

# THE DYNAMICS OF A THIN FERROMAGNETIC FILM NEAR THE PHASE TRANSITION POINT FROM THE STATE OF HOMOGENEOUS MAGNETIZATION TO THE DOMAIN STRUCTURE INDUCED BY A CHANGE IN THE EXTERNAL MAGNETIC FIELD

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We considered the dynamics of a thin ferromagnetic film near the point  $h_c$  of the phase transition from the state of homogeneous magnetization in the plane of the film to the state of inhomogeneous magnetization, i.e., to the domain structure. The easy magnetization axis of a ferromagnet is perpendicular to the film. In the calculations the Landau-Lifshitz motion equations with a damping term of the Gilbert type were used. The demagnetization energy of a thin film was taken into consideration. The motion equations were solved simultaneously with Maxwell equations in a magnetostatic approximation. The solutions for the motion equations satisfy the boundary conditions laid on the magnetization vector and potential of a magnetic field on the surface of a thin film. In the first approximation, with respect to the small parameters,  $\frac{h-h_c}{h_c}$  and  $\frac{L_c}{L}$ , the critical value  $h_c$  having an intensity of an external magnetic field was obtained. The dispersion relation for spin waves in a state of homogeneous magnetization also was obtained. The description of the dynamics of a phase transition allowed us to consider relaxation processes below and above the phase transition point. The analytical expressions for the values which characterise the phase transition process and the values which characterise an appearing domain structure for  $h < h_c$  are given.

## 1. Introduction

In a thin ferromagnetic film the phase transition from the homogeneous to the inhomogeneous magnetization state occurs for both fixed values of the external magnetic field and thickness  $L$  of the film. The  $x, y$  dimensions of the thin film are much larger than its thickness and they can be treated as infinite. The easy magnetization axis is perpendicular

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ular to the surfaces  $z = \pm \frac{1}{2}L$  of the film. The magnetization of the film is homogeneous for  $L > L_c$  when  $H^e > H_c^e(L)$ . In this case the field  $H^e$  and the magnetization vector point in the  $y$  direction.

Let us assume that at  $t = 0$  the field  $H^e$  is varied from  $H^e > H_c^e(L)$  to  $H^e < H_c^e(L)$  in a stepwise fashion and that condition  $|H^e - H_c^e|/H_c^e \ll 1$  is fulfilled.

The relaxation of the magnetization can be represented as a superposition of two processes: the relaxation of the magnetization vector magnitude,  $M(H^e)$ , and the inhomogeneous relaxation of the magnetization vector orientation. The first process takes place rapidly for temperatures much lower than the Curie temperature. The relaxation time depends slightly on the magnitude of  $H^e - H_c^e$ , and the magnetization magnitude is varied only slightly. We shall now consider the inhomogeneous relaxation process of the magnetization vector orientation.

If the layer is much thicker than  $L_c$  it is possible to use the expansion method with respect to the small parameter  $L_c/L$ . We will show that solutions describing the surface modes can be omitted only in the first approximation with respect to  $L_c/L$ . In addition, the universal boundary conditions can be obtained for solutions describing the volume mode unimodal approximation.

## 2. Equations of motion. Boundary conditions

We may use the phenomenological theory because the characteristic frequencies and dimensions do not rule out this treatment [1]. The relaxation is described by the Landau-Lifshitz equation

$$\frac{\partial \vec{M}}{\partial t} = g[\vec{M} \times \vec{H}_{\text{ef}}]; \quad \vec{H}_{\text{ef}} = -\frac{\delta W}{\delta \vec{M}}, \quad (1)$$

where the energy  $W(\vec{M})$  of the system is a functional of  $\vec{M}(\vec{r}, t)$ . We shall study the simplest model for which [1]

$$W = \int \left\{ \frac{C}{2} (\nabla \vec{M})^2 - \frac{\beta}{2} M_z^2 - \frac{1}{2} \vec{H}^m \vec{M} - \vec{H}^e \vec{M} \right\} dV, \quad (2)$$

where  $H^m$  is the demagnetization field,  $C$  — isotropic exchange constant and  $\beta$  — uniaxial anisotropy constant ( $\beta < 4\pi$ ). The equation of motion (1) should be solved simultaneously with Maxwell's equations, which have the following form in quasistatic approximation:

$$\vec{H}^m + \vec{H}^e = \vec{H}^i = -\vec{\nabla} \phi, \quad 4\pi \operatorname{div} \vec{M} - \Delta \phi = 0. \quad (3)$$

The corresponding boundary conditions should be added.

