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THE ORBITAL DYNAMICS IN THE SPIN FLUCTUATION FEEDBACK APPROXIMATION: THE OUT-OF-PHASE MODE IN SUPERFLUID ^{3}He-A

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A theory is proposed, which can permit a description of an out-of-phase mode in ^{3}He-A which results when the spin fluctuation feedback effect is taken into account. A simple, phenomenological theory was developed and a kinetic equation technique was used for microscopic description. Some of the experimental consequences are discussed briefly.

1. Introduction

In their classical paper, Anderson and Morel [1] built the p-wave theory of superfluidity in the fermion systems where pairing between ^{3}He atoms with parallel spins takes place. In such a model, we have two-spin subsystems which act completely independently and all the physical quantities can be described considering only one group of spins. Subsequently, after the discovery of the superfluid phases of ^{3}He it has been found that this model, despite of the fact that it possesses the required symmetry from the NMR point of view, is not stable and must be generalized. This generalization [2] known as the feedback effect (FE), is a particular example of the theory called the strong-coupling (SC) theory, and consists in taking into account correlations between the up and down spin populations. In such a description, the FE is responsible for an existence of the stable ABM phase. The question arises of how important are the FE or SC effects on the nonequilibrium properties of the ABM phase, such as, e.g. sound propagation. Trivial absorption of the SC corrections into renormalized value of the order parameter is not sufficient. Wölfle and Koch [3] claim for the first time to find the SC effects important to nonequilibrium phenomena and to estimate their effect to be about 10–20% of the total sound absorption. For the present paper we define the FE theory as the one in which two coupled $\boldsymbol{l}$ vectors,

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each for one group of spins, are being introduced. In the FE theory the most stable configuration is the one where $\boldsymbol{l}_\uparrow$ and $\boldsymbol{l}_\downarrow$ are parallel [2]. It should be mentioned that both vectors have their own dynamics. We expect that their orbital dynamics is very similar to the usual one and can be described by a slight generalization of the existing theories which involve only one $\boldsymbol{l}$ vector.

In Section 2, we develop a simple phenomenological theory based on the Leggett–Takagi [4] theory of orbital dynamics and in Section 3 we make microscopic calculations, based on the kinetic equation technique, for such situation. Section 4 contains some speculations on the possible experimental consequences.

2. *Simple phenomenological theory*

As we have already stated, the essence of the phenomenological theory consists in introducing two $\boldsymbol{l}$ vectors, each of the two with its own dynamics. The first obvious question is that, what sort of motion one should expect, i.e. do the $\boldsymbol{l}$ vectors have some sort of processional motion against each other, or if one can visualize their dynamics in terms of two coupled oscillators (pendulums). It is readily noticed, that the first possibility takes place only if one takes into account the particle-hole asymmetry, because the $\boldsymbol{l}$ vectors commute in the approximation of the particle-hole symmetry, see: Ref. [4], with the special attention to the formula (4.3), and the remarks that follow. Then, in absence of the static magnetic field we are left with the coupled pendulums model, where feedback effects are responsible for the coupling of those two oscillators. It is well known, that general motion of two coupled oscillators may be considered as the superposition of two normal modes of the oscillation. In the first of the normal modes, two oscillators move in phase with equal amplitudes, whereas in the second normal mode they move in opposition — the out-of-phase motion — again with equal amplitudes.

To make qualitative the above intuitive description, let us generalize the Leggett–Takagi theory [4] of the orbital dynamics. The energy of the whole system can be divided into three parts, that refer to: the spins up, the spins down, and the coupling between them. Therefore, one can write

$$E(\boldsymbol{l}_\uparrow, \boldsymbol{l}_\downarrow) = E(\boldsymbol{l}_\uparrow)+E(\boldsymbol{l}_\downarrow)-\gamma \boldsymbol{l}_\uparrow \cdot \boldsymbol{l}_\downarrow, \tag{2.1}$$

where $E(\boldsymbol{l}_\uparrow)$ contains all the terms depending on the $\boldsymbol{l}_\uparrow$ vector only. This part of energy contains the normal-locking energy, i.e.

$$E_{\mathrm{nl}\uparrow} = -\tfrac{1}{2}\, g_{\mathrm{n}\uparrow}(T)\, (\boldsymbol{l}_\uparrow \cdot \boldsymbol{l}_\mathrm{n})^2,$$

where we neglect the dipolar energy.

