{49}$$

In the second case if

$$r_3^0 = r_2^0 > r_1^0$$

one can note that (41) and (44) are fulfilled for $v^0 > 0$ or $v^0 < 0$ and $M < M_c$, where

$$M_c = \frac{(r_3^0-r_1^0)\,(u^0+v^0)}{4|v^0|\,(5u^0+3v^0)}\,. \tag{50}$$

Concluding, for the tetragonal symmetry the solution of the C_1 class exists if

1. $r_i^0 = r_j^0 < r_k^0$, $v^0 > 0$; σ_i is a critical variable,

or if

2. $r_j^0 < r_i^0 = r_k^0$, for $v^0 > 0$ or $v^0 < 0$ and $M < M_c$.

It is easy to see that there is no C_1 solution if $r_j^0 = r_k^0$, thus, on the basis of the previous section we come to the conclusion that for $r_j^0 = r_k^0$ the tricritical point can occur at $h = h_c$.

Similarly as above, one can check the conditions (8) for the field of an arbitrary direction. Because the complete analysis of this case demands rather complicated but obvious calculations, we do not show them here. However, in the last section we will present all main results for any direction of the field.

4. *Conclusions*

By using the conditions derived in our previous paper [1], for the existence of the critical points in the anisotropic ferromagnets of an arbitrary symmetry in the external magnetic field, some concrete cases were considered. Since the maximum number of the critical variables, in the case under consideration (three component vector model with field) is two, we took into account only the *XY*-like and Ising-like critical behavior. For convenience, we considered these two types of critical behavior separately.

Carrying out the systematic analysis of the uniaxial, orthorhombic, cubic and tetragonal ferromagnets in the field of the arbitrary direction besides previously known, some new results were obtained. Furthermore some general conclusions for ferromagnets of

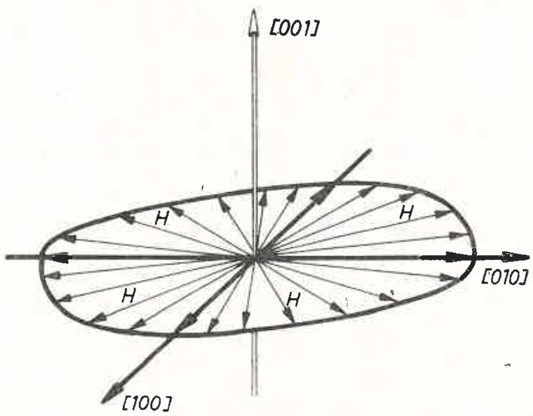

Fig. 1. The directions in which the directed external magnetic field does not destroy the continuous phase transition in the uniaxial and orthorhombic ferromagnets with easy axis along [001]. Bold curve represents critical lines of Ising character

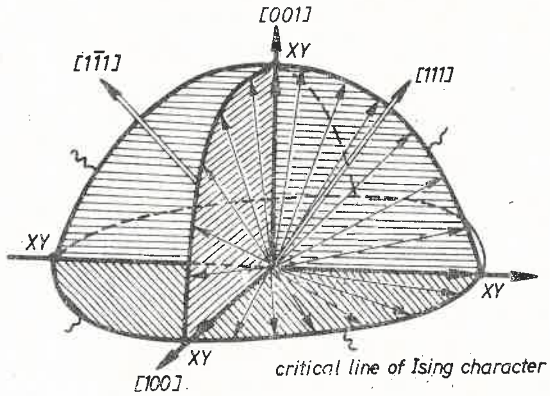

Fig. 2. The ferromagnet with easy plane (001). If the field is directed along a hard direction [001] the critical point should be of *XY* character, whereas, if the field is directed in the other directions — Ising character. There is no critical point for the field parallel to the easy plane

arbitrary symmetry was formulated. The directions in which the directed external magnetic field allows the critical points to occur in the uniaxial, orthorhombic and cubic (four- and three-axial ferromagnets) are shown in Figs. 1–4.