The system of Eqs (1), (3) has the following form for small deviations  $\delta \vec{M}$  from homogeneous magnetization along the  $y$  axis

$$i\Omega m_x + (Cq^2 - \beta + h)m_z + ik\varphi = 0, \quad (h + Cq^2)m_x - i\Omega m_z + ik\varphi = 0, \\ 4\pi i\kappa m_x + 4\pi i\kappa m_z + q^2\varphi = 0, \quad (4)$$

where  $\Omega = \frac{\omega}{gM_0}$ ;  $h = \frac{H_y^e}{M_0}$ ;  $\vec{m} = \frac{\delta \vec{M}}{M_0}$ ;  $\varphi = \frac{\phi}{M_0}$ . The values  $m_x, m_z, \varphi$  depend on  $x, z$ , and  $t$  in the following manner

$$\begin{aligned} m_l(x, z, t) &= m_l(\kappa, k, \omega) \exp \{i(\omega t + \kappa x + kz)\}; \quad l = x, z \\ \varphi(x, z, t) &= \varphi(\kappa, k, \omega) \exp \{i(\omega t + \kappa x + kz)\}, \end{aligned} \quad (5)$$

and  $q^2 = \kappa^2 + k^2$ . From the solubility condition of system (4) one can obtain

$$(i\Omega)^2 = q^{-2} \{4\pi\beta\kappa^2 - q^2(Cq^2 + h - \beta + 4\pi)(Cq^2 + h)\}. \quad (6)$$

Equation (6) has three roots with respect to  $k^2$  or six roots with respect to  $k$  to fixed values of  $\kappa^2$  and  $(i\Omega)^2$ :  $k_1, k_2 = -k_1, k_3, k_4 = -k_3, k_5, k_6 = -k_5$ .

Let us write  $m_z$  in the following manner

$$m_z = \sum_{j=1}^6 a_j \exp(ik_j z). \quad (7)$$

The values  $m_x$  and  $\varphi$  can be represented in a similar way. The coefficients with  $\exp(ik_j z)$  can be computed using the set of Eqs (4). These coefficients depend linearly on coefficients  $a_j$  in (7). The solutions of Eqs (1), (3) would satisfy the respective boundary conditions on the surfaces  $z = \pm \frac{1}{2}L$  where both the potential  $\varphi$  and the normal component of the magnetic induction vector are continuous. Outside the film the magnetization is equal to zero and the potential  $\varphi$  satisfies Laplace's equation. Its solutions are equal to zero at infinity and have the following form in the  $(\kappa, z, \omega)$  representation

$$\varphi^e = \varphi_{\pm} e^{\pm |\kappa|z}. \quad (8)$$

Using (8), the continuity condition for the normal component of the magnetic induction vector  $B_z = H_z + 4\pi M_z$  can be presented in the following way

$$4\pi m_z - \frac{\partial \varphi}{\partial z} \pm |\kappa| \varphi = 0; \quad z = \pm \frac{L}{2}. \quad (9)$$

The boundary conditions for  $\vec{m}$  have the form [1, 3]

$$\frac{\partial m_x}{\partial z} \pm dm_x = 0; \quad z = \pm \frac{L}{2}, \quad \frac{\partial m_z}{\partial z} \pm dm_z = 0; \quad z = \pm \frac{L}{2}, \quad (10)$$

where  $d$  is the surface anisotropy constant. After substituting  $m_x, m_z, \varphi$  expressed in the form (7) into (9), (10) we find the equations describing  $a_j$  to be

$$\sum_{j=1}^6 A_{ij} a_j = 0. \quad (11)$$

The condition  $\text{Det } |A| = 0$  gives us the connection between  $k_1, k_3, k_5, \kappa$  and  $\Omega$ . Each of the roots of the dispersion equation (6) is a function of  $\kappa$  and  $\Omega$ . Conditions (11) and (6) define the dependence  $\Omega = \Omega(\kappa)$ . The magnitude of  $Y = (i\Omega)^2$  is less than zero for  $h > h_c$  and has a maximum at the point  $\kappa^2 = \kappa_0^2$  for an arbitrary  $h$ .

