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# INFLUENCE OF SAMPLE THICKNESS ON MAGNETIC DOMAIN STRUCTURE IN FERROMAGNETIC SUPERCONDUCTOR

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The phenomenological approach is applied to ferromagnetic superconductors with uniaxial anisotropy. The Bloch wall energy and the dependence of the domain width on sample thickness are obtained. The magnetic domain structure of the Landau-type (plate-like domain structure) and the chess-board pattern of the domains are considered and the comparison of its energies per unit area as a function of the sample thickness is presented. Also the influence of the sample thickness on reduced and averaged magnetic induction vector is shown.

## 1. *Introduction*

The coexistence of ferromagnetism and superconductivity has been considered in many theoretical papers [1–6]. Experimental evidence of this phenomenon has also been presented in [7, 8]. Although the microscopic mechanism of this coexistence is not yet known; it seems to us that from the successful application of the Ginzburg–Landau theory [9] in superconductivity and the micromagnetic theory in ferromagnetism, the combination of both these theories can yield a good approach towards the problem. Of course, the applicability of the theory is restricted to temperatures near the superconducting critical temperature $T_c$ and far away from the magnetic critical temperature $T_m$.

Generally, the free energy function depends on the vector potential $\vec{A}(\vec{B} = \mathrm{rot}\,\vec{A})$ and the magnetization vector $\vec{J}(0, J_s \cos\varphi, J_s \sin\varphi)$, $J_s = |\vec{J}|$ for the case of the uniaxial ferromagnetic superconductor of type II [10, 11].

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By substituting the solutions of the minimization equations in the free energy function it is possible to obtain among others the 180°-Bloch wall energy for a magnetic domain structure of the Landau-type. By taking into account the energy of Bloch walls in the unit cell of the periodic domain structure and the demagnetization energy produced by free

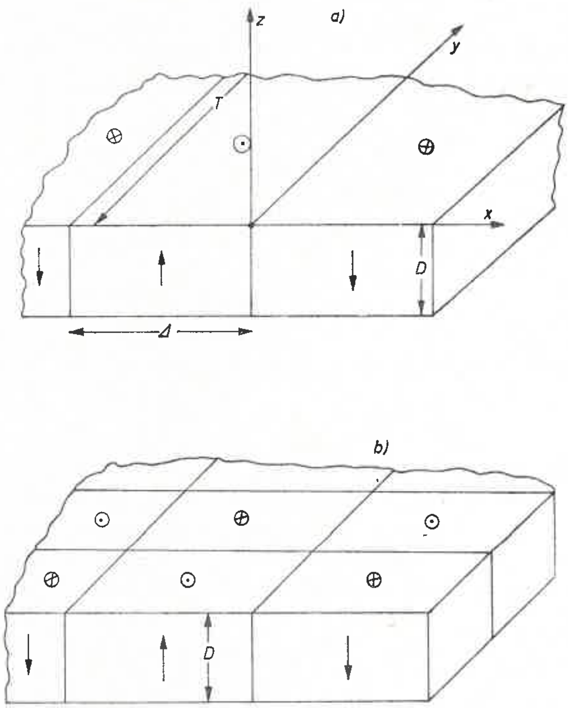


Fig. 1a) Landau-type domain structure. b) Chess-board-type domain structure

magnetic poles on the surface of the sample one can obtain the total energy of the ferromagnetic superconductor. The domain structure of the Landau-type L (Fig. 1a) and the chess-board-type C (Fig. 1b) are considered in the phenomenological approach and first of all the question of the stability of the system with respect to the domain width is studied at different values of sample thickness.

## 2. *The energy of the system*

### a. The energy of Bloch walls

The difference between the free energy of the ferromagnetic superconductor with domain structure $E_s[\varphi]$ and the free energy of the uniformly magnetized superconductor $E_s[\pi/2]$ can be treated as the energy of a single Bloch wall

$$E = E_s[\varphi] - E_s[\pi/2], \tag{1}$$

where

$$E_s[\varphi] = TD \int_{-\Delta/2}^{\Delta/2} \Big\{ \frac{1}{2\mu_0} [(B_z(x) - J_s \sin \varphi(x))^2 + (B_y(x) - J_s \cos \varphi(x))^2]$$

$$+ \lambda^{-2}(A_y^2(x) + A_z^2(x)) + \tilde{K} \cos^2 \varphi(x) + \tilde{A}\dot{\varphi}^2(x) + 2^{-1} q_D \sin^2 \varphi(x) - \varepsilon_c \Big\} dx, \tag{2a}$$

$$E_s[\pi/2] = TD(J_s^2/2\mu_0 + 2^{-1} q_D - \varepsilon_c) \tag{2b}$$

and $T$, $D$ are breadth and thickness of the sample respectively, $\Delta$ — the domain width, $\tilde{A}$ — the exchange integral, $\tilde{K}$ — the exchange anisotropy constant, $\mu_0$ — the vacuum permeability constant, $\varepsilon_c$ — the condensation energy, $\lambda$ — the penetration depth and $q_D = 4\tilde{K}(1-k^2)$, where $k$ denotes the elliptic modulus [13].

After standard and simple calculations it can be easily shown that the energy of single Bloch wall has the form [14]

$$E = TD\Delta \Big\{ \tilde{K}K^{-1}k^{-2} \Big[ (3k^2-1)E(k) + (1-2k^2)(1-k^2)K(k) - J_s^2/2\mu_0$$

$$+ (J_s^2/2\mu_0) \frac{\Delta}{2\lambda} \frac{\pi^2}{K^2(k)k^2} \frac{\tanh(\Delta/2\lambda)}{\tanh(\Delta K'(k)/\lambda K(k)) \sinh(\Delta K'(k)/\lambda K(k))}. \tag{3}$$

Here, $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and the second kind, respectively, and $K'(k) = K(k')$ where $k'^2 = 1-k^2$. Hence, for the asymptotic value of the Bloch wall energy (3) for the case of large domain width ($k \to 1$) we have

$$E = TD[4(\tilde{A}\tilde{K})^{1/2} - (J_s^2/2\mu_0)\Delta + (J_s^2/2\mu_0)(2\lambda)\tanh(\Delta/2\lambda)]. \tag{4}$$

The first term in Eq. (4) is the well-known energy of a Bloch wall in the normal ferromagnetic material and the next terms are due to the superconductivity. The number of Bloch walls in the crystal depends on the type of the domain structure and is equal to $\eta R/\Delta$ where

$$\eta = \begin{cases} 1 & \text{for Landau-type structure} \\ 2 & \text{for a chess-board-type structure} \end{cases} \tag{5}$$

and $R$ denotes the sample length (Fig. 1).

Consequently, the energy of Bloch walls per unit area of the sample can be written in the following way

$$\gamma_B = \eta D/\Delta[4\sqrt{\tilde{A}\tilde{K}} - (J_s^2\lambda/\mu_0)\{(\Delta/2\lambda) - \tanh(\Delta/2\lambda)\}]. \tag{6}$$

b. The demagnetization energy

In our case, we assume that the magnetization is practically constant in the domain and equal to its average value $\bar{J}$. The demagnetization energy per unit area can be expressed in the following way [12]

$$\gamma_D = (4\pi/\mu_0)\varrho\bar{J}^2\Delta, \tag{7}$$

where

$$\varrho = \begin{cases} 1.705 \text{ for Landau-type structure} \\ 1.060 \text{ for chess-board-type structure.} \end{cases} \tag{8}$$

According to the definition we have

$$\bar{J} = \bar{B}_z = 2/\Delta \int_0^{\Delta/2} B_z(x)dx. \tag{9}$$

It can be easily seen that the $z$-component of the magnetic induction resulting from the minimization of the free energy of the system (2a) has the form (see e.g., [14])

$$B_z = J_s \left[ \operatorname{sn}(2Kx/\Delta) + \frac{\pi\Delta}{4kK\lambda} \frac{\cosh(|x|/\lambda - \Delta/2\lambda) - \cosh(\Delta/2\lambda)}{\sinh(\Delta K'/2\lambda K)\cosh(\Delta/2\lambda)} \operatorname{sign} x \right], \tag{10}$$

where sn is the Jacobi elliptic function. Hence, the average value of the quantity (10) according to (9) gives us

$$\bar{J} = J_s \left[ \frac{1}{kK} \ln \sqrt{\frac{1+k}{1-k}} + \frac{\pi\Delta}{4kK} \frac{\lambda \sinh(\Delta/2\lambda) - (\Delta/2)\cosh(\Delta/2\lambda)}{\sinh(\Delta K'/2\lambda K)\cosh(\Delta/2\lambda)} \right]. \tag{11}$$

This formula becomes much simpler for $k$ tending to 1

$$\bar{J} = J_s \tanh(\Delta/2\lambda)/(\Delta/2). \tag{12}$$

Let us notice that the average value of the magnetization $\bar{J}$ reaches the maximum value $J_s$ when the penetration depth goes to infinity. Finally by inserting (12) into (7) we obtain the demagnetization energy

$$\gamma_D = 8\pi\varrho\Delta(J_s^2/2\mu_0)[\tanh(\Delta/2\lambda)/(\Delta/2\lambda)]^2. \tag{13}$$

### *3. The stability condition of the system*

In order to obtain the stability condition let us consider the total energy of the system as a function of the domain width $\Delta$ holding the thickness of the sample constant. This energy consists of two parts

$$\gamma_{tot} = \gamma_B + \gamma_D. \tag{14}$$

The explicit form of the total energy can be expressed by means of Eqs. (6) and (13) as follows

$$\gamma_{tot} = 8\pi\lambda\varrho J_s^2 \mu_0^{-1}(\varepsilon - \tilde{D}), \tag{15a}$$

where

$$\varepsilon = \tilde{\Delta}^{-1}\{\tanh^2 \tilde{\Delta} + (\alpha + \tanh \tilde{\Delta})\tilde{D}\}, \tag{15b}$$

$$\alpha = 4\mu_0 \sqrt{\tilde{A}\tilde{K}}/\lambda J_s^2, \quad \beta = \eta/16\pi\varrho, \quad \tilde{\Delta} = \Delta/2\lambda, \quad \tilde{D} = D\beta/\lambda. \tag{15c}$$

The necessary condition for the minimum of (15b) can be written in the following way

$$\tilde{D}\{\alpha - (\tilde{\Delta}\operatorname{sech}^2 \tilde{\Delta} - \tanh \tilde{\Delta})\} = \tanh \tilde{\Delta}(2\tilde{\Delta}\operatorname{sech}^2 \tilde{\Delta} - \tanh \tilde{\Delta}). \tag{16}$$

## *4. Numerical results and discussion*

The dependence of the total energy (15b) on the (reduced) domain width is plotted in Fig. 2 for different values of the parameter $\tilde{D} = 0.2, 0.5, 1$ and for $\alpha = 0.1$. In order to find a solution of Eq. (16) it is necessary to solve it numerically. The results for the chosen (reduced) value of the parameter $\alpha = 0.1$ and both domain structure (see Fig. 1) are pre-

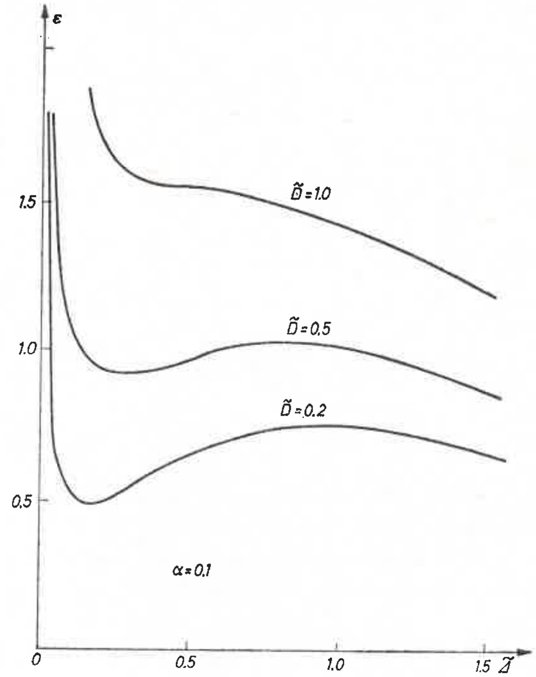


Fig. 2. Dependence of the reduced energy of the ferromagnetic superconductor on the (reduced) domain width for the selected values of parameters $\tilde{D} = 0.2, 0.5, 1.0$ and $\alpha = 0.1$

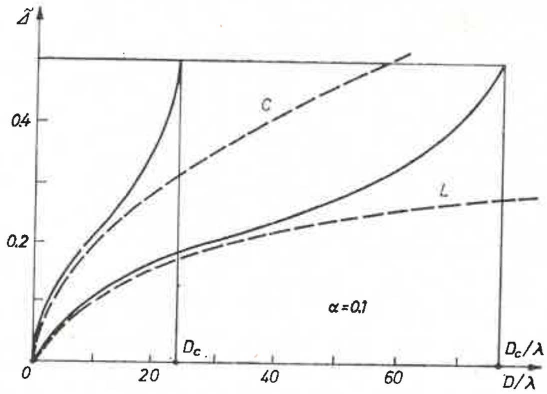


Fig. 3. Dependence of the domain width on the thickness of the sample (solid line) for the Landau-type (L) and the chess-board-type (C) domain structure, respectively. Dashed lines describe the same dependence for the normal ferromagnetic material

sented in Fig. 3. In the case when the domain width $\Delta$ is much smaller than the penetration depth $\lambda$ we have

$$\alpha\tilde{D} = \tilde{\Delta}^2. \tag{17}$$

This dependence is shown by dashed lines in Fig. 3. Finally, it is easy to see from Fig. 4 that both domain structures are stable up to the respective critical thickness of the sample $D_c$ and that the Landau-type domain structure is more favourable than the chess-board-type

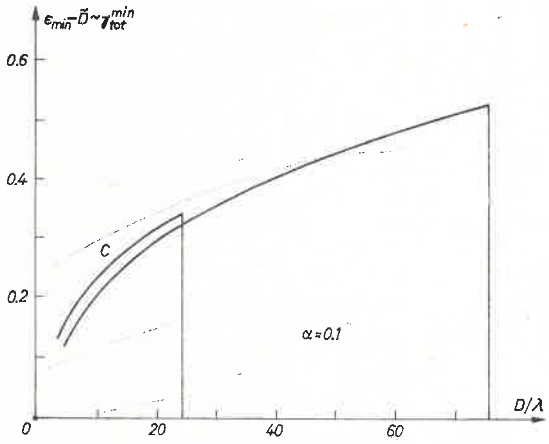


Fig. 4. Dependence of the (reduced) total energy of the ferromagnetic superconductor on the (reduced) thickness of the sample for C and L domain structures

one. It is necessary to emphasize that the phenomenological approach presented here is not valid for thin films.

By introducing normal regions it seems possible to us to extend the applicability of the above model to a larger thickness for the sample than $D_c$.

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