THE WAVE POTENTIAL OF MIYAMOTO AND WOLF FOR AN ELECTROMAGNETIC FIELD IN A GENERALIZED UNIAXIAL ANISOTROPIC MEDIUM

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(Received June 5, 1978; final version received July 4, 1979)

This paper contains the wave potential of Miyamoto and Wolf for an electromagnetic field in a generalized uniaxial anisotropic medium. The wave potential of Miyamoto and Wolf is of fundamental importance in Kirchhoff's Diffraction Theory. This gives a possibility of presenting the diffraction field in agreement with Young–Rubinowicz's model.

1. *Introduction*

In this paper, which is an extension of the work by Petykiewicz and Rynkowski [1], we consider a medium that can be defined by a dielectric tensor, $\vec{\vec{\varepsilon}}$, and a magnetic permeability tensor, $\vec{\vec{\mu}}$. These tensors have a property that the tensor

$$\vec{\vec{\gamma}} = \vec{\vec{\varepsilon}} \cdot \vec{\vec{\mu}}^{-1} \tag{1.1}$$

is the uniaxial anisotropic one where $\vec{\vec{\mu}}^{-1}$ is the tensor inverse to the magnetic permeability tensor.

We have assumed the coordinate system: x_1, x_2, x_3 in which the axes of the system are the principal axes of the tensor $\vec{\vec{\gamma}}$. This can be represented as:

$$\vec{\vec{\gamma}} = \gamma_0 \vec{\vec{I}} + (\gamma_e - \gamma_0)\vec{k}\vec{k}, \tag{1.2}$$

where $\vec{\vec{I}}$ stands for a unit tensor and $\vec{k}\vec{k}$ for the dyadic. The unit vector $\vec{k}$ lies in the direction of the x_3-axis, which corresponds to the optical axis. The principal constants

$$\gamma_0 = \frac{\varepsilon_1}{\mu_1} = \frac{\varepsilon_2}{\mu_2},$$

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and

$$\gamma_e = \frac{\varepsilon_3}{\mu_3}, \tag{1.3}$$

are described by ε_i and μ_i which are the principal constants for tensors $\overleftrightarrow{\varepsilon}$ and $\overleftrightarrow{\mu}$, respectively. We assume that the principal axes of these are the same and

$$\varepsilon_1 \neq \varepsilon_2 \neq \varepsilon_3, \quad \mu_1 \neq \mu_2 \neq \mu_3. \tag{1.4}$$

In this work the SI units used are ε_0 and μ_0 which stand for the dielectric constant and magnetic permeability of the vacuum, respectively and

$$k_0^2 = \omega^2 \varepsilon_0 \mu_0, \quad \overleftrightarrow{\varepsilon} = \varepsilon_0 \overleftrightarrow{\varepsilon}_r, \quad \overleftrightarrow{\mu} = \mu_0 \mu_r \overleftrightarrow{I}, \tag{1.5}$$

where ω defines the time dependence of the field: $\exp(-i\omega t)$.

The Larmor–Lorentz principle was used as the basis for this report, since it enabled us to express the state of the monochromatic field at point P by values of the field and its derivatives at all points Q at the surface surrounding the point considered.

This principle has been widely recognized by Maue and Westfahl [2] discussing the isotropic medium, while for the anisotropic medium it was formulated by Wünsche [3].

We can write the Larmor–Lorentz principle for the generalized uniaxial anisotropic medium in the form

$$E_j = \oint\!\!\oint df\{-(\vec{n}\times\vec{E})\cdot\overleftrightarrow{\mu}^{-1}\cdot \operatorname{curl}\vec{G}_j^{(E)} + (\vec{n}\times\vec{G}_j^{(E)})\cdot\overleftrightarrow{\mu}^{-1}\cdot \operatorname{curl}\vec{E}\}, \tag{1.6a}$$

$$H_j = \oint\!\!\oint df\{+(\vec{n}\times\vec{G}_j^{(H)})\cdot\overleftrightarrow{\varepsilon}^{-1}\cdot \operatorname{curl}\vec{H} - (\vec{n}\times\vec{H})\cdot\overleftrightarrow{\varepsilon}^{-1}\cdot \operatorname{curl}\vec{G}_j^{(H)}\}, \tag{1.6b}$$

where the subscript $j = 1, 2, 3$ refers to the particular components of the field. The evident feature of the vector $\vec{G}_j^{(E)}$ (or $\vec{G}_j^{(H)}$) is the j's column of tensor $\overleftrightarrow{G}^{(E)}$ (or $\overleftrightarrow{G}^{(H)}$)