Near the maximum the value  $Y$  can be presented in the following form

$$Y(\kappa^2) = Y_0(\kappa_0^2) - \gamma(\kappa^2 - \kappa_0^2)^2. \quad (12)$$

The solutions of Eqs (4) increasing as  $\exp(\sqrt{Y_0}t)$  exist for  $Y_0 > 0$ . The fastest growth appears for the harmonics where  $\kappa = \pm \kappa_0$ . The value  $h_c$  is defined by condition

$$Y_0 = 0 \quad \text{for} \quad h = h_c. \quad (13)$$

We can obtain the solutions of Eqs (4) with  $Y_0 = 0$ ,  $h = h_c$ ,  $\kappa^2 = \kappa_0^2$  if we take into account the symmetry of this static solution with regard to the substitution  $z \rightarrow -z$  ( $k_i \rightarrow -k_i$ ). In this case, in place of (11) we shall obtain three equations with three unknown quantities.

### 3. The solution of the motion equations near the phase transition point

The computations for  $h \approx h_c(L)$  are much simpler if  $L \gg L_c$  ( $h_c(L_c) = 0$ ). In this case for  $\kappa^2 \rightarrow \kappa_0^2$  one of the roots  $k_1^2, k_3^2, k_5^2$  is small and the ratio  $k_1/k_i$  tends to zero if  $h \rightarrow h_c$  and  $L \rightarrow \infty$ . The coefficients with  $\exp(ik_i z)$  in (7) are as small as  $k_1/k_i$  for  $i = 3, 4, 5, 6$ . In the domain of the values  $\kappa, h, L$  the relation  $k_1^2 \ll \kappa^2$  holds for small  $k_1$ . By omitting the small components in (4) one can obtain

$$\begin{aligned} i\Omega m_x + (C\kappa^2 - \beta + h)m_z + ik\varphi &= 0, & hm_x - i\Omega m_z + ik\varphi &= 0, \\ 4\pi i\kappa m_x + 4\pi i\kappa m_z + \kappa^2 \varphi &= 0. \end{aligned} \quad (14)$$

Hence,

$$k^2 = \frac{\kappa^2}{4\pi h} \{-Y - (4\pi + h)(C\kappa^2 - \beta + h)\}, \quad (15)$$

and

$$m_x = \frac{i\Omega\kappa^2 - 4\pi\kappa k}{\kappa^2(4\pi + h)} m_z, \quad \varphi = \frac{-4\pi i(hk + i\kappa\Omega)}{\kappa^2(4\pi + h)} m_z. \quad (16)$$

The solutions of (14) should satisfy the boundary conditions (9), (10). In the full solutions for  $m_z, m_x, \varphi$  we should take into account terms such as  $k_j \exp(ik_j z)$  and  $j = 3, 4, 5, 6$  because the roots  $k_j$  are not small quantities. However, the derivatives with respect to  $z$  of the small terms are, in general, of the same order of magnitude as the derivative of the main part. After taking into account that  $k_j > \kappa \sim \sqrt{k_1}$  one can obtain the connection between the respective parts of the solution when the roots are large. From Eq. (4) one can obtain

$$\begin{aligned} i\Omega m_x + Ck^2 m_z + ik\varphi &= 0, & (Ck^2 + h)m_x - i\Omega m_z &= 0, \\ 4\pi i\kappa m_x + k^2 \varphi &= 0. \end{aligned} \quad (17)$$