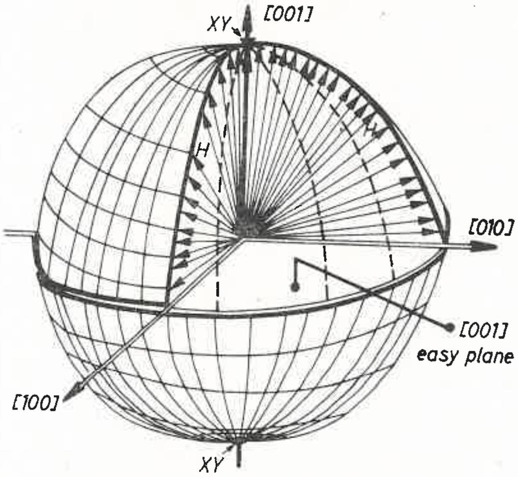

Fig. 3. The cubic four-axial ferromagnet. If the field is along the axis [100] the critical point should be of XY character in the other cases of Ising character

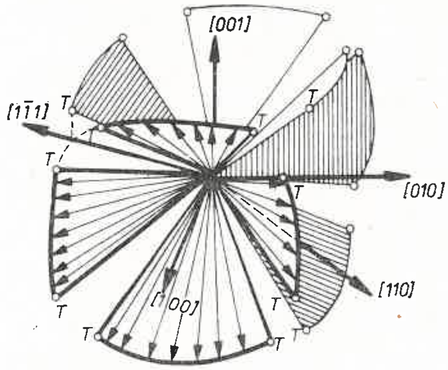

Fig. 4. The cubic three-axial ferromagnet. All critical points should be of Ising character. The change from the continuous to first order phase transition occurs for the field directed along the OT direction (tricritical point) [6]

The detailed results of this paper are presented below.

A. The external magnetic field is along one of the directions $\langle 100 \rangle$.

1. The XY-like critical points exist;

1.1. in the systems described by the Hamiltonian which is invariant in respect to interchange of the critical variables $\sigma^i \leftrightarrow \sigma^j$,

1.2. if (1.1) is not fulfilled the XY-like critical point can exist only for a certain value of the field, depending on the interaction parameters,

1.3. the discussion carried out for some concrete ferromagnets confirms the results known before that the XY-like critical points can occur in

(i) uniaxial ferromagnet in the field perpendicular to the easy plane [2, 7],
(ii) cubic, four-axial ferromagnet ($v^0 > 0$), [5],
(iii) tetragonal ferromagnet in the field along the hard axis (e.g., if $v^0 > 0$, $r_1^0 > r_2^0 = r_3^0$ the field must be directed along [100] direction).
1.4. besides it was shown that:
(i) XY-like critical points can also occur in the tetragonal ferromagnets for $v^0 > 0$, $r_1^0 < r_2^0 = r_3^0$ if $M > M_c$, or $v^0 < 0$, $r_1^0 > r_2^0 = r_3^0$ if $M < M_c$ where

$$M_c = |r_1^0 - r_2^0|/12|v^0|,$$

(ii) there are no XY-like critical points in the orthorhombic ferromagnets,
(iii) there are no bi- and tetracritical points for the systems described by the interaction parameters which fulfilled the following relations $u_{ii}^0 = u^0 + v^0$ and $u_{ij}^0 = u^0$ for $i \neq j$, in the field under consideration ($\boldsymbol{h} \parallel [100]$).
2. The Ising-like critical points
2.1. According to the previous papers such points occur in:
(i) uniaxial ferromagnet when the field is perpendicular to the easy axis [2],
(ii) orthorhombic ferromagnet in the field perpendicular to the easy axis [3].
2.2. Moreover, it was shown that the Ising-like critical points can occur in the tetragonal ferromagnet if
(i) $v^0 > 0$, $r_1^0 = r_2^0 < r_3^0$ or $r_1^0 = r_3^0 < r_2^0$,

or

(ii) $v^0 > 0$, $r_3^0 < r_1^0 = r_2^0$ or $r_2^0 < r_1^0 = r_3^0$,

or

(iii) $v^0 < 0$, $r_1^0 = r_2^0 > r_3^0$ or $r_1^0 = r_3^0 > r_2^0$ for $M < M_c$,

where

$$M_c = \frac{(u^0 + v^0)(r_1^0 - r_j^0)}{4|v^0|(3v^0 + 5u^0)}, \quad j = 2, 3,$$

for the field directed along the [100] direction in all cases ((i)–(iii)).
2.3. Comparing the results which were presented in the points 2.2 and 1.4 we can conclude that the classical tricritical points can occur in the tetragonal ferromagnets, for the field directed, say, along the [100] direction if

