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ELASTIC SCATTERING OF ELECTRONS BY ATOMS AT INTERMEDIATE ENERGY

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The elastic scattering of electrons at intermediate energy from different light atoms was studied using a partial wave phase-shift analysis in a modified Coulomb field. The calculated differential scattering cross-sections are compared both with the theoretical method, obtained by different approaches, and with experimental results. They were found to be in good agreement.

The study of the elastic scattering of non-relativistic electrons has supplied much information regarding the structure of atoms. In elastic electron scattering the screening has been taken into account by different statistical models and exponentially screened Coulomb potentials. Allis and Morse [1], while explaining the quasi-periodic behaviour of partial cross-sections at very low energy, have proposed an atomic field with a finite cut-off parameter and have evaluated partial cross-sections by the W.K.B. method and have obtained a good agreement with the experiment. We have attempted to study the elastic scattering of electrons by Helium, atomic nitrogen, atomic oxygen and Neon at an intermediate energy using Allis-Morse potential. We have, however, used the exact method to evaluate the differential cross-sections. The model we choose is the Schrödinger equation of an electron in the electrostatic Coulomb field which is effectively screened by the atomic electrons. The potential considered is given by

$$V(r) = \begin{cases} -Ze^2\left(\dfrac{1}{r} - \dfrac{1}{a}\right) & \text{for} \quad r \leqslant a, \\ 0 & \text{for} \quad r \geqslant a, \end{cases} \tag{1}$$

and the Schrödinger equation is given by

$$\nabla^2\psi + (k^2 - U(r))\psi = 0, \tag{2}$$

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where

$$U(r) = \frac{2mV(r)}{\hbar^2},$$

and "a" is the "screening parameter" which is determined by Slater's rules [2].
The solution of Eq. (2) in the interior region gives

$$\psi_{\text{int}} = N \frac{(r\sqrt{a_0-k^2})^{l+1}}{r} e^{-r\sqrt{a_0-k^2}} F(l+1-\eta;\, 2l+2;\, 2r\sqrt{a_0-k^2}) \tag{3}$$

where

$$\eta = \frac{\alpha k}{\sqrt{a_0-k^2}}; \quad \alpha = \frac{Ze^2}{\hbar v}; \quad a_0 = \frac{2mZe^2}{a\hbar^2}$$

and N is the normalization constant.
The wave-function in the external region is given by

$$\psi_{\text{ext}} = A_l j_l(kr) + B_l \eta_l(kr) \tag{4}$$

where j_l and η_l denote the spherical Bessel and Neumann functions, respectively, and A_l and B_l are constants.

Now, the continuity of the wave-functions and their derivatives at the boundary gives

$$\tan(\delta_l) = \frac{2Rj_l(ka) - k(j_{l-1}(ka) - j_{l+1}(ka))}{2R\eta_l(ka) - k(\eta_{l-1}(ka) - \eta_{l+1}(ka))} \tag{5}$$

where

$$R = \left(\frac{\frac{d}{dr}(\psi_{\text{int}})}{\psi_{\text{int}}} \right)_{r=a},$$

δ_l is the phase shift of the l-th partial wave.

Then the differential cross-sections are calculated from the partial wave phase-shift series

$$f(\vartheta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1)(e^{2i\delta_l} - 1) P_l(\cos\vartheta) \tag{6}$$

and

$$\sigma(\vartheta) = |f(\vartheta)|^2 \tag{7}$$

The angular distributions for the differential cross-sections are calculated using equations (3), (5), (6) and (7) and are plotted in figures 1–4. In all events the angular distributions are decisively in favour of other previous results and show better agreement at

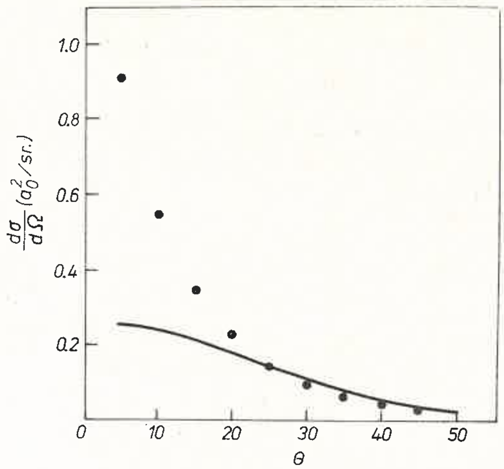

Fig. 1. Differential cross-sections for the elastic scattering of electrons at 500 eV from Helium. The points show experimental results [3] and the curve represents the present calculations

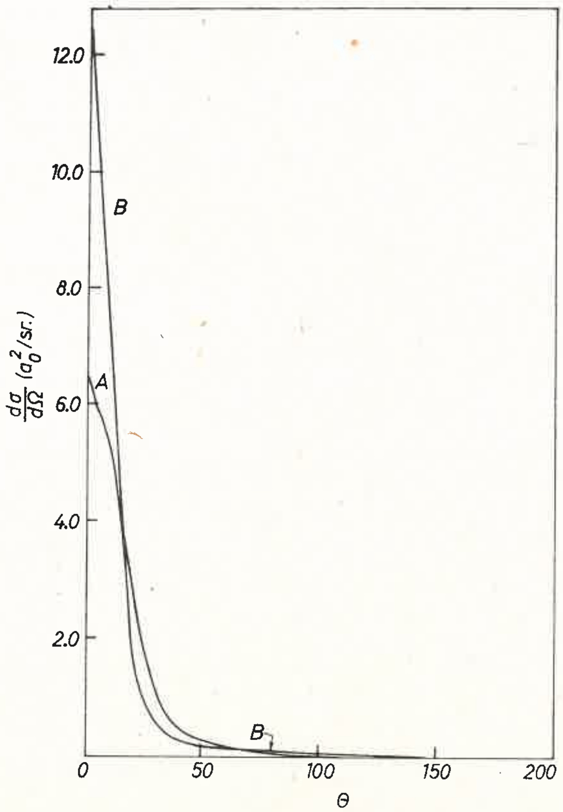

Fig. 2. Differential cross-sections for the elastic scattering of electrons at 500 eV from atomic Nitrogen. The curve A represents the present calculations while curve B is the theoretical work of Blaha et al. [4] calculated by a modified distorted-wave approximation

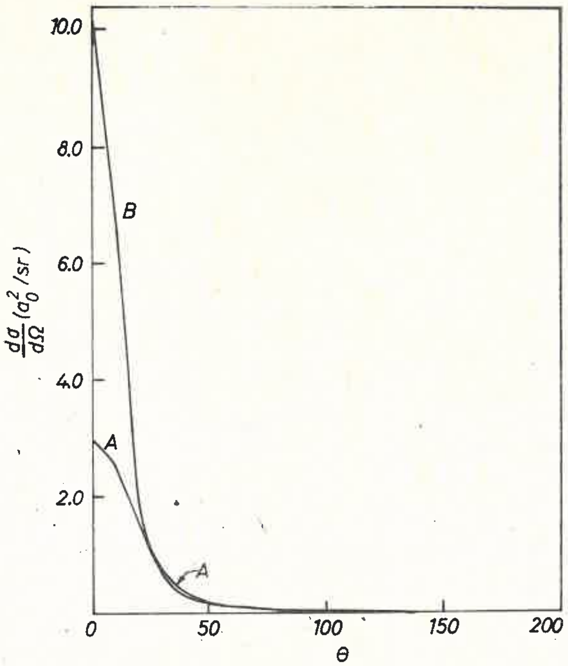

Fig. 3. Differential cross-section for the elastic scattering of electrons at 500 eV from atomic oxygen. Curve *A* represents the present calculations while curve *B* is the theoretical work of Blaha et al. [4] calculated by a modified distorted-wave approximation

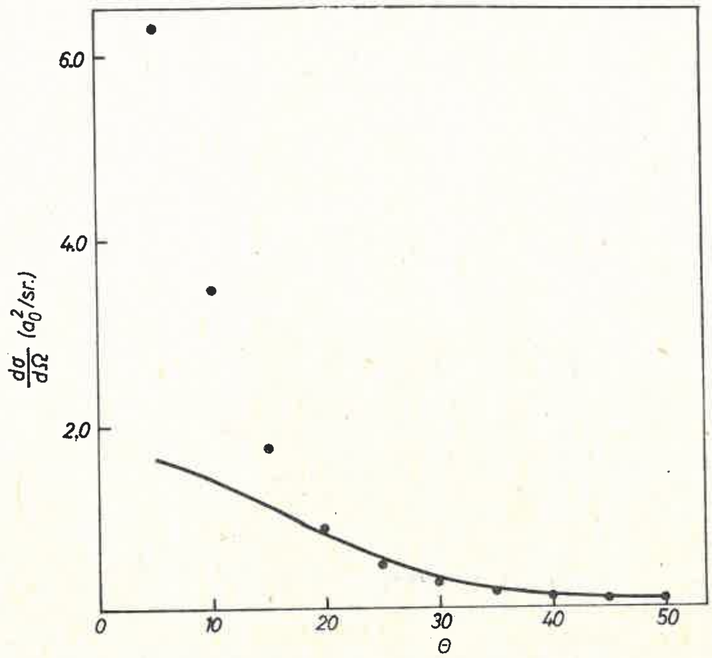

Fig. 4. Differential cross-sections for the elastic scattering of electrons at 1 keV from Neon. The points give experimental results [3] and the curve represents the present calculations

large angles than at forward angles. This conforms that the screening effect dominates at large angles. A careful observation of the angular distribution curves also reveals that we have succeeded in reproducing the angular distributions in a better way than those obtained by Allis and Morse [1] at very low energy, perhaps, because the screening is more effective at intermediate energy. Again, while in most cases the screening by atomic electrons is taken into account by additional form factors or different statistical models, the use of a simplified model enables us to visualize the screening effect directly. The abrupt cut-off nature of the potential, in particular, can be used to explain any possible anomalous behaviour, which other types of continuously varying screened potentials cannot be expected to do.

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