The solubility condition for the set of Eqs (17) is as follows

$$(i\Omega)^2 = -(Ck^2 + h)(Ck^2 + 4\pi). \quad (18)$$

We obtain the following results for  $|i\Omega| \ll 1$  with an accuracy of the order of terms small in comparison with  $\Omega$

$$k_3^2 = -\frac{h}{C}, \quad k_5^2 = -\frac{4\pi}{C}. \quad (19)$$

It follows from (17) that

$$m_x = \frac{i\Omega}{Ck^2+h} m_z, \quad \varphi = -\frac{4\pi i}{k} m_z. \quad (20)$$

The order of magnitude of the coefficients with  $\exp(ik_j z)$ ,  $j = 1, 2$  can be estimated by utilizing boundary conditions (9), (10). Let us present  $m_z$  in the following form

$$m_z = m_z^{(0)} + m_z^{(1)}, \quad m_z^{(0)} = a \cos k_1 z + b \sin k_1 z, \\ m_z^{(1)} = \sum_{j=3}^6 c_j e^{ik_j z}, \quad (21)$$

and similarly for  $m_x$  and  $\varphi$ . The precise relations between  $m_x$ ,  $\varphi$  and  $m_z$  are defined by formulas (16) and (20)

$$m_x^{(0)} = \frac{i\Omega\kappa^2}{\kappa^2(4\pi+h)} m_z^{(0)} + \frac{4\pi i}{\kappa(4\pi+h)} \frac{\partial m_z^{(0)}}{\partial z}, \\ \varphi^{(0)} = \frac{i\Omega}{\kappa(4\pi+h)} m_z^{(0)} - \frac{4\pi h}{\kappa^2(4\pi+h)} \frac{\partial m_z^{(0)}}{\partial z}, \quad (22)$$

and

$$m_x^{(1)} = i\Omega \sum_{j=3}^6 \frac{c_j}{Ck_j^2+h} e^{ik_j z}, \quad \varphi^{(1)} = -4\pi i \sum_{j=3}^6 \frac{c_j}{k_j} e^{ik_j z}. \quad (23)$$

Let us assume that the contribution of  $m_z^{(1)}$ ,  $m_x^{(1)}$ ,  $\varphi^{(1)}$  is sufficiently small not only in comparison with  $m_z$ ,  $m_x$ ,  $\varphi$  but also in comparison with their derivatives with respect to  $z$ . This means that  $m_z^{(0)}$ ,  $m_x^{(0)}$  and  $\varphi^{(0)}$  should satisfy the boundary conditions (9), (10) which have the following form after taking into account formula (22)

$$m_z^{(0)} + \lambda_j \frac{\partial m_z^{(0)}}{\partial z} = 0, \quad z = \pm \frac{L}{2}, \quad (24)$$

where the  $\lambda_j$  ( $j = 1, 2, \dots, 6$ ) have different values for different conditions (9), (10). However, in general, Eqs. (24) are consistent only if first order terms in  $k_1$  are neglected. Their non-consistency when first order terms in  $k_1$  are included means that it is necessary to take into account the terms which arise from  $m_z^{(1)}$ ,  $m_x^{(1)}$ ,  $\varphi^{(1)}$ . Hence, we find that in the zero approximation the condition  $m_z^{(0)} = 0$  should be fulfilled for  $z = \pm \frac{1}{2}L$  which leads to

$$k_{1n} = n\pi/L, \quad n = 1, 2, \dots. \quad (25)$$



The  $L^{-2}$  order corrections in the expression defining  $k_{1n}$  can be obtained only after taking into consideration the remaining roots  $k_3^2$  and  $k_5^2$ .

