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COUPLING OF MAGNONS TO PLASMONS AND TO LA-PHONONS IN THE *s-d* MODEL*

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The possibility of interactions between magnons and plasmons as well as between magnons and longitudinal acoustical (LA) phonons was investigated in the concept of the *s-d* model. It was found in this model that the scattering of the magnon related to the creation (absorption) of the plasmon or LA-phonon is a process of the lowest order occurring in the ferromagnetic semiconductors.

1. *Introduction*

The large interest in magnetic semiconductors is due to their specific properties that depend on the large influence of the magnetic structure on the nonmagnetic properties such as the conductivity, light absorption etc. It is well known that this influence is due to the strong exchange interaction between the conduction electrons or the electrons localized in the donor states and the electrons localized in the partially filled *d* or *f* shells of the magnetic atom, i.e., to the so-called *s-d* or *s-f* exchange interaction [1, 2].

The magnetic semiconductors give a possibility to investigate the coupling between the magnetic excitations and the plasmons. Recently, a model was proposed for this coupling based on the spin-orbit interaction [3]. A similar mechanism was proposed for the magnon-phonon coupling [4]. It is shown here, that the coupling between magnons and plasmons and between magnons and LA-phonons can exist as a result of the *s-d* interaction. However, this interaction cannot give the hybridization of the interacting excitations recently predicted [5].

In Section 2 are given the contributions calculated for the magnon energy and for the lifetime due to the interaction with the electron system and with the LA-phonons. Some essential conclusions are discussed in Section 3.

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2. Contributions to the magnon energy and lifetime due to the interaction with the electron system

Let us consider a simple cubic ferromagnetic semiconductor (FS) with the wide conduction band; i.e., $W \gg AS$, where A is the *s-d* exchange parameter, S is the spin of the magnetic atom and W is the conduction band width. For simplicity we restrict the considerations to cases of the large spin. In the spin wave approximation a Hamiltonian of the FS has the following form [6]:

$$H = H_s + H_d + H_{s\text{-}d}, \tag{1}$$

where

$$H_s = \sum_{k,\sigma} \mathscr{E}_{k\sigma} a^\dagger_{k\sigma} a_{k\sigma}, \tag{2}$$

$$H_d = \sum_q E_q \alpha^\dagger_q \alpha_q, \tag{3}$$

$$H_{s\text{-}d} = -\left(\frac{2S}{N}\right)^{1/2} \sum_{p,k} A(k)\,(a^\dagger_{p-k\uparrow} a_{p\downarrow} \alpha^\dagger_k + a^\dagger_{p-k\downarrow} a_{p\uparrow} \alpha_{-k})$$

$$+\frac{1}{N} \sum_{p,k,k'} A(k-k')\,(a^\dagger_{p+k-k'\uparrow} a_{p\uparrow} - a^\dagger_{p+k-k'\downarrow} a_{p\downarrow}) \alpha^\dagger_{k'} \alpha_k. \tag{4}$$

H_s is the Hamiltonian of the conduction electron system with the diagonal part of the *s-d* exchange included. H_d denotes the Hamiltonian of the non-interacting magnon system and $H_{s\text{-}d}$ is the non-diagonal part of the *s-d* interaction. In Eqs. (2–4), $a_{k\sigma}$ and α_q ($a^\dagger_{k\sigma}$ and $\alpha^\dagger_q$) are the annihilation (creation) operators of the electron with the wave vector $\boldsymbol{k}$ and spin $\sigma = \uparrow, \downarrow$, and magnon with the wave vector $\boldsymbol{q}$, respectively. N denotes the total number of the magnetic atoms and $A(\boldsymbol{k})$ is the Fourier $\boldsymbol{k}$-component of the *s-d* exchange integral. The single electron energies, $\mathscr{E}_{k\sigma}$, are given by the expressions:

$$\mathscr{E}_{k\uparrow(\downarrow)} = \mathscr{E}_k \mp A(0)S, \tag{5}$$

where $\mathscr{E}_k$ is the Bloch energy. The upper sign in Eq. (5) refers to the case of $\uparrow$-spin and the lower one to the case of a $\downarrow$-spin.