$$\overleftrightarrow{G}^{(E)} = -\frac{1}{4\pi}\left\{\left[\frac{1}{k_0^2\sqrt{\gamma_0}}\nabla\nabla + (\gamma_0\sqrt{\gamma_0})^{-1}|\overleftrightarrow{\varepsilon}|\overleftrightarrow{\varepsilon}^{-1}\right]\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\right.$$

$$\left. + \frac{(\sqrt{\gamma_0})^{-1}}{ik_0}\overleftrightarrow{\varepsilon}^{-1}\cdot \operatorname{curl}\left[\vec{k}\,\frac{\vec{\varrho}\times\vec{k}}{\varrho_E^2}(\exp(-ik_0R_\varepsilon) - \exp(-ik_0R_\mu))\right]\right\},$$

$$\overleftrightarrow{G}^{(H)} = -\frac{1}{4\pi}\left\{\left[\frac{\sqrt{\gamma_0}}{k_0^2}\nabla\nabla + \gamma_0\sqrt{\gamma_0}\,|\overleftrightarrow{\mu}|\overleftrightarrow{\mu}^{-1}\right]\frac{\exp(-ik_0R_\mu)}{R_\mu}\right.$$

$$\left. + \frac{\sqrt{\gamma_0}}{ik_0}\overleftrightarrow{\mu}^{-1}\cdot \operatorname{curl}\left[\vec{k}\,\frac{\vec{\varrho}\times\vec{k}}{\varrho_\mu^2}(\exp(-ik_0R_\mu) - \exp(-ik_0R_\varepsilon))\right]\right\}, \tag{1.7}$$

where

$$\varrho_E^2 = \frac{x_1^2}{\varepsilon_1} + \frac{y_1^2}{\varepsilon_2}, \qquad \varrho_M^2 = \frac{x_1^2}{\mu_1} + \frac{y_1^2}{\mu_2},$$

$$R_\varepsilon = \sqrt{\gamma_0\gamma_\varepsilon|\vec{\mu}|\left(\frac{x^2}{\varepsilon_1} + \frac{y^2}{\varepsilon_2} + \frac{z^2}{\varepsilon_3}\right)}, \qquad R_\mu = \sqrt{\gamma_0|\vec{\mu}|\left(\frac{x^2}{\mu_1} + \frac{y^2}{\mu_2} + \frac{z^2}{\mu_3}\right)}. \tag{1.8}$$

In the uniaxial anisotropic medium, we can resolve any arbitrarily polarized incident wave into two waves, one of which is a wave of the TE-type and the other — a wave of the TM-type. This division is taken with regard to the distinguished axis of anisotropy, which is the x_3 axis (see-Kujawski and Przeździecki [4]).

The evident feature of the vector $\vec{H}$ for a wave of the TE-type satisfies the equation

$$\frac{1}{\varepsilon_2\mu_3}\frac{\partial^2}{\partial x^2}H_j + \frac{1}{\varepsilon_1\mu_3}\frac{\partial^2}{\partial y^2}H_j + \frac{1}{\varepsilon_1\mu_2}\frac{\partial^2}{\partial z^2}H_j = -k_0^2 H_j. \tag{1.9}$$

The evident feature of the vector $\vec{E}$ for a wave of the TM-type satisfies the equation

$$\frac{1}{\varepsilon_3\mu_2}\frac{\partial^2}{\partial x^2}E_j + \frac{1}{\varepsilon_3\mu_1}\frac{\partial^2}{\partial y^2}E_j + \frac{1}{\varepsilon_1\mu_2}\frac{\partial^2}{\partial z^2}E_j = -k_0^2 E_j \tag{1.10}$$

(see Przeździecki [5]).

We want to show that the formula (1.6) can be transformed into

$$E_j = \oiint \vec{n} \cdot \operatorname{curl} W_j^{(E)} df,$$

$$H_j = \oiint \vec{n} \cdot \operatorname{curl} W_j^{(H)} df, \tag{1.11}$$

where the vectors $\vec{W}_j^{(E)}$ and $\vec{W}_j^{(H)}$ are the j's columns of the tensor potentials for an electromagnetic field in the medium examined.

The method applied in this work is similar to that used by Petykiewicz and Rynkowski [1], who only considered the case of the uniaxial electrical medium.

We resolve any arbitrarily polarized incident wave into two waves, one of which is a wave of the TE-type and the other's a wave of TM-type

$$\vec{E} = \vec{E}^{\mathrm{TE}} + \vec{E}^{\mathrm{TM}}, \qquad \vec{H} = \vec{H}^{\mathrm{TE}} + \vec{H}^{\mathrm{TM}}, \tag{1.12}$$

and every case of the field TE or TM can be treated separately

a) in the case of the incident of the field of the TE-type, we find the field $\vec{H}_{\mathrm{I}}$ (using the expressions (1.6b)) and the suitable electrical field is computed using Maxwell's equations,

b) in the case of the incident of the field of the TM-type, we find the field $\vec{E}_{\mathrm{II}}$ (using the expressions (1.6a)) and the suitable magnetic field is computed using Maxwell's equations.

The arbitrary field is found by superposition of these two fields, which can be written in the form:

$$\vec{E}(P) = \vec{E}_{\mathrm{II}}(P) + \frac{j}{\omega_0\varepsilon_0}\vec{\vec{\varepsilon}}^{\,-1}\operatorname{curl}_P \vec{H}_{\mathrm{I}}. \tag{1.13}$$

2. The Kottler's formulation of the Larmor-Lorentz principle

In this paragraph we present the transformation of the Larmor–Lorentz Principle (1.6) to the form (1.11) and to obtain the tensors of wave potentials $\vec{\vec{W}}^{(H)}$ and $\vec{\vec{W}}^{(E)}$.

When the field in the medium is of the TE-type, the Larmor–Lorentz Principle gives us the magnetic field $\vec{H}_{\mathrm{I}}$ at point P in the form:

$$H_{\mathrm{I}l} = \oiint df\{\vec{H}^{\mathrm{TE}} \cdot (\vec{n} \times \vec{\vec{\varepsilon}}^{-1} \cdot \operatorname{curl} G_l^{(H)}) - \vec{G}_l^{(H)} \cdot (n \times \vec{\vec{\varepsilon}}^{-1} \cdot \operatorname{curl} H^{\mathrm{TE}})\}, \tag{2.1}$$

where the subscript $l = 1, 2, 3$ defines the different components of the field. The evident feature of the vector $\vec{G}_l^{(H)}$ is the l's column of tensor $\vec{\vec{G}}^{(H)}$ (see formula (1.7)).

When the field in the medium is of the TM-type, the electric vector of a field E_{II} at the point P can be expressed using the Larmor–Lorentz principle in the form:

$$E_{\mathrm{III}}(P) = \oiint df\{-(\vec{n} \times \vec{E}^{\mathrm{TM}}) \cdot \vec{\vec{\mu}}^{-1} \cdot \operatorname{curl} \vec{G}_l^{(E)} + (\vec{n} \times \vec{G}_l^{(E)}) \cdot \vec{\vec{\mu}}^{-1} \cdot \operatorname{curl} \vec{E}^{\mathrm{TM}}\}, \tag{2.2}$$

where the subscript $l = 1, 2, 3$ defines the different components of the field and the evident feature of the vector $\vec{G}_l^{(E)}$ is the l's column of tensor $\vec{\vec{G}}^{(E)}$ (see formula (1.7)).

We can write the expressions (2.1) and (2.2) using the common vector identities in the form

$$H_{\mathrm{I}l}(P) = ik_0\sigma_2 \oiint df\left[\frac{\exp(-ik_0R_\mu)}{R_\mu}(\vec{n} \cdot \vec{\vec{\mu}} \cdot \nabla)H_l^{\mathrm{TE}} - H_l^{\mathrm{TE}}(\vec{n} \cdot \vec{\vec{\mu}} \cdot \nabla)\frac{\exp(-ik_0R_\mu)}{R_\mu}\right]$$

$$+ \oiint df\vec{n} \cdot \operatorname{curl}\left\{\left[\vec{\vec{\mu}}\vec{H}^{\mathrm{TE}} \times \vec{l}\frac{\exp(-ik_0R_\mu)}{R_\mu}\right]\sigma_2 ik_0 + \frac{i\sigma_2}{k_0}\vec{\vec{\varepsilon}} \cdot \operatorname{curl} \vec{H}^{\mathrm{TE}}\frac{\partial}{\partial x_l}\left(\frac{\exp(-ik_0R_\mu)}{R_\mu}\right)\right.$$

$$\left. + \frac{\sigma_2}{\mu_2\varepsilon_1}\left[\vec{H}^{\mathrm{TE}}\frac{\partial\alpha_l^*}{\partial x_3} - (\vec{k} \cdot \vec{H}) \operatorname{grad} \alpha_l^* + \alpha_l^* \operatorname{curl}\frac{\partial H^{\mathrm{TE}}}{\partial x_3}\right]\right\}, \tag{2.3a}$$

and

$$E_{\mathrm{III}}^{\mathrm{TM}}(P) = ik_0\sigma_1 \oiint df\left[\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}(\vec{n} \cdot \vec{\vec{\varepsilon}} \cdot \nabla)E_l^{\mathrm{TM}} - E_l^{\mathrm{TM}}(\vec{n} \cdot \vec{\vec{\varepsilon}} \cdot \nabla)\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\right]$$

$$+ \oiint \vec{n} \cdot \operatorname{curl}\left\{\frac{i\sigma_1}{k_0}\left[(\vec{\vec{\mu}}^{-1} \operatorname{curl} \vec{E}^{\mathrm{TM}})\frac{\partial}{\partial x_l}\left(\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\right)\right]\right.$$

$$+ ik_0\sigma_1\left(\vec{\vec{\varepsilon}} \cdot \vec{E}^{\mathrm{TM}} \times \frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\vec{l}\right)$$

$$\left. + \frac{\sigma_1}{\mu_1\varepsilon_2}\left[\vec{E}^{\mathrm{TM}}\frac{\partial\alpha}{\partial x_3} - (\vec{k} \cdot \vec{E}^{\mathrm{TM}})(\operatorname{grad} \alpha_l) - \frac{\partial E^{\mathrm{TM}}}{\partial x_3}\alpha_l\right]\right\} df, \tag{2.3b}$$

where

$$\alpha_l = \frac{\vec{\varrho} \cdot (\vec{k} \times \vec{l})}{\varrho_\varepsilon^2} [\exp(-ik_0 R_\varepsilon) - \exp(-ik_0 R_\mu)],$$

$$\alpha_l^* = \frac{\vec{\varrho} \cdot (\vec{k} \times \vec{l})}{\varrho_\mu^2} [\exp(-ik_0 R_\mu) - \exp(-ik_0 R_\varepsilon)].$$

In the case of lack of lines of discontinuity on the integration surface, all integrals in the expression (2.3a) and (2.3b), which have the integrand from $\vec{n} \cdot$ curl... equal to zero and the field at the point of observation P can be expressed by the formula:

$$H_l^{\mathrm{TE}}(P) = ik_0\sigma_2 \oiint df \left[\frac{\exp(-ik_0 R_\mu)}{R_\mu} (\vec{n} \cdot \vec{\vec{\mu}} \cdot \nabla) H_l^{\mathrm{TE}} - H_l^{\mathrm{TE}} (\vec{n} \cdot \vec{\vec{\mu}} \cdot \nabla) \frac{\exp(-ik_0 R_\mu)}{R_\mu} \right], \tag{2.4a}$$

and

$$E_l^{\mathrm{TM}}(P) = ik_0\sigma \oiint df \left[\frac{\exp(-ik_0 R_\varepsilon)}{R_\varepsilon} (\vec{n} \cdot \vec{\vec{\varepsilon}} \cdot \nabla) E_l^{\mathrm{TM}} - E_l^{\mathrm{TM}} (\vec{n} \cdot \vec{\vec{\varepsilon}} \cdot \nabla) \frac{\exp(-ik_0 R_\varepsilon)}{R_\varepsilon} \right]. \tag{2.4b}$$

When the lines of discontinuity appear on the integration surface, as in the case of diffraction of a field of the TE-type on the object with the edge (see Kirchhoff's theory of diffraction [7]), then the field of the TE-type at the point of observation P will be

$$H_l^{\mathrm{TE}}(P) = ik_0\sigma_2 \oiint df \left[\frac{\exp(-ik_0 R_\mu)}{R_\mu} (\vec{n} \cdot \vec{\vec{\mu}} \cdot \nabla) H_l - H_l (\vec{n} \cdot \vec{\vec{\mu}} \cdot \nabla) \frac{\exp(-ik_0 R_\mu)}{R_\mu} \right] + \oiint \vec{n}\, \mathrm{curl}\, (\vec{S}_l) df, \tag{2.5a}$$

where

$$\vec{S}_l = \frac{i\sigma_2}{k_0} \left[(\vec{\vec{\varepsilon}}^{-1}\, \mathrm{curl}\, \vec{H}) \left(\frac{\partial}{\partial x_l} \frac{\exp(-ik_0 R_\mu)}{R_\mu} \right) \right] + ik_0\sigma_2 \left[\vec{\vec{\mu}} \vec{H} \times \vec{l} \frac{\exp(-ik_0 R_\mu)}{R_\mu} \right]$$

$$+ \frac{\sigma_2}{\mu_2 \varepsilon_1} \left\{ \vec{H} \frac{\partial}{\partial x_3} \left[\frac{(\vec{\varrho} \times \vec{k}) \cdot \vec{l} (\exp(-ik_0 R_\mu) - \exp(-ik_0 R_\varepsilon))}{\varrho_\mu^2} \right] \right.$$

$$- (\vec{k} \cdot \vec{H})\, \mathrm{grad} \left[\frac{(\vec{\varrho} \times \vec{k}) \cdot \vec{l} (\exp(-ik_0 R_\mu) - \exp(-ik_0 R_\varepsilon))}{\varrho_\mu^2} \right]$$

$$\left. - \frac{\partial \vec{H}}{\partial x_3} \left[\frac{(\vec{\varrho} \times \vec{k}) \cdot \vec{l} (\exp(-ik_0 R_\mu) - \exp(-ik_0 R_\varepsilon))}{\varrho_\mu^2} \right] \right\}.$$

For the case of the TM-type, we obtain:

$$E_l^{\mathrm{TM}}(P) = ik_0\sigma \oiint df\left[\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}(\vec{n}\cdot\vec{\vec{\varepsilon}}\cdot\nabla)E_l - E_l(\vec{n}\cdot\vec{\vec{\varepsilon}}\cdot\nabla)\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\right]$$

$$+\oiint \vec{n}\,\mathrm{curl}\,(\vec{R}_l)df, \tag{2.5b}$$

where

$$\vec{R}_l = \frac{i\sigma_1}{k_0}\left[(\vec{\vec{\mu}}^{-1}\,\mathrm{curl}\,\vec{E}\frac{\partial}{\partial x_l}\left(\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\right)\right] + ik_0\sigma_1\left(\vec{\vec{\varepsilon}}\vec{E}\times\vec{l}\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\right)$$

$$+\frac{\sigma_1}{\mu_1\varepsilon_2}\left\{\vec{E}\frac{\partial}{\partial x_3}\left[\frac{(\vec{\varrho}\times\vec{k})\cdot\vec{l}(\exp(-ik_0R_\varepsilon)-\exp(-ik_0R_\mu))}{\varrho_E^2}\right]\right.$$

$$-(\vec{k}\cdot\vec{E})\,\mathrm{grad}\left[\frac{(\vec{\varrho}\times\vec{k})\cdot\vec{l}(\exp(-ik_0R_\varepsilon)-\exp(-ik_0R_\mu))}{\varrho_E^2}\right]$$

$$\left.-\frac{\partial\vec{E}}{\partial x_3}\left[\frac{(\vec{\varrho}\times\vec{k})\cdot\vec{l}(\exp(-ik_0R_\varepsilon)-\exp(-ik_0R_\mu))}{\varrho_E^2}\right]\right\}.$$

The expressions (2.5a) and (2.5b) are connected with Kottler's formulae [6] which have been formulated in Kirchhoff's theory of diffraction of the electromagnetic waves for an isotropic medium.

The expressions (2.5a) and (2.5b) are extensions of Kottler's formulae for an anisotropic medium and for the uniaxial electrical anisotropy are equivalent to those derived by Petykiewicz and Rynkowski [1].

Kottler's form of the Larmor–Lorentz principle is very useful in Kirchhoff's theory of diffraction, since it provides a very simple interpretation of its results according to the Young–Rubinowicz [7] model of diffraction phenomena. This is mainly due to the fact that Kottler's form of the Larmor–Lorentz principle permits one to obtain in a very simple way the wave potential of Miyamoto and Wolf [8], which in turn is the basis for this model.

3. The tensor potential of the Miyamoto and Wolf type

By introducing the tensor potential we can present the Larmor–Lorentz principle in the form

$$E_l^{\mathrm{TM}}(P) = \oiint_S \vec{n}\cdot\mathrm{curl}\,\vec{W}_l^{(\mathrm{TM})}df \tag{3.1}$$

if the incident field is of the TM-type and in the form

$$H_l^{\mathrm{TE}}(P) = \oiint_S \vec{n}\cdot\mathrm{curl}\,\vec{W}_l^{(\mathrm{TE})}df, \tag{3.2}$$

when the incident field is of the TE-type.

The integrands in (2.5a) and (2.5b) are already partially presented in the required form while only the integrals (2.4a) and (2.4b) should be transformed. But in this case we use the results of [8], and these integrands can be shown in the forms

$$\left[\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}(\vec{n}\cdot\vec{\vec{\varepsilon}}\cdot\nabla)E_l^{\mathrm{TM}}-\vec{E}_l^{\mathrm{TM}}(\vec{n}\cdot\vec{\vec{\varepsilon}}\cdot\nabla)\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\right]=\vec{n}\cdot\operatorname{curl}(\vec{W}_l^{(E)}) \quad (3.3a)$$

and

$$\left[\frac{\exp(-ik_0R_\mu)}{R_\mu}(\vec{n}\cdot\vec{\vec{\mu}}\cdot\nabla)H_l^{\mathrm{TE}}-H_l^{\mathrm{TE}}(\vec{n}\cdot\vec{\vec{\mu}}\cdot\nabla)\frac{\exp(-ik_0R_\mu)}{R_\mu}\right]=\vec{n}\cdot\operatorname{curl}(\vec{W}_l^{(H)}), \quad (3.3b)$$

where

$$\vec{W}_l^{(E)}=\frac{1}{4\pi}\sqrt{\frac{\gamma_\varepsilon}{\gamma_0}}\,|\vec{\vec{\mu}}|^{1/2}\vec{\vec{\varepsilon}}^{1/2}\left\{\nabla R_\varepsilon\times\frac{1}{R_\varepsilon}\int_{R_\varepsilon}^{\infty}\frac{\exp(-ik_0R_\varepsilon)}{R_\varepsilon}\nabla(E_l)dR_\varepsilon\right\}, \quad (3.4)$$

and

$$\vec{W}_l^{(H)}=\frac{1}{4\pi}\sqrt{\gamma_0}\,\gamma_0|\vec{\vec{\mu}}|^{1/2}\vec{\vec{\mu}}^{1/2}\left\{\nabla R_\mu\times\frac{1}{R_\mu}\int_{R_\mu}^{\infty}\frac{\exp(-ik_0R_\mu)}{R_\mu}\nabla(H_l)dR_\mu\right\}. \quad (3.5)$$

The tensor $\vec{\vec{\varepsilon}}^{1/2}$ (or $\vec{\vec{\mu}}^{1/2}$) has a property that $\vec{\vec{\varepsilon}}^{1/2}\cdot\vec{\vec{\varepsilon}}^{1/2}=\varepsilon$ (or $\vec{\vec{\mu}}^{1/2}\cdot\vec{\vec{\mu}}^{1/2}=\mu$).

Thus, when the electromagnetic field is of the TM-type, then the explicit expression of the potential for l-component of field can be written in the form

$$\vec{W}_l^{(\mathrm{TM})}=\vec{W}_l^{(E)}+\vec{R}_l, \quad (3.6)$$

where $\vec{R}_l$ is defined by (2.5b). When the field in the medium is of the TE-type, then the explicit expression of the potential for the l-component of the field is given by following expressions:

$$\vec{W}_l^{(\mathrm{TE})}=\vec{S}_l+\vec{W}_l^{(H)}, \quad (3.7)$$

where $\vec{S}_l$ is defined by (2.5a). The above formula may be interpreted in the Young–Rubinowicz's model of diffracted phenomena [9]. This model gives a possibility to present the diffraction field as a superposition of the so-called diffracted waves arising as a result of the reflection of the incidence field from the edge and the so-called geometrical waves.

By introducing the tensor potential into the Larmor–Lorentz principle integral, we can present the results of Kirchhoff's diffraction theory as the sum of the curvillinear integrals

$$E_l^{\mathrm{TM}}(P)=\oint_D\vec{W}^{\mathrm{TM}}\cdot d\vec{s}+\sum_i\oint_{C_i}\vec{W}^{\mathrm{TM}}\cdot d\vec{s} \quad (3.8)$$

in the case of the field of TM-type and as

$$H_l^{\mathrm{TM}}(P)=\oint_D\vec{W}^{\mathrm{TE}}\cdot d\vec{s}+\sum_i\oint_{C_i}\vec{W}^{\mathrm{TE}}\cdot d\vec{s} \quad (3.9)$$

for a field of the TE-type.

The first of the integrals is taken along the diffracting edge D (diffracted waves) and the remaining integrals are calculated around the singular points of the potential $\vec{W}^{TM}$ (or $\vec{W}^{TE}$) lying on the integration surface (geometrical wave).

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