Using (25) we can obtain the following form of formula (15)

$$Y = (i\Omega)^2 = -\frac{4\pi\beta n^2 \pi^2}{\kappa^2 L^2} - (4\pi + h)(C\kappa^2 - \beta + h). \quad (26)$$

No assumptions about the magnitude of  $\Omega$  and  $k_{1n}$  were made in the derivation of (26). The magnitude of  $Y$  is limited by the possibility of the appearance of an additional small root in (18), which occurs when  $Y = 4\pi h$ . From (26), near the maximum of  $Y$  one can obtain

$$Y = Y_0 - \gamma(\kappa^2 - \kappa_{0n}^2)^2, \quad Y_0 = -(4\pi + h)(h - \beta) - 4k_{1n}\sqrt{\pi h C(4\pi + h)},$$

$$\gamma = \frac{1}{2k_{1n}} \left[ \frac{(4\pi + h)^3 C^3}{\pi h} \right]^{1/3}, \quad (27)$$

$$\kappa_{0n}^2 = k_{1n} \left[ \frac{4\pi h}{C(4\pi + h)} \right]^{1/2}. \quad (28)$$

The phase transition defined by the condition  $Y_0 = 0$  appears for the critical field  $h_c$  which is defined as follows

$$h_c = \beta - \frac{4k_{11}}{4\pi + \beta} [\pi\beta C(4\pi + \beta)]^{1/2}. \quad (29)$$

Formulas (28), (29) were given in [2]. In that paper the calculations were done in the approximation (14) without a detailed analysis of accuracy, i.e., without taking into consideration solutions which are connected with the remaining four roots of relation (6). According to (27) the domain of the increasing excitations appears for  $h < h_c$ . The character of nonstability of the homogeneous magnetization is similar to the situation that occurs in spinoidal decay [4, 5].

#### 4. The relaxation of a system to the equilibrium state

In the process of the magnetization relaxation near  $h = h_c$  the damping processes play an essential part. An additional term has to be introduced into the Landau-Lifshitz equation (1) in order to account for the damping of the spin waves. The analysis of the approximations carried out above is not altered when damping is taken into account. The Gilbert equation of motion with the damping term has the following form [1]

$$\frac{\partial \vec{M}}{\partial t} = g[\vec{M} \times \vec{H}_{\text{eff}}] - \frac{\alpha}{M_0} \left[ \vec{M} \times \frac{\partial \vec{M}}{\partial t} \right]. \quad (30)$$

We are also going to take into consideration the nonlinear terms which limit the increase of the magnetization amplitudes. Near  $h = h_c$  the value of  $m_z$  is small (of the order of

$(h_c - h)/h_c$ ) and hence we take into consideration only the first nonlinear term as in the static case [2]. With the above assumptions only the form of the first equation of set (14) is changed

$$i\Omega m_x + \left( -C \frac{\partial^2}{\partial x^2} - \beta + h + i\alpha\Omega \right) m_z + \frac{\partial \varphi}{\partial z} + \frac{\beta}{2} m_z^3 = 0. \quad (31)$$

In the same approximation with respect to  $L/L_c$  and  $(h_c - h)/h_c$  we obtain the following equation for  $m_z$

$$\begin{aligned} & \left[ \frac{1}{gM_0} \frac{\partial^2}{\partial t^2} + \alpha(4\pi + \beta) \frac{\partial}{\partial t} \right] \frac{\partial^2 m_z}{\partial x^2} \\ &= (4\pi + \beta) \frac{\partial^2}{\partial x^2} \left[ C \frac{\partial^2 m_z}{\partial x^2} + (\beta - h)m_z - \frac{\beta}{2} m_z^3 \right] - 4\pi\beta \frac{\partial^2 m_z}{\partial z^2}. \end{aligned} \quad (32)$$

We are looking for solutions of Eq. (32) having the form

$$m_z = \sum_{\pm n=1}^{\infty} A_n(x, t) \cos k_{1n}z, \quad k_{1n} = nk_{11} = n\pi/L. \quad (33)$$

From (32) the following set of equations results for  $A_n(x, t)$

$$\hat{G}_n^{-1}(x, t) A_n = \frac{\beta}{8} (4\pi + \beta) \frac{\partial^2}{\partial x^2} \sum_{n_1+n_2+n_3=n} A_{n_1} A_{n_2} A_{n_3}, \quad (34)$$

where  $\hat{G}_n$  is a linear operator. It is significant that the solutions of the linearized equations

$$\hat{G}_n^{-1}(x, t) A_n = 0 \quad (35)$$

are increasing functions for  $n = 1$  only (with  $h \rightarrow h_c$ ). This means that the ratio  $A_m/A_1 \sim (h_c - h)/h_c$  for  $m \neq \pm 1$ , and the solution of Eq. (34) may be found using the successive approximation method. In the first approximation we obtain the equation for  $A_1$

$$\begin{aligned} & \left[ \frac{1}{gM_0} \frac{\partial^2}{\partial t^2} + \alpha(4\pi + \beta) \frac{\partial}{\partial t} \right] \frac{\partial^2 A_1}{\partial x^2} = 4\pi\beta k_{11}^2 A_1 \\ & + (4\pi + \beta) \frac{\partial^2}{\partial x^2} \left[ C \frac{\partial^2}{\partial x^2} + (\beta - h) - \frac{3}{8} \beta A_1^2 \right] A_1. \end{aligned} \quad (36)$$

Let us consider the solution of Eq. (36) in the following form

$$A_1 = \int_{-\infty}^{\infty} A_1(\kappa, t) \cos \kappa x d\kappa. \quad (37)$$

From considerations similar to those above we conclude that it is possible to retain in Eq. (37) only the terms arising from the domain of  $\kappa$ , for which the magnetization is unstable when  $(h_c - h)/h_c \rightarrow 0$ . This domain is reduced to the point  $\kappa_0$  which is described by (28),

that is

$$\kappa_0^2 = \left[ \frac{4\pi\beta}{C(4\pi+\beta)} \right]^{1/2}; \quad D = \frac{2\pi}{\kappa_0} = \left[ \frac{4C(4\pi+h)}{\pi^3 h} \right]^{1/4} \sqrt{L}. \quad (38)$$

where  $D$  is the period of the appearing domain structure. This is the reason that the value  $m_z$  may be described by

$$m_z(x, z, t) = A(t) \cos \kappa_0 x \cos k_{11} z. \quad (39)$$

where  $A(t)$  fulfills the following equation

$$\frac{d^2 A}{dt^2} + \gamma \frac{dA}{dt} = - \frac{dU(A)}{dA}. \quad (40)$$

The values  $U(A)$  and  $\gamma$  are defined in the following manner

$$U(A) = -\frac{1}{2} g M_0 Y_0 A^2 + \frac{9}{128} g M_0 (4\pi + \beta) \beta A^4, \quad (41)$$

$$\gamma = \alpha(4\pi + \beta) g M_0. \quad (42)$$

The coefficient appearing with  $A^2$  in expression (41), proportional to  $Y_0$ , changes the sign for  $h = h_c$  ( $Y_0(h > h_c) < 0$ ;  $Y_0(h = h_c) = 0$ ;  $Y_0(h < h_c) > 0$ ). The dependence  $U = U(A)$  is illustrated by Fig. 1.

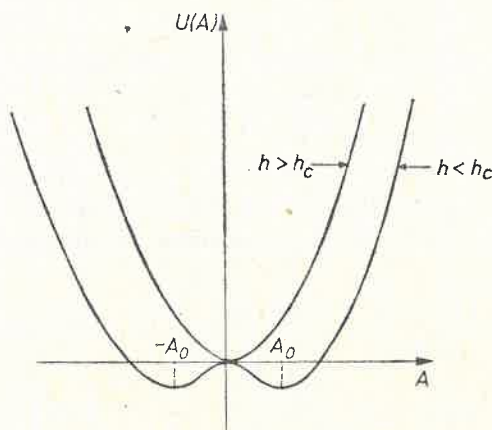


Fig. 1

Eq. (40) is the equation of motion of a nonlinear damped oscillator. The magnitude of  $A(0)$  is defined for  $t = 0$  by the thermodynamic fluctuations in the initial stable state. Near the phase transition point the frequency of the eigen vibrations of the oscillator is small for  $h < h_c$  and the relaxation is described by the monotonic function