In the theory presented here, the coefficient $\gamma(T)$ is of the main importance and the way of how to calculate it forms some difficulties. We used in calculations the free energy expression in general form [5], assuming that the SC values of β_i parameters are still valid for the kind of dynamics discussed here, which should be true for a sufficiently slow motion. Assuming the A-phase to be described by the ABM order parameter and by some straightforward calculations, one gets (see: [6] for details in notation)

$$\gamma = \tfrac{3}{5} N(0) \left(\frac{3.06}{1.42}\right)^2 \left(\frac{\Delta C_v}{C_N}\right)^2 (k_B T_C)^2 t^2 (2\beta_1+\beta_3+\beta_4+\beta_5)$$

$$\equiv \tfrac{3}{5} \gamma_0 (2\beta_1+\beta_3+\beta_4+\beta_5), \tag{2.2}$$

where $t \equiv 1-T/T_C$, and $\Delta C_v/C_N$ is the ratio of the specific heat jump at T_C and the specific heat just above T_C. It is easy to verify that $\gamma = 0$ in the weak coupling approximation, since we do not expect any coupling between the up and down spin populations. For SC values of β_i coefficients [6], we find that

$$\gamma^{SC} = \tfrac{18}{25} \gamma_0 \delta', \tag{2.3}$$

where $\delta' > \frac{1}{4}$.

To construct the theory of the orbital dynamics which takes into account the feedback effects, we assume that both systems are fully characterized by the existence of the $\boldsymbol{l}_\uparrow$ and $\boldsymbol{l}_\downarrow$ vectors. Following Leggett and Takagi [4], let us introduce the $\boldsymbol{K}_\uparrow$ and $\boldsymbol{K}_\downarrow$, which we can visualize as pseudo-angular momenta of the Cooper pairs with the up and down paired spins, respectively. Then, the equations of motion are, α stands for $\uparrow$ and $\downarrow$

$$\dot{\boldsymbol{l}}_\alpha = -\boldsymbol{l}_\alpha \times \boldsymbol{K}_\alpha / \chi_{\mathrm{orb},\alpha},$$

$$\dot{\boldsymbol{K}}_\alpha = -\boldsymbol{l}_\alpha \times \frac{\partial E}{\partial \boldsymbol{l}_\alpha} - \frac{\boldsymbol{K}_\alpha}{\tau_{K,\alpha}}, \qquad \dot{\boldsymbol{l}}_n = -\frac{\boldsymbol{l}_n - \boldsymbol{l}_\alpha}{\tau_\alpha}. \tag{2.4}$$

When writing the last equation we postulate, that both spin subsystems are relaxing towards the same equilibrium configuration with the normal component. Using expression (2.1) for the $\boldsymbol{l}$-dependent part of the energy of the system, together with (2.4), after Fourier transformation and linearization, we arrive at the equation which gives the collective modes of the system, assuming that $\delta \boldsymbol{d} = 0$

$$g^2(\omega) + 2\gamma g(\omega) = 0, \tag{2.5}$$

where we have assumed, that all parameters which describe both systems, are equal, e.g. $\tau_{K\uparrow} = \tau_{K\downarrow} = \tau_K$ etc., and the function $g(\omega)$ is defined as follows:

$$g(\omega) = i\omega \chi_{\mathrm{orb}} \frac{1+i\omega\tau_K}{\tau_K} + g_n \frac{i\omega\tau}{1+i\omega\tau}. \tag{2.6}$$

Let us discuss the consequences of equation (2.5). First, in the weak-coupling limit, i.e. when $\gamma = 0$, we can re-derive all the results of Leggett and Takagi [4]. In the collisionless limit, i.e. $\omega\tau \gg 1$ and $\omega\tau_K \gg 1$, we find two flapping modes. The one with the frequency $\omega^2 = g_n/\chi_{\mathrm{orb}}$ is the usual mode, which has been discovered by Wölfle [7], and it represents in-phase motion. The other mode is [8]

$$\omega^2 = \frac{g_n}{\chi_{\mathrm{orb}}} + 2\frac{\gamma}{\chi_{\mathrm{orb}}} \tag{2.7}$$

and represents out-of-phase motion; the second term in (2.7) is due to the feedback effects.

3. *The kinetic equation approach in zero magnetic field*

In the ABM phase, the equilibrium order parameter is

$$\Delta^{\mathrm{eq}}_{\alpha\beta}(\boldsymbol{n}) = \Delta_0(T)\,(n_x+in_y)\,(\sigma_y\sigma_i)_{\alpha\beta}d_i, \tag{3.1}$$

where we have chosen the $\boldsymbol{l}_{\mathrm{eq}}$ vector along the z axis. If we take the $\boldsymbol{d}$ vector along the y axis, then

$$\Delta^{\mathrm{eq}}_{\alpha\beta}(\boldsymbol{n}) = \Delta_0(T)\,(n_x+in_y)\delta_{\alpha\beta}. \tag{3.2}$$

Both spin populations are then decoupled.

