LINE SHAPE OF THE QUASI-LOCAL LEVEL*

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A line shape of the quasi-local level induced at the impurity by a quantizing magnetic field, in a weakly doped crystal, is studied.

In papers [1, 2] the influence of impurities on the electron energy spectrum of a semimetal in a quantizing magnetic field was considered. Within the framework of a simple model for a single-band system with a parabolic dispersion law for electrons, by means of the local-perturbation method and within a linear approximation with respect to the impurity concentration (n_i) it was shown that a magnetic field induces the local and quasi-local levels at the impurities. Positions of those levels are determined by the following expressions[1]:

$$E_{\mathrm{loc}} = \tfrac{1}{2}\hbar\omega - \varepsilon, \quad E_{qN} = (N+\tfrac{1}{2})\hbar\omega - \varepsilon,$$

$$\varepsilon \cong \frac{4\pi\hbar\omega}{L}\left(\frac{ma^2u_0}{\hbar^2}\right)^2, \quad L = \tfrac{1}{2}(l/a)^2, \quad N = 1, 2, \ldots . \tag{1}$$

Above, the electron spin has been neglected for simplicity, and the notation is: ω — the cyclotron frequency, m – the electron effective mass, u_0 — depth of the potential well associated with the impurity, a — range of action of the potential, $l = \sqrt{\hbar/m\omega}$ — the cyclotron radius, N — the magnetic quantum number.

In Ref. [1] the following expression determining the electron density of states was found

$$\varrho(E) = \varrho_H(E) + \Delta\varrho(E), \tag{2}$$

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[1] For the case when at the potential well there are no bound states, for $H = 0$, i.e. $4ma^2|u_0| < \hbar^2$.

where $\varrho_H(E)$ is the electron density of states of a perfect crystal in the magnetic field and

$$\Delta\varrho(E) = \frac{2N_i}{\pi}\frac{\Gamma(E)+\pi g'(E)A(E)/f'(E)}{\Gamma^2(E)+A^2(E)} \tag{3}$$

is the contribution to the density of states due to N_i impurities. The functions $A(E)$ and $\Gamma(E)$ which occur in (3) are of the form $\Gamma(E) = -\pi g(E)/f'(E)$; $A(E) = -[1-u_0 f(E)]/u_0 f'(E)$. A prime sign over the functions $g(E)$ and $f(E)$ denotes their derivatives with respect to E, and $f(E)$ is a real part of the Green function with the imaginary part given by

$$g(E) \cong \frac{2\sqrt{2}}{\sqrt{\pi\hbar\omega}}(a/l)^3 \sum_{N=0}^{N_m-1} \frac{1}{\sqrt{E-(N+1/2)\hbar\omega}}, \qquad N_m = \text{Entier}\left(\frac{E}{\hbar\omega}-\tfrac{1}{2}\right). \tag{4}$$

It was assumed in [1] that peaks of the density of states, due to impurities and described by the expression (3), had a Lorentzian shape. In the present paper we investigate more accurately a behaviour of the electron density of states in the vicinity of the quasi-local levels.

Taking into account that

$$\varrho_H(E) = \frac{m^{3/2}\omega V}{\sqrt{2}\,\hbar^2\pi^2} \sum_{N=0}^{N_m-1} \frac{1}{\sqrt{E-(N+1/2)\hbar\omega}}, \tag{5}$$

(with V being a volume of the system) we rewrite the formula (4) in the form

$$g(E) \cong \tfrac{1}{2}\left(\frac{2\pi\hbar}{m\omega L}\right)^{3/2} \varrho_H(E). \tag{6}$$

The function $g(E)$ has no singularities in the region of the energy electron spectrum of our interest, therefore below the Landau levels we can put in (6)

$$\varrho_H(E) \to \varrho_{H=0}(E) = \varrho_0(E) = \frac{V}{2\pi^2}(2m/\hbar^2)^{3/2}\sqrt{E} \tag{7}$$

and get

$$g(E) \cong \frac{2}{\sqrt{\pi}}(1/\hbar\omega L)^{3/2}\sqrt{E}\,. \tag{8}$$

In the regions of the energy spectrum we are interest in, a main contribution to the real part of the Green function $f(E)$ is given by the following formula (see Ref. [1] for details)

$$f(E) \cong -\frac{\sqrt{\pi}}{\sqrt{\hbar\omega}\,L^{3/2}}\frac{1}{\sqrt{F_N(E)}}, \qquad F_N(E) = (N+1/2)\hbar\omega - E. \tag{9}$$