$$A^{-2}(t) = A_0^{-2} + (A^{-2}(0) - A_0^{-2}) \exp(-t/t_0), \quad (43)$$



where the magnitude  $A_0$  of the amplitude of the equilibrium state is given by

$$A_0^2 = \frac{32Y_0}{9\beta(4\pi + \beta)}, \quad (44)$$

and the relaxation time is given by

$$t_0 = \frac{528}{81} \alpha \kappa_0 (h_c - h). \quad (45)$$

The vibrations of the system are damped for  $h > h_1$  under the condition  $(h - h_c)/h_c \ll 1$ . The friction should be small ( $\alpha^2 \ll 1$ ) in order to realize this case. The limiting magnitude of the magnetic field  $h_1$  is as follows

$$h_1 = h_c + \frac{1}{4} \alpha^2 (4\pi + \beta). \quad (46)$$

In this case the frequency of the damped vibrations and the relaxation time  $t_0$  are given by

$$\omega_0^2 = gM_0|Y_0|, \quad t_0 = 2\gamma^{-1}. \quad (47)$$

For the value of the intensity of an external magnetic field in a range  $h_1 > h > h_c$  the dependence  $A(t)$  is described by functions exponentially diminishing in time. The soft mode of the spin waves shows decreasing values of  $h > h_c$ . When  $(h - h_c)/h_c \ll 1$  the mode with  $\kappa = \kappa_0$  has the smallest frequency. The frequency of these spin waves in the homogeneous phase decreases near the phase transition point. In the domain  $|h - h_c| \sim |h_1 - h_c|$  strong damping appears. For  $h > h_c$  the lowest value in the frequency spectrum of spin waves is given by formula (47).

### 5. Concluding remarks

In a thin ferromagnetic film whose thickness  $L > L_c(0)$  with one axis of anisotropy described by a constant  $\beta < 4\pi$  the domain structure appears. The case in which a thin film is in an external magnetic field lying in the plane of the film and perpendicular to the easy magnetization axis has been considered. For the values of an external magnetic field  $h > h_c$ , the thin film is homogeneously magnetized in the direction of external magnetic field action. When  $h < h_c$  in the thin film, the domain structure appears. At the point  $h = h_c$  there is a continuous phase transition from the state of homogeneous magnetization to the domain structure. The value of the critical field  $h_c$  is a function of the thickness  $L$  of the film.

In this paper the scheme which describes the phenomenon occurring in the phase transition region in support of Landau-Lifshitz formalism has been given. In the range of external magnetic field values  $h < h_c$ , the dispersion relation (6) was obtained for the spin waves. The scheme for the solution of the motion equations (1) and Maxwell equations (3) in a magnetostatic approximation with respect to boundary conditions (9), (10) for arbitrary values of an external magnetic field  $h > h_c$  and the thicknesses of a film  $L > L_c(0)$  was given. The detailed analysis of the solutions of the set of equations (1), (3) for  $(h - h_c)/h_c \ll 1$  and  $L_c(0)/L \ll 1$  with an estimation of the accomplished approximation

has been carried out. This has allowed the determination of the critical value  $h_c$  given by formula (29). We considered the relaxation process of a system to the equilibrium state, which is, in dependence on the values  $h$ , the state of homogeneous magnetization, or the domain structure. The relaxation process is described by equation (32) in which the damping term and nonlinear term have been considered. The scheme for the application of the perturbation calculations with respect to the small parameters  $|h - h_c|/h_c$ ,  $L_c/L$  for the solution of equation (32) has been given. In the first approximation of the perturbation calculations the relaxation process of the system is described by equation (40) which is analogous to the equation describing the motion of the nonlinear oscillator with damping. The analysis of coefficients appearing in equation (40), being functions of  $h$  and the damping parameter  $\alpha$ , shows the existence of different types of relaxation processes. The calculations carried out allow for the determination of period  $D$  of the domain structure (38) and magnetization in domains defined by equilibrium amplitude  $A_0$  given by (44). The model presented is a selfconsistent description of the domain structure in a thin film near the phase transition point.

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