DOMAIN STRUCTURE PARAMETERS IN A UNIAXIAL FERROMAGNETIC SUPERCONDUCTOR

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By using the Ginzburg–Landau theory and micromagnetic theory of magnetism a magnetic domain structure for a finite sample of a uniaxial ferromagnetic superconductor is studied. The distribution of the magnetic induction vector in domain structure of the Landau-type is derived. The dependence of the domain wall width on the sample thickness and penetration depth is obtained.

1. *Introduction*

Coexistence of superconductivity and magnetic ordering has been a subject of many experimental and theoretical papers [3]. At first Mathias et al. demonstrated the existence of the superconducting phase transition (T_c) below the temperature of magnetic order (T_M) in $Gd_xCe_{1-x}Ru_2$ and $Gd_xY_{1-x}Os_2$ alloys [1–2]. Besides, several experiments gave evidence of coexistence of both these phenomena in the following alloys: Gd_xLa_{1-x} [5], $Tb_xCe_{1-x}Ru_2$ [6] and $Gd_xCe_{1-x}Ru_2$ [6], [7]. Up to now the main question to answer in connection with the problem of coexistence is a type of magnetic ordering. Steiner et al. [16] have pointed out the possibility of coexistence of the spin-glass type order with superconductivity in Eu_xLa_{1-x}. Barz [17] has shown that the mixture $Mo_5SnGa_{0.5}S_6$ is a ferromagnetic superconductor. Recently, Fischer [8], Moncton [11], McCallum [10] and Ishikawa [9] have demonstrated experimentally that antiferromagnetic or ferromagnetic order can coexist with the superconductivity in Chevrel phases. The first theoretical investigation

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connected with superconductors which have magnetic ordering have been worked out by Gorkov and Rusinov [4] in the framework of the Abrikosov-Gorkov theory. Apart from microscopic attitudes towards the problem in [13–14] a very interesting phenomenological approach has been proposed by Krey [18]. This approach was applied to describe a ferromagnetic superconductor where among others the magnetic domain structure with asymptotic restrictions has been treated.

The main aim of our paper is to investigate the existence of the periodic domain structure in the finite ferromagnetic superconductor sample within the phenomenological approach [26].

2. *Free energy of a ferromagnetic superconductor*

The combination of the Ginzburg–Landau theory and micromagnetic theory of magnetism was previously applied by Krey [18] to the idealized one-dimensional magnetic domain structure of the Landau type (see Fig. 1) in a uniaxial ferromagnetic II type superconductor. However, the free energy functional has not contained the demagnetization energy produced by the demagnetizing field due to magnetic poles on the sample surfaces.

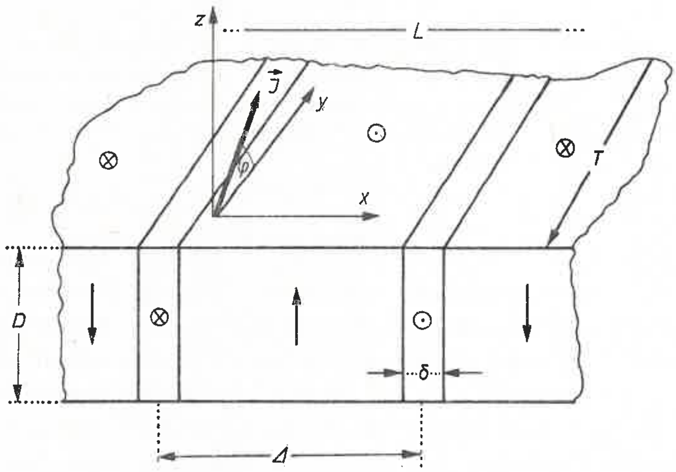

Fig. 1. The domain structure of the Landau type. Angle describes the rotation of the magnetization vector $\vec{J}$ in the sample, δ and Δ is width of the Bloch wall and domain, respectively and D, T and L denotes thickness, width and length of the sample, respectively

Odożyński [19] has recently taken demagnetization energy into account in the case of a pure ferromagnet and he obtained some formula describing the behaviour of the magnetization vector as a function of the position in the domain structure and the dependence of the domain width Δ on the sample thickness D. Let us consider the density of the demagnetization energy for the ferromagnetic superconductor similarly as in [19, 20] as follows

$$\varepsilon_d = 2^{-1} q_D \sin \varphi(x)\, \mathrm{sn}\,(2K(k)x/\Delta), \tag{1}$$

where $q_D = 8\pi^2\mu_0^{-1}J_s^2 \exp(-\pi D/2\Delta)$, μ_0 is the vacuum permeability, $J_s \equiv |\vec{J}|$ denotes the length of the magnetization vector $\vec{J} = J_s[0, \cos\varphi(x), \sin\varphi(x)]$, sn is the elliptic sine and $K(k)$ is the complete elliptic integral of the first kind and k denotes the elliptic modulus.

Besides, the density of the anisotropy and exchange energy, respectively, can be expressed in the following way

$$\varepsilon_a = \tilde{K}\cos^2\varphi(x), \tag{2}$$

$$\varepsilon_e = \tilde{A}\dot{\varphi}^2(x), \tag{3}$$

where $\tilde{A}$ denotes the exchange integral and $\tilde{K}$ is the anisotropy constant.

The three above-mentioned energies (1)–(3) are connected with a part of the free energy due to ferromagnetic ordering [19]. Now, we have to include to the total free energy of the system some additional terms due to the existence of the superconductivity. Namely, the condensation energy density ε_c which is assumed to be constant because the penetration depth λ is much larger than the coherence length ξ (and $B \approx \mu_0 H_{c1}$ where H_{c1} is so-called lower critical field in superconductor). Then, it means that the order parameter is practically independent of position in the sample.

The kinetic energy density of the superconducting currents can be written down as

$$\varepsilon_k = (2\mu_0)^{-1}\lambda^{-2}\{A_y^2(x)+A_z^2(x)\}. \tag{4}$$

Here, $\vec{A} \equiv [0, A_y(x), A_z(x)]$ denotes the vector potential of the magnetic field. Finally, it is necessary to include the magnetic energy density

$$\varepsilon_m = (2\mu_0)^{-1}\{[B_y(x)-J_s\cos\varphi(x)]^2+[B_z(x)-J_s\sin\varphi(x)]^2\}, \tag{5}$$

where $\vec{B} \equiv [0, B_y(x), B_z(x)]$ is the magnetic induction vector ($\vec{B} = \operatorname{rot}\vec{A}$). By combining Eqs. (1)–(5), we obtain the free energy functional of the ferromagnetic superconductor

$$F = 2TD\int_0^{\Delta/2}\Big\{\frac{1}{2\mu_0}[(\dot{A}_y(x)-J_s\sin\varphi(x))^2+(\dot{A}_z(x)+J_s\cos\varphi(x))^2$$
$$+\lambda^{-2}(A_y^2(x)+A_z^2(x))]+\tilde{K}\cos^2\varphi(x)+\tilde{A}\dot{\varphi}^2(x)-\varepsilon_c$$
$$+2^{-1}q_D\sin\varphi(x)\,\mathrm{sn}\,(2K(k)x/\Delta)\Big\}\,dx, \tag{6}$$

where T is the dimension of the sample in the plane perpendicular to the z-axis (Fig. 1).

3. *Solutions of minimization equations*

The minimization of the free energy functional with respect to the rotation angle $\varphi(x)$ of the magnetization vector and the vector potential $\vec{A}$ gives us

$$2\tilde{A}\ddot{\varphi}(x)+\tilde{K}\sin 2\varphi(x)-\tfrac{1}{2}q_D\cos\varphi(x)\,\mathrm{sn}\,(2K(k)x/\Delta)$$
$$= -\mu_0^{-1}J_s[\dot{A}_y(x)\cos\varphi(x)+\dot{A}_z(x)\sin\varphi(x)] \tag{7a}$$

$$\ddot{A}_y(x) - \lambda^{-2} A_y(x) = J_s \frac{d}{dx} \sin \varphi(x) \tag{7b}$$

$$\ddot{A}_z(x) - \lambda^{-2} A_z(x) = -J_s \frac{d}{dx} \cos \varphi(x). \tag{7c}$$

A rigorous solution to Eqs. (7) is practically impossible. Let us try to solve these equations by means of an iteration procedure. The first step is based on the assumption that the right hand side of Eq. (7a) is negligible (the following condition has to be fulfilled: $\pi \tilde{A} J_s^2 K \ll 4\mu_0 \lambda^2 \tilde{K}^2 k^3$) and Eq. (7a) has the form

$$2\tilde{A}\ddot{\varphi}(x) + \tilde{K} \sin 2\varphi(x) - 2^{-1} q_D \cos \varphi(x) \operatorname{sn}(2K(k)x/\Delta) = 0. \tag{8}$$

The solution to Eq. (8) is well-known [19]

$$\cos \varphi(x) = \operatorname{cn}(2K(k)x/\Delta), \qquad \sin \varphi(x) = \operatorname{sn}(2K(k)x/\Delta) \tag{9}$$

with the additional condition. (Condition (10) can be obtained in the following way: from Eq. (9) one has to calculate $\ddot{\varphi}$ and insert it together with solutions (9) into Eq. (8).)

$$4k^2 K^2(k) \Delta^{-2} = \tilde{K} \tilde{A}^{-1} (1 - q_D / 4\tilde{K}). \tag{10}$$

Furthermore, the so-called transversality condition [23] leads to the additional formula

$$4K^2(k)/\Delta^2 = \tilde{K}/\tilde{A} = r_0^{-2}. \tag{11}$$

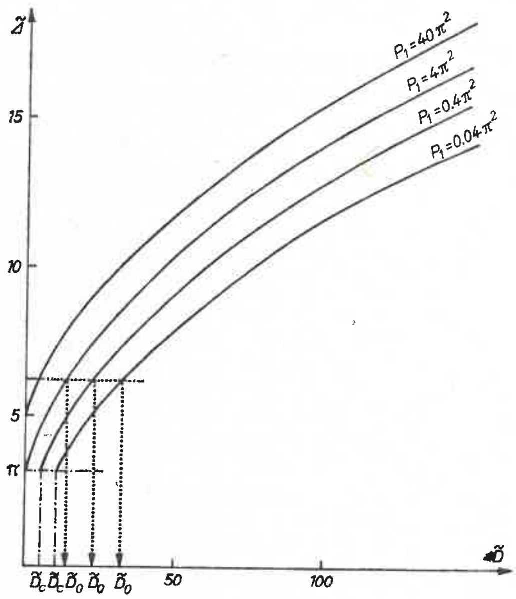

Fig. 2. The dependence of reduced domain width $\tilde{\Delta}$ on the reduced sample thickness $\tilde{D}$ for $(P_1/\pi^2) = 0.04$, 0.4, 4, 40. The (reduced) sample thickness $\tilde{D}_0$ corresponds to the elliptic modulus $k = k_0 = 0.9848$ and $\tilde{D}_c$ corresponds to $k = 0$

The dependence of the domain width Δ on the sample thickness D can be easily seen from Eqs. (1), (10) and (11). Some numerical results $\Delta(D)$ for various values of the parameter $P_1 = 2\mu_0\tilde{K}J_s^{-2}$ are shown in Fig. 2. Let us notice from Fig. 2 that the dependence $\Delta(D)$ holds for arbitrary thicknesses of the sample $0 < \tilde{D} = D/r_0 < \infty$ if $P_1 \geqslant 4\pi^2$ and $0 < \tilde{D}_c \leqslant \tilde{D} < \infty$ when $P_1 < 4\pi^2$ (a helical magnetic structure exists for $\tilde{D} = \tilde{D}_c$ (i.e., $k \to 0$)).

One way to solve Eqs. (7b), (7c) is to expand the elliptic functions in Fourier series [21]

$$\operatorname{sn}(2K(k)x/\Delta) = \frac{\pi}{K(k)k} \sum_{n=1}^{\infty} \frac{\sin\{(2n-1)\pi x/\Delta\}}{\sinh(2n-1)\pi K'(k)/2K(k)}, \tag{12}$$

$$\operatorname{cn}(2K(k)x/\Delta) = \frac{\pi}{K(k)k} \sum_{n=1}^{\infty} \frac{\cos\{(2n-1)\pi x/\Delta\}}{\cosh(2n-1)\pi K'(k)/2K(k)}, \tag{13}$$

where $K'(k) = K(k')$ and $k'^2 = 1-k^2$. Then, we obtain solutions of Eq. (7b) and (7c) as follows

$$A_y(x) = -\frac{\Delta J_s}{K(k)k} \sum_{n=1}^{\infty} \frac{(2n-1)}{(2n-1)^2+(\Delta/\pi\lambda)^2} \frac{\cos(2n-1)\pi x/\Delta}{\sinh(2n-1)\pi K'(k)/2K(k)},$$

$$A_z(x) = -\frac{\Delta J_s}{K(k)k} \sum_{n=1}^{\infty} \frac{(2n-1)}{(2n-1)^2+(\Delta/\pi\lambda)^2} \frac{\sin(2n-1)\pi x/\Delta}{\cosh(2n-1)\pi K'(k)/2K(k)}. \tag{14}$$

Formula (14) after simple calculations gives a concise and analytical expression for the magnetic induction:

$$-\frac{\dot{A}_z(x)}{J_s} = \frac{B_y(x)}{J_s} = \operatorname{cn}\frac{2K(k)x}{\Delta} + \frac{\pi\Delta}{4\lambda K(k)k} \frac{\sinh\left(\frac{|x|}{\lambda} - \frac{\Delta}{2\lambda}\right)}{\cosh \Delta K'(k)/2\lambda K(k) \cosh \Delta/2\lambda},$$

$$\frac{\dot{A}_y(x)}{J_s} = \frac{B_z(x)}{J_s} = \operatorname{sn}\frac{2K(k)x}{\Delta} + \frac{\pi\Delta}{4\lambda K(k)k} \frac{\cosh\left(\frac{|x|}{\lambda} - \frac{\Delta}{2\lambda}\right) - \cosh\frac{\Delta}{2\lambda}}{\sinh \Delta K'(k)/2\lambda K(k) \cosh \Delta/2\lambda}. \tag{15}$$

Some numerical calculations of the (reduced) magnetic induction components for the parameter $P_1 = 4\pi^2$ and two extreme values of the sample thickness D are presented in Figs. 3a, 4a ($\tilde{D} = 70$, $\tilde{\Delta} = \Delta/r_0 = 12$) and in Figs. 3b, 4b ($\tilde{D} = 1$, $\tilde{\Delta} = 3.5$). It is clearly seen from Figs. 3, 4 that the values of the magnetic induction components are reduced in comparison to the pure ferromagnet and the effect is more significant if the parameter $P_2 = r_0/\lambda$ is larger.

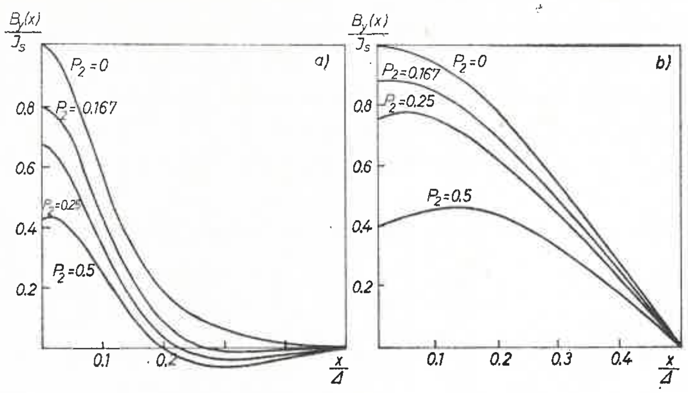

Fig. 3. The (reduced) magnetic induction $B_y(x)/J_s$ as a function of x/Δ in domain structure for $P_1 = 4\pi^2$ and $P_2 = 0$, 0.167, 0.25, 0.5 at $\tilde{D} = 70$ (Fig. 3a) and $\tilde{D} = 1$ (Fig. 3b), respectively

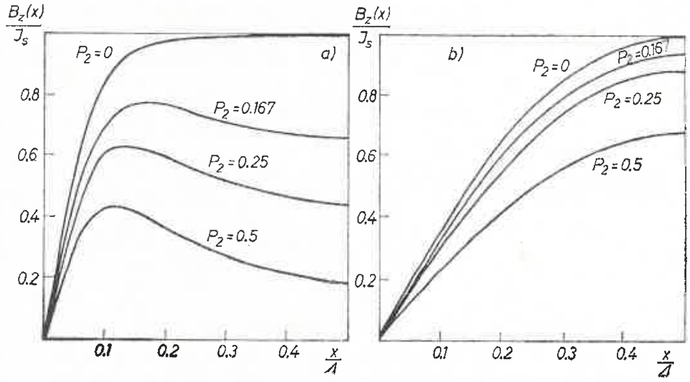

Fig. 4. The (reduced) magnetic induction $B_z(x)/J_s$ as a function of x/Δ in domain structure for $P_1 = 4\pi^2$ and $P_2 = 0$, 0.167, 0.25, 0.5 at $\tilde{D} = 70$ (Fig. 4a) and $\tilde{D} = 1$ (Fig. 4b), respectively

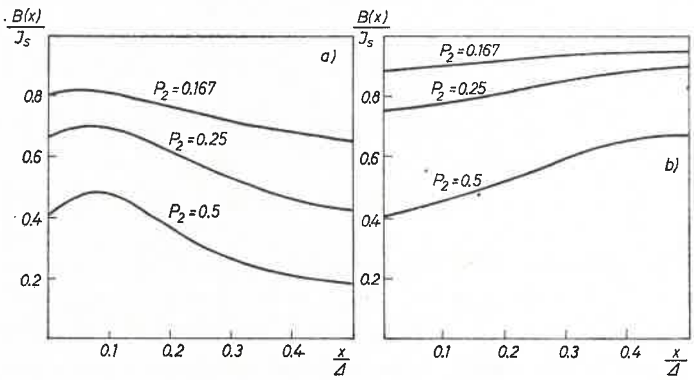

Fig. 5. The absolute value of the (reduced) magnetic induction $B(x)/J_s$ as a function of x/Δ in domain structure for $P_1 = 4\pi^2$ and $P_2 = 0.167$, 0.25, 0.5 at $\tilde{D} = 70$ (Fig. 5a) and $\tilde{D} = 1$ (Fig. 5b), respectively

Now let us calculate the quantity $\Delta B = B_y(0) - B_z(\Delta/2)$ by using Eq. (15)

$$\Delta B = \frac{\pi \Delta J_s \tanh(\Delta/2\lambda)}{4\lambda k K(k) \sin \Delta K'(k)/2\lambda K(k)} \left\{ \tanh \frac{\Delta}{4\lambda} - \tanh \frac{\Delta K'(k)}{2\lambda K(k)} \right\}. \quad (16)$$

Expression (16) is negative for $k < k_0$ where $k_0 \approx 0.9848$ satisfies the equation $2K'(k_0) = K(k_0)$. On the other hand, ΔB is positive for $k > k_0$. The examples of both extreme cases are plotted in Fig. 5b and 5a, respectively. The values of the sample thickness $\tilde{D}_0$ correspond to k_0.

4. Bloch walls in a ferromagnetic superconductor

According to the well-known definition, the width of the 180° Bloch wall, which is preferable in a rather thicker sample [24], has the form (see e.g. Fig. 6)

$$\delta = \pi |\dot{\varphi}(0)|^{-1}. \quad (17)$$

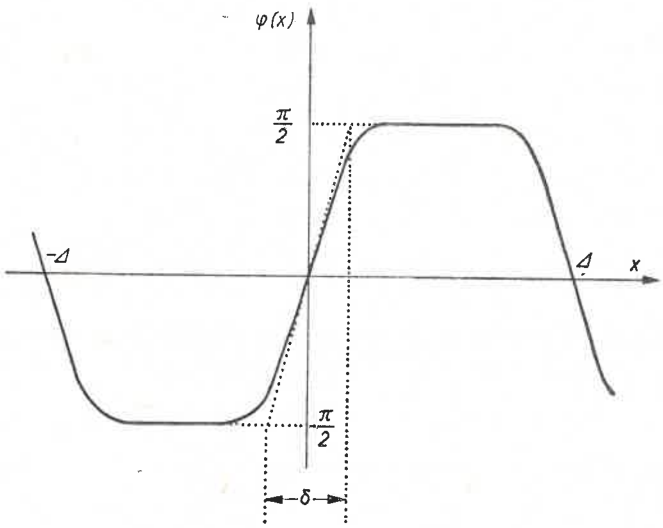

Fig. 6. The angle φ which describes the rotation of the magnetization vector $\vec{J}$ as a function of the position x in domain structure (Δ and δ denote respectively domain width and Bloch wall width)

By using Eqs. (9) and (11) we get $\dot{\varphi}_F(0) = r_0^{-1}$ and then the width of the Bloch wall in a pure ferromagnet [19] is

$$\delta_F = \pi r_0. \quad (18)$$

If we want to obtain the width of the Bloch wall in the ferromagnetic superconductor it is necessary to calculate $\dot{\varphi}_{SF}(x)$ in the next step of the iteration procedure of the solutions to Eqs. (7). By inserting Eqs. (12), (13) and (15) to the right-hand side of Eq. (7a) in the vicinity of the domain wall centre we have

$$\varphi_{SF}(x) = \varphi_F(x)+(2P_1)^{-1}\frac{\Delta}{2\lambda}\tanh\frac{\Delta}{2\lambda}\left[\lambda^{-1}\sinh^{-1}\frac{\Delta K'(k)}{2\lambda K(k)}\right.$$
$$\left.-2K(k)\Delta^{-1}\cosh^{-1}\frac{\Delta K'(k)}{2\lambda K(k)}\right]x. \tag{19}$$

Finally, by using Eqs. (11), (17) we obtain

$$\delta_{SF} = \delta_F\{1+(2P_1)^{-1}K(k)P_2^2\tanh(P_2K(k))\,[\sinh^{-1}(P_2K'(k))-P_2^{-1}\cosh^{-1}(K'(k)P_2)]\}^{-1}. \tag{20}$$

The numerical calculations δ_{SF}/δ_F as a function of the thickness $\tilde{D}$ are presented in Figs. 7a, 7b. The general conclusion which can be drawn from Figs. 7 concerns the fact

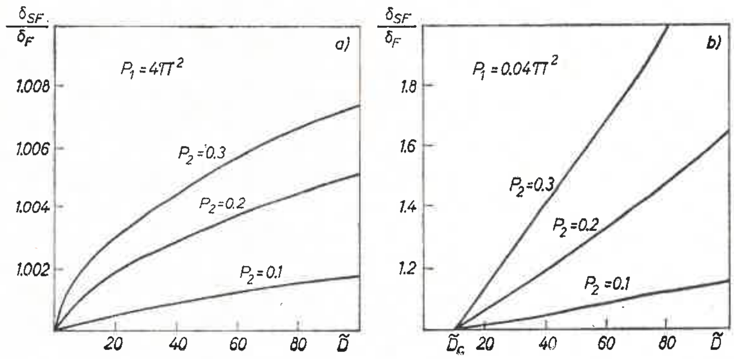

Fig. 7. Dependence of the ratio δ_{SF}/δ_F on the (reduced) thickness of the sample $\tilde{D}$ for $P_1 = 4\pi^2$ and $P_2 = 0.1, 0.2, 0.3$ (Fig. 7a) and for $P_1 = 0.04\pi^2$ and $P_2 = 0.1, 0.2, 0.3$ (Fig. 7b); $\tilde{D}_c$ corresponds to $k = 0$

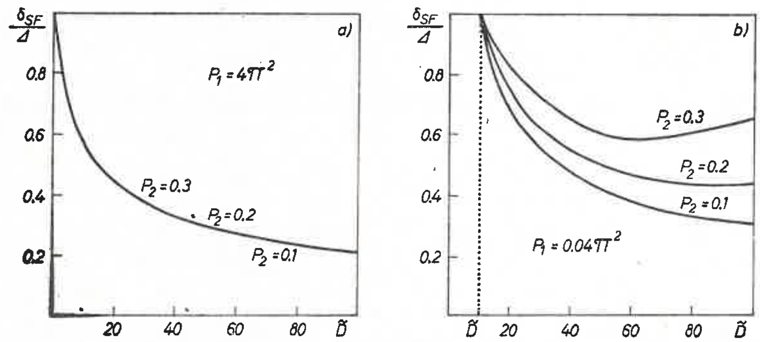

Fig. 8. Dependence of the ratio δ_{SF}/Δ on the (reduced) thickness $\tilde{D}$ for $P_1 = 4\pi^2$ and $P_2 = 0.1, 0.2, 0.3$ (Fig. 8a) and for $P_1 = 0.04\pi^2$ and $P_2 = 0.1, 0.2, 0.3$ (Fig. 8b); $\tilde{D}_c$ corresponds to $k = 0$

that the width of the Bloch walls in the ferromagnetic superconductor is larger than in pure ferromagnet. This effect becomes more significant in a rather thick sample and for larger P_2 and $P_1 \ll 4\pi^2$. The ratio δ_{SF}/Δ presented in Fig. 8a and 8b shows that the quantity δ_{SF} is comparable to Δ when the parameter P_2 is large. Then, it means that the magnetic arrangement is some type of helical structure.

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