Now taking into account (8) and (9) we get (see also Fig. 1a)

$$A(E) = 2F_N(E)\left[1-(F_N(E)/\varepsilon)^{1/2}\right], \tag{10}$$

$$\Gamma(E) = \frac{4\sqrt{E}}{\hbar\omega} F_N^{3/2}(E), \tag{11}$$

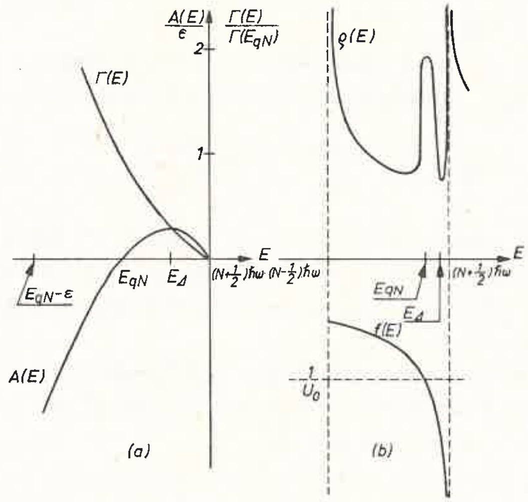

Fig. 1. (a) – Behaviour of $\Gamma(E)$ and $A(E)$ near the bottom of the $(N+1)$-th Landau level (b) – $\varrho(E)$ and $f(E)$ vs E for the N-th Landau level and the graphical solutions of the equation $1\text{-}u_0 f(E) = 0$ (schematically)

By substituting (10), (11) and the derivatives of the functions $g(E)$ and $f(E)$ into (3) we find

$$\Delta\varrho(E) = \frac{2N_i}{\pi\sqrt{EF_N(E)}} \frac{E-\left[1-(F_N(E)/\varepsilon)^{1/2}\right]F_N(E)}{4\dfrac{EF_N(E)}{\hbar\omega}+\left[1-(F_N(E)/\varepsilon)^{1/2}\right]^2\hbar\omega}. \tag{12}$$

The formula (12) gives a shape of the function describing a contribution to the electron density of states which are induced by a magnetic field at N_i impurities in a high energy part of the N-th Landau band. Thus, $\Delta\varrho(E)$ takes its maximum for $E = E_{qN}$ (see Fig. 1b) and the minimum for $E = E_\Delta = (N+1/2)\hbar\omega - \frac{4}{9}\varepsilon$, where

$$\Delta\varrho(E_{qN}) \cong \frac{2N_i}{\pi} \frac{1}{\Gamma(E_{qN})}, \tag{13}$$

$$\Delta\varrho(E_\Delta) \cong \frac{27N_i}{\pi}\left(\frac{N}{\varepsilon\hbar\omega}\right)^{1/2}, \quad \text{(for } N \ll L\text{)}. \tag{14}$$

After taking the minimum value for $E = E_\Delta$ the function $\Delta\varrho(E)$ increases and reaches infinity in the points where the Landau levels are located, i.e. $\Delta\varrho(E = (N+1/2)\hbar\omega) = \infty$. The peak of the function $\Delta\varrho(E)$ in the vicinity of $E = E_{qN}$ is asymmetric what is due to a mechanism of its formation — by the separation of the energy levels from a bottom of the Landau band as the magnetic field increases, since $\varepsilon \sim H^2$.

The peaks in the density of states due to impurities will be profound against the background of the density of states of a perfect crystal if

$$\frac{\Delta\varrho(E_{qN})}{\varrho_H(E_{qN})} \cong \frac{\pi n_i l^3}{2\sqrt{2}}\left(\frac{\hbar\omega}{\varepsilon}\right)^{3/2}\left(\frac{\hbar\omega}{\varepsilon_F}\right) \gg 1. \tag{15}$$

where ε_F is the Fermi energy.

From (15) it is easy to find a condition for a concentration of impurities for which the quasi-local levels play a substantial role in physical phenomena. The condition reads

$$n_i > 24\pi^{5/2}\left(\frac{ma^2|u_0|}{\hbar^2}\right)^3\left(\frac{\hbar\omega}{\varepsilon_F}\right)^{1/2}\frac{n_e}{L^{3/2}}. \tag{16}$$

There are two small parameters in (16) : $(\hbar\omega/\varepsilon_F)^{1/2}$ and $L^{-3/2}$, hence for a small electron concentration n_e a small amuont of impurities is required to exhibit the quasi-local levels. The above inequality is satisfied for values of the parameters characterizing semimetals and degenerate semiconductors.

The behaviour of $\varrho(E)$ in the interval of the N-th Landau level as well as the graphical solution of the equation $1-u_0(f(E) = 0$ which determines a position of the quasi-local levels, are depicted schematically on Fig. 1b.

It follows from the above discussion that, to a good approximation, the peak in the electron density of states due to the quasi-local levels can be described by a Lorentzian curve and the remaining part, on the right hand side of E_Δ, can be taken into account as smearing out of the Landau level, caused by impurity-electron scattering. That part is what can be described by the Dingle factor.

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