(i) $v^0 > 0$, $r_1^0 < r_2^0 = r_3^0$ and $M = (r_2^0 - r_1^0)/12v^0$

or

(ii) $v^0 < 0$, $r_1^0 > r_2^0 = r_3^0$ and $M = (r_2^0 - r_1^0)/12v^0$

or

(iii) $v^0 < 0$, $r_1^0 = r_2^0 > r_3^0$ and $M = \dfrac{(r_1^0 - r_2^0)(u^0 + v^0)}{4v^0(3v^0 + 5u^0)}$.

3. According to the previous papers we obtained that there are no continuous phase transitions in the uniaxial ferromagnet in the field parallel to the easy axis and in the cubic three-axial ferromagnet when the field is along one of the edges of the cube.

B. External magnetic field parallel to the plane {100} apart from axes ⟨100⟩.

1. It was shown that the *XY*-like critical points for the symmetry given by the relations (9) occur only if

$$r_3^0 < r_1^0 = r_2^0, \quad v_0 < -\frac{u^0}{6} < 0,$$

for the field directed along the direction ⟨011⟩, at the point $M = M^*$, where

$$(M^*)^2 = \frac{r_1^0 - r_3^0}{6|v^0|}.$$

It is clear that this point corresponds to the bicritical behavior.

2. The Ising-like critical points.

2.1. According to the previous papers such points can occur in:

(*i*) uniaxial [2] and orthorhombic [3] ferromagnets when the field is directed in any direction in the plane perpendicular to the easy axis,

(*ii*) cubic, four-axial ferromagnet ($v^0 > 0$) in the field parallel to any direction in the plane {100}, [5],

(*iii*) cubic, three-axial ferromagnet ($v^0 < 0$) in the field parallel to the diagonal of the face (⟨100⟩ directions) [5].

2.2. Moreover it was shown that

(*i*) the Ising-like critical points exist in the uniaxial ferromagnets with easy plane if the field is directed in any direction neither parallel nor perpendicular to this plane,

(*ii*) for the tetragonal ferromagnets in the field parallel to the diagonal of the face (e.g., $\boldsymbol{h} \,||\, [011]$) if $r_3^0 < r_1^0 = r_2^0$ and $v^0 < 0$ two distinct solutions corresponding to two distinct critical directions exist. One of them is stable for $M < M^*$ whereas the other for $M > M^*$, where

$$(M^*)^2 = \frac{r_1^0 - r_3^0}{6|v^0|}.$$

In that case there is the bicritical point (*XY*-like) at $M = M^*$.

C. External field non-parallel to any of the planes {100}.

1. It was shown that there are no *XY*-like critical points in the uniaxial, orthorhombic, cubic and tetragonal ferromagnets.

2. The Ising-like critical points.

2.1. There are such points in the uniaxial ferromagnets with easy plane when the field is directed in any direction neither parallel nor perpendicular to the easy plane.

2.2. There are no critical points in cubic, four-axial ferromagnets ($v^0 > 0$) when the field is directed in any direction non-parallel to the {100} planes.

2.3. According to the papers [6], in the cubic three-axial ferromagnets ($v^0 < 0$) the Ising-like critical point can occur only if the field is directed along some directions parallel to the plane {110} type (see Fig. 4). There is no continuous phase transitions in such a system if the field is directed along the [111] direction and in some of its vicinities.

Concluding, by using the general formalism all critical points which can occur in the uniaxial, orthorhombic, cubic and tetragonal ferromagnets were found. By systematizing the results known previously this formalism allows us to find some new ones, likewise draw some conclusions for the general case. It is clear that a similar analysis as was presented for the mentioned above ferromagnets can be carried out for any symmetry although sometimes it needs some effort and, most of all patience.

APPENDIX

The coefficients of equations (28) and (30).

$$A = -\tilde{r}_{11}\sin^2\varphi+\tilde{r}_{22}(\sin^2\theta-\cos^2\theta\cos^2\varphi)+\tilde{r}_{33}(\cos^2\theta-\sin^2\theta\cos^2\varphi)$$
$$+2\tilde{r}_{12}\cos\theta\cos\varphi\sin\varphi+2\tilde{r}_{13}\sin\theta\cos\varphi\sin\varphi-2\tilde{r}_{23}\cos\theta\sin\theta(1+\cos^2\varphi);$$
$$B = -\tilde{r}_{22}\cos\theta\sin\theta\cos\varphi+\tilde{r}_{33}\cos\theta\sin\theta\cos\varphi+\tilde{r}_{12}\sin\theta\sin\varphi$$
$$-\tilde{r}_{13}\cos\theta\sin\varphi+\tilde{r}_{23}\cos\varphi(\cos^2\theta-\sin^2\theta);$$
$$C = \tilde{r}_{11}\cos\varphi\sin\varphi-\tilde{r}_{22}\cos^2\theta\cos\varphi\sin\varphi-\tilde{r}_{33}\sin^2\theta\cos\varphi\sin\varphi$$
$$+\tilde{r}_{12}\cos\theta(\sin^2\varphi-\cos^2\varphi)+\tilde{r}_{13}\sin\theta(\sin^2\varphi-\cos^2\varphi)-2\tilde{r}_{23}\cos\theta\sin\theta\cos\varphi\sin\varphi;$$
$$D = -\tilde{r}_{22}\cos\theta\sin\theta\sin\varphi+\tilde{r}_{33}\cos\theta\sin\theta\sin\varphi-\tilde{r}_{12}\cos\varphi\sin\theta$$
$$+\tilde{r}_{13}\cos\theta\cos\varphi+\tilde{r}_{23}\sin\varphi(\cos^2\theta-\sin^2\theta);$$
$$a = M\sin\varphi\cos\theta\sin\theta(u^0_{22}\sin^2\theta+u^0_{33}\cos^2\theta+u^0_{23});$$
$$b = M\cos\varphi\sin\varphi\{u^0_{12}\sin^2\theta+u^0_{13}\cos^2\theta-3(u^0_{22}+u^0_{33})\cos^2\theta\sin^2\theta$$
$$+u^0_{23}(6\cos^2\theta\sin^2\theta-1)\};$$
$$c = M\cos\theta\sin\theta\sin\varphi\{u^0_{12}(3\cos^2\varphi-1)-u^0_{13}(3\cos^2\varphi-1)-3u^0_{22}\cos^2\theta\cos^2\varphi$$
$$+3u^0_{33}\sin^2\theta\cos^2\varphi+3u^0_{23}\cos^2\varphi(\cos^2\theta-\sin^2\theta)\};$$
$$d = M\cos\varphi\sin\varphi\{u^0_{11}\sin^2\varphi-u^0_{22}\cos^4\theta\cos^2\varphi-u^0_{33}\sin^4\theta\cos^2\varphi$$
$$+u^0_{12}\cos^2\theta(\cos^2\varphi-\sin^2\varphi)+u^0_{13}\sin^2\theta(\cos^2\varphi-\sin^2\varphi)-2u^0_{23}\cos^2\theta\sin^2\theta\cos^2\varphi\}.$$

REFERENCES

[1] G. Ritter, J. Sznajd, *Acta Phys. Pol.* **A57**, 819 (1980).
[2] H. Thomas, *Phys. Rev.* **187**, 630 (1969).
[3] K. Durczewski, *Acta Phys. Pol.* **A40**, 687 (1971).
[4] E. Domany, M. E. Fisher, *Phys. Rev.* **B15**, 3510 (1977).
[5] J. Sznajd, *Acta Phys. Pol.* **A47**, 61 (1975).
[6] D. Mukamel, M. E. Fisher, E. Domany, *Phys. Rev. Lett.* **37**, 565 (1976).
[7] J. Sznajd, *Acta Phys. Pol.* **A51**, 145 (1977).