The influence of the electron excitations on the magnon system will be investigated by calculating the lifetime and renormalized energy of the magnon. For this purpose we define the Green function $\langle\langle \alpha_q | \alpha^\dagger_{q'} \rangle\rangle$ [6, 7]. Taking into account the equations of motion for this and higher order Green functions one can find that:

$$\langle\langle \alpha_q | \alpha^\dagger_{q'} \rangle\rangle_E = \frac{i}{2\pi}\,\delta_{q,q'} \Bigg\{ E - E_q - \Delta - \frac{2S}{N}\,|A(q)|^2 \mathscr{R}(q, E)$$

$$- \frac{1}{N^2} \sum_{k(\neq q)} |A(k-q)|^2 N_k [\mathscr{K}^+(q-k, E-\bar{E}_k) + \mathscr{K}^-(q-k, E-\bar{E}_k)] \Bigg\}^{-1}, \tag{6}$$

where

$$\Delta = \frac{A(0)}{N}\sum_{p}(n_{p\uparrow}-n_{p\downarrow}), \tag{7}$$

$$\mathscr{R}(q, E) = \sum_{p}\frac{n_{p-q\uparrow}-n_{p\downarrow}}{E+\bar{\mathscr{E}}_{p-q\uparrow}-\bar{\mathscr{E}}_{p\downarrow}}, \tag{8}$$

$$\mathscr{K}^{+(-)}(k-q, E-\bar{E}_k) = \sum\frac{n_{p+k-q\uparrow(\downarrow)}-n_{p\uparrow(\downarrow)}}{E+\bar{\mathscr{E}}_{p+k-q\uparrow(\downarrow)}-\bar{\mathscr{E}}_{p\uparrow(\downarrow)}-\bar{E}}, \tag{9}$$

$$\bar{\mathscr{E}}_{p\uparrow(\downarrow)} = \mathscr{E}_{p\uparrow(\downarrow)}\pm\frac{A(0)}{N}\sum_{k}N_k, \tag{10}$$

$$\bar{E}_k = E_k+\Delta+\frac{2S}{N}|A(k)|^2\mathscr{R}(k, \bar{E}_k). \tag{11}$$

In Eqs. (6–9) N_k and $n_{k\sigma}$ denote the occupation numbers of the magnons and electrons, respectively.

The interaction between the electrons was neglected in the considerations above. However, since the existence of the collective excitations in the electron system is a consequence of this interaction we must take it into account. This can be done in the following manner. Using the Green function method and the decoupling procedure which was used when deriving Eq. (6), one can show that the charge density fluctuation $\varrho_{k\sigma} = \sum_{p}\langle a^{\dagger}_{p+k\sigma}a_{p\sigma}\rangle$ is proportional to the $\mathscr{K}$, i.e.,

$$\varrho_{k\uparrow(\downarrow)} \sim \sum_{k'}A(k)\mathscr{K}^{+(-)}(k, \bar{E}_{k'}-\bar{E}_{k'+k}). \tag{12}$$

Such a fluctuation is screened due to the Coulomb interaction and this screening can be taken into account by introducing the dynamical dielectric constant $\varepsilon(k, \omega)$, calculated in the RPA [8]. Thus we can write:

$$\varrho_{k\uparrow(\downarrow)} \sim \sum_{k'}A(k)\frac{\mathscr{K}^{+(-)}(k, \bar{E}_{k'}-\bar{E}_{k'+k})}{\varepsilon(k, \hbar^{-1}\bar{E}_{k'}-\hbar^{-1}\bar{E}_{k'+k})}. \tag{13}$$

The Coulomb interaction will be taken into account, according to the above, if we replace $\mathscr{K}^{+(-)}(k, E)$ in Eq. (5) by $\mathscr{K}^{+(-)}(k, E)/\varepsilon(k, \hbar^{-1}E)$ i.e.

$$\langle\langle\alpha_q|\alpha^{\dagger}_{q'}\rangle\rangle = \frac{i}{2\pi}\delta_{q,q'}\Big\{E-E_q-\Delta-\frac{2S}{N}|A(q)|^2\mathscr{R}(q, E)$$

$$-\frac{1}{N^2}\sum_{k(\neq q)}|A(k-q)|^2N_k\frac{\mathscr{K}^{+}(q-k, E-\bar{E}_k)+\mathscr{K}^{-}(q-k, E-\bar{E}_k)}{\varepsilon(q-k, \hbar^{-1}E-\hbar^{-1}\bar{E}_k)}\Big\}^{-1}. \tag{14}$$