Let us see now, how the order parameter changes when we start with the small vibrations of the $\boldsymbol{l}$ vectors. We can describe such vibrations by performing small rotations with the angle $\delta\theta$, ($\delta\theta \ll 1$) of both imaginary parts of the order parameter (3.2). This gives us

$$\Delta_\uparrow = \Delta_0(T)\,(n_x+in_y+in_z\delta\theta),$$
$$\Delta_\downarrow = \Delta_0(T)\,(n_x+in_y-in_z\delta\theta), \tag{3.3}$$

or in terms of the tensor $d_{\alpha i}$

$$d_{\alpha i} = \Delta_0(T)\begin{pmatrix} 0 & 0 & -i\delta\theta \\ -i & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.4}$$

Small variation of the order parameter $\delta\Delta_{\alpha\beta}(\boldsymbol{n})$ for such rotation is ($\sigma^z_{\alpha\beta}$ — the Pauli matrix)

$$\delta\Delta_{\alpha\beta}(\boldsymbol{n}) = i\Delta_0(T)n_z\sigma^z_{\alpha\beta}\delta\theta. \tag{3.5}$$

Both results, i.e. (3.2) and (3.5), tell us that near the ABM phase the order parameter is diagonal in spin space, although is not proportional to the unity. This makes possible a discussion of the up and down spin populations separately, remembering that they are coupled by the feedback term.

For the description of the FE in the ABM phase, we will use the usual BCS formalism supplemented by the term which preserves the stability of the ABM phase. Such a Hamiltonian reads

$$H = \sum_{k\alpha}\xi_k a^+_{k\alpha}a_{k\alpha}+\tfrac{1}{2}\sum_{kk'}\sum_{\alpha\beta\gamma\delta}(V_{kk'}\delta_{\alpha\delta}\delta_{\beta\gamma}-\chi^{ij}_{kk'}\sigma^i_{\alpha\delta}\sigma^j_{\beta\gamma})a^+_{k\alpha}a^+_{-k\beta}a_{-k'\gamma}a_{k'\delta}, \tag{3.6}$$

where $a^+_{k\alpha}$ and $a_{k\alpha}$ are the creation and the destruction operators of a ^{3}He atom, respectively. Performing Hartree–Fock factorization of the interaction term, we arrive at

$$H = \sum_{k\alpha}\xi_k a^+_{k\alpha}a_{k\alpha}+\tfrac{1}{2}\sum_{k\alpha\beta}\{a^+_{k\alpha}a^+_{-k\beta}\Delta_{\beta\alpha}(\boldsymbol{n})+\mathrm{h.c.}\}, \tag{3.7}$$

where the gap equation has now the form

$$\Delta_{\beta\alpha}(\boldsymbol{n}) = \sum_{k'\gamma\delta}V^{\alpha\beta\gamma\delta}_{kk'}\langle a_{-k'\gamma}a_{k'\delta}\rangle \tag{3.8}$$

and

$$V_{kk'}^{\alpha\beta\gamma\delta} \equiv V_{kk'}\delta_{\alpha\delta}\delta_{\beta\gamma} - \chi_{kk'}^{ij}\sigma_{\alpha\delta}^{i}\sigma_{\beta\gamma}^{j}.$$

The mentioned Hamiltonian, but with the different gap equation, is the usual one which permits a description of the ordinary orbital dynamics [9]. The kinetic equation, which follows directly from the form (3.6), preserves its usual form. At this stage, the main question is that, what is the true susceptibility in (3.6). At the first glance, one might be forced to use the usual total susceptibility. And indeed, in the static case it is a quite successful approach, see: e.g. [10] and [11] to explain stability of the ABM phase. Anyhow, it should be remembered that in the static problem the superfluid and normal components are in equilibrium. In non-equilibrium situations, the choice of total susceptibility might not be the case as it was indicated in [8]. At this stage, the problem has been left to a more microscopic approach and we treat $\chi_{kk'}^{ij}$ as a phenomenological parameter. When we think of $\chi_{kk'}^{ij}$ as referring to the Cooper pairs only, we can use the Leggett–Takagi result [12], which is for the ABM phase

$$\chi_{p0}^{ij} = \chi_{n0}[1-f(T)]\,(\delta_{ij}-d_i d_j) \tag{3.9}$$

and $\chi_{n0} = \frac{1}{4}\gamma^2\hbar^2\left(\frac{dn}{d\varepsilon}\right)$. We ought to emphasize, that χ_{p0}^{ij} has a different temperature dependence near T_C than the total susceptibility.

Our main purpose in this section will be to find the dispersion relation of the out-of--phase mode (2.7) based on the kinetic equation technique. The collective excitations are described by the equation for variation of the order parameter, which comes from (3.8), and near the ABM phase is

$$\delta\Delta_{\alpha\alpha}(\boldsymbol{n}) = \sum_{k'}\{(V_{kk'}-\chi_{kk'}^{zz})\delta n_{k'\alpha\alpha}^{(01)} - (\chi_{kk'}^{xx}-\chi_{kk'}^{yy})\delta n_{k'\bar{\alpha}\bar{\alpha}}^{(01)}\} - \sum_{k'\beta}\theta_{k'}\Delta_{\beta\beta}(\boldsymbol{n}')\delta V_{kk'}^{\alpha\alpha\beta\beta}. \tag{3.10}$$

Equation (3.10) must be supplemented by a kinetic equation, which describes the time evolution of $\delta \boldsymbol{n}_k^{(01)}$ as follows:

$$\omega\delta\tilde{n}_k - \tilde{\varepsilon}_k^0\delta\tilde{n}_k + \delta\tilde{n}_k\tilde{\varepsilon}_k^0 = \delta\tilde{\varepsilon}_k\tilde{n}_k^0 - \tilde{n}_k^0\delta\tilde{\varepsilon}_k. \tag{3.11}$$

Here, the quantities with a tilde denote 4×4 matrices in the Nambu space of particle-hole and spin space. Underscored quantities will denote 2×2 matrices in spin space. The quantities which have appeared in (3.11), are defined as follows:

the equilibrium matrix distribution function

$$\left(\theta_k \equiv \frac{1}{2E_k}\tanh\tfrac{1}{2}\beta E_k\right),$$

$$\tilde{n}_k^0 = \begin{pmatrix} \frac{1}{2}-\xi_k\theta_k & -\theta_k\Delta_k \\ -\theta_k\Delta_k^* & \frac{1}{2}+\xi_k\theta_k \end{pmatrix}, \tag{3.12}$$

the equilibrium energy matrix

$$\tilde{\varepsilon}_k^0 = \begin{pmatrix} \xi_k & \Delta_k \\ \Delta_k^* & -\xi_k \end{pmatrix}, \tag{3.12a}$$

the change in the matrix distribution function

$$\delta \tilde{n}_k = \begin{pmatrix} \delta n_k^{(00)} & \delta n_k^{(01)} \\ \delta n_k^{(10)} & \delta n_k^{(11)} \end{pmatrix}, \tag{3.13}$$

and the change in the quasi-particle energy matrix

$$\delta \tilde{\varepsilon}_k = \begin{pmatrix} 0 & \delta \Delta_k \\ \delta \Delta_k^* & 0 \end{pmatrix}. \tag{3.14}$$

We have neglected Fermi-liquid effects and external fields. Remembering the fact that matrices Δ_k and $\delta\Delta_k$ are diagonal, Eq. (3.11) can be solved easily and in the approximation of particle–hole symmetry, one finds

$$\sum_{|k|} \delta n_k^{(01)} = \varphi(n)\delta\Delta(n) - (\tfrac{1}{2}\omega^2 - |\Delta(n)|^2) f(n, \omega)\delta\Delta(n) + \Delta^2(n) f(n, \omega)\delta\Delta^*(n), \tag{3.15}$$

where we have defined

$$\varphi(n) = \sum_{|k|} \frac{1}{2E_k} \tanh \tfrac{1}{2}\beta E_k, \tag{3.16}$$

$$f(n, \omega) = \sum_{|k|} \frac{1}{E_k(4E_k^2 - \omega^2)} \tanh \tfrac{1}{2}\beta E_k. \tag{3.17}$$

Here and in the following we have made the replacement

$$\sum_k \cdots \to \int \frac{d\Omega}{4\pi} \sum_{|k|} \cdots \equiv \Big\langle \sum_{|k|} \cdots \Big\rangle.$$

Assuming p-wave pairing and $\delta\Delta_\alpha = n_z d_\alpha$, from (3.10) and (3.15), one finds

$$d_\alpha + \sum_{\beta = \alpha, \bar{\alpha}} V^\beta d_\beta \langle n_z^2 \varphi \rangle = \sum_\beta V^\beta \langle n_z^2 f(n, \omega)(\omega^2 - 2|\Delta|^2)\rangle d_\beta, \tag{3.18}$$

where the last term in (3.10) vanishes due to angular integration. In Eq. (3.18) the coefficient V^β is defined as follows:

$$V^\beta = \begin{cases} V - \chi^{zz}, & \beta = \alpha \\ -(\chi^{xx} - \chi^{yy}), & \beta = \bar{\alpha} \end{cases}$$

where V, χ^{xx}, χ^{yy} and χ^{zz} are appropriate p-wave coefficients.

The gap equation now takes the form

$$1 = -\frac{1}{2\Delta_0^2} \sum_\beta V^\beta \langle |\Delta|^2 \varphi \rangle. \tag{3.19}$$

Using Eq. (3.19), the lhs of Eq. (3.18) can be rewritten as follows:

$$\text{LHS} = \tfrac{1}{2}\{V^\alpha d_\alpha(3\langle n_z^2\varphi\rangle - \langle\varphi\rangle) + V^{\bar{\alpha}}[(2d_{\bar{\alpha}} + d_\alpha)\langle n_z^2 \varphi\rangle - d_\alpha\langle\varphi\rangle]\}. \tag{3.20}$$

Performing a partial integration in the angular variable, the first term in Eq. (3.20) takes the form

$$3\langle n_z^2\varphi\rangle-\langle\varphi\rangle = \left\langle n_z^2 \sum_{|k|} \frac{|\Delta_k|^2}{E_k^2}\,\theta_k\right\rangle -2\langle n_z^2 g(\boldsymbol{n})\rangle, \tag{3.21}$$

where

$$g(\boldsymbol{n}) \equiv -\sum_{|k|} \frac{|\Delta_k|^2}{2E_k^2}\frac{dn_k^0}{dE_k}, \tag{3.22}$$

and

$$n_k^0 = (e^{\beta E_k}+1)^{-1}.$$

In the weak-coupling limit, i.e. when both $\boldsymbol{l}$ vectors oscillate together ($\delta\Delta_\uparrow = \delta\Delta_\downarrow$), Eqs. (3.18) and (3.20) are reduced to the Leggett and Takagi result [4]. Using Eq. (3.21), we can explicitly identify the frequency dependent orbital susceptibility

$$\chi_{\mathrm{orb}}(\omega) = \tfrac{3}{4}\,\Delta_0^2(T)\int\frac{d\Omega}{4\pi}\,n_z^2\sum_{|k|}\frac{\tanh\frac{1}{2}\beta E_k}{E_k}\,\frac{1}{4E_k^2-\omega^2}\left(1+\frac{\xi_k^2}{E_k^2}\right) \tag{3.23}$$

and the normal locking term

$$g_n(T) = -3\Delta_0^2(T)\left\langle n_z^2\sum_{|k|}\frac{|\Delta_k|^2}{2E_k^2}\,\frac{dn_k^0}{dE_k}\right\rangle. \tag{3.24}$$

For the out-of-phase mode when the change of the order parameter is given by (3.5), Eq. (3.18) gives

$$\omega^2\chi_{\mathrm{orb}}(\omega)-g_n(T)-2\gamma(T) = 0, \tag{3.25}$$

where the coefficient $\gamma(T)$ is

$$\gamma(T) = \tfrac{3}{4}\,\Delta_0^2(T)\,\frac{\eta(T)}{1-\eta(T)}\left\langle(n_z^2-1)\sum_{|k|}\frac{1}{E_k}\tanh\tfrac{1}{2}\beta E_k\right\rangle \tag{3.26}$$

and

$$\eta(T) = -\frac{\chi^{xx}-\chi^{yy}}{V-\chi^{zz}}. \tag{3.27}$$

4. Conclusions

We have developed a theory for a new orbital mode in superfluid ^{3}He–A for which the feedback effect is essential. The temperature dependence of the frequency of the out-of--phase mode directly reflects the properties of the coefficient $\gamma(T)$, which is related to the susceptibility in (3.6). Thus, from the dispersion relation of the out-of-phase mode we can obtain information concerning the susceptibility, entering the Hamiltonian (3.6).

We can offer two experimental possibilities for the detection of the out-of-phase mode, discussed here. One is the A phase in a magnetic field near the A–N transition, where the coupling of the out-of-phase mode to zero sound would be possible due to the difference between the magnitudes of the order parameters for the up and down spins. Another possibility is the A–B interphase boundary, where according to current theories [13], the system forces the $\boldsymbol{l}$ vectors to rotate against each other in order to shift from the A phase to the 2D phase.

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