The poles of the Green function above give the renormalized energy $\bar{\bar{E}}_q$ and lifetime τ_q of the magnon:

$$\bar{\bar{E}}_q = E_q + \Delta + \frac{2S}{N}|A(\boldsymbol{q})|^2 \mathscr{P}\{\mathscr{R}(\boldsymbol{q}, \bar{\bar{E}}_q)\} + \frac{1}{N^2}\mathscr{P}\sum_{\boldsymbol{k}(\neq \boldsymbol{q})} |A(\boldsymbol{k}-\boldsymbol{q})|^2$$

$$\times N_{\boldsymbol{k}} \frac{\mathscr{K}^+(\boldsymbol{q}-\boldsymbol{k}, \bar{\bar{E}}_q - \bar{E}_k) + \mathscr{K}^-(\boldsymbol{q}-\boldsymbol{k}, \bar{\bar{E}}_q - \bar{E}_k)}{\varepsilon(\boldsymbol{q}-\boldsymbol{k}, \hbar^{-1}\bar{\bar{E}}_q - \hbar^{-1}\bar{E}_k)}, \tag{15}$$

$$\hbar\tau_q^{-1} = \pi\frac{2S}{N}|A(\boldsymbol{q})|^2 \sum_{\boldsymbol{p}} (n_{\boldsymbol{p}-\boldsymbol{q}\uparrow} - n_{\boldsymbol{p}\downarrow})\delta(\bar{\bar{E}}_q - \bar{\mathscr{E}}_{\boldsymbol{p}-\boldsymbol{q}\uparrow} - \bar{\mathscr{E}}_{\boldsymbol{p}\downarrow})$$

$$+\frac{\pi}{N^2}\mathscr{P}\sum_{\boldsymbol{k}(\neq \boldsymbol{q})} \frac{|A(\boldsymbol{k}-\boldsymbol{q})|^2 N_{\boldsymbol{k}}}{\varepsilon(\boldsymbol{q}-\boldsymbol{k}, \hbar^{-1}\bar{\bar{E}}_q - \hbar^{-1}\bar{E}_k)} \sum_{\boldsymbol{p},\sigma} (n_{\boldsymbol{p}+\boldsymbol{k}-\boldsymbol{q}\sigma} - n_{\boldsymbol{p}\sigma})\delta(\bar{\bar{E}}_q + \bar{\mathscr{E}}_{\boldsymbol{p}+\boldsymbol{k}-\boldsymbol{q}\sigma} - \bar{\mathscr{E}}_{\boldsymbol{p}\sigma} - \bar{E}_k)$$

$$+\frac{\pi}{N^2}\sum_{\boldsymbol{k}(\neq \boldsymbol{q})} |A(\boldsymbol{k}-\boldsymbol{q})|^2 N_{\boldsymbol{k}} \mathscr{P}\{\mathscr{K}^+(\boldsymbol{q}-\boldsymbol{k}, \bar{\bar{E}}_q - \bar{E}_k) + \mathscr{K}^-(\boldsymbol{q}-\boldsymbol{k}, \bar{\bar{E}}_q - \bar{E}_k)\}$$

$$\times \left\{ \frac{\partial\varepsilon(\boldsymbol{q}-\boldsymbol{k}, \omega)}{\partial\omega}\bigg|_{\omega=\omega^c_{\boldsymbol{q}-\boldsymbol{k}}} \right\}^{-1} \delta(\bar{\bar{E}}_q - \bar{E}_k - \hbar\omega^c_{\boldsymbol{q}-\boldsymbol{k}}), \tag{16}$$

where $\mathscr{P}$ denotes the principal value. When deriving Eqs. (15, 16) we have assumed $\varepsilon(\boldsymbol{k}, \omega)$ to be a real function. If we restrict ourselves to the case of the pure electron system, i.e., if we neglect a contribution to the dielectric constant due to the ionic motion, then (in the RPA) this assumption is fulfilled in the region where plasmons exist as a well defined excitations [8]. In the general case $\varepsilon(\boldsymbol{k}, \omega)$ contains a contribution due to the ionic system and is a complex function. For simplicity we neglected the imaginary part of the $\varepsilon(\boldsymbol{k}, \omega)$

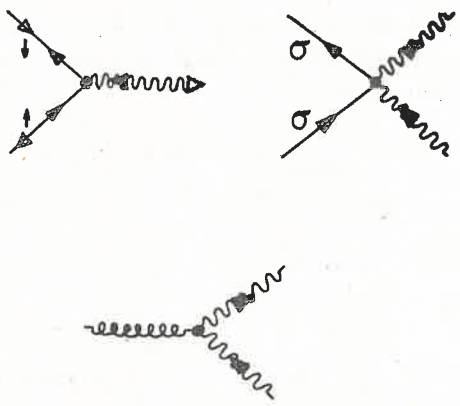

Fig. 1. Graph of the processes giving contributions to the energy and to the lifetime of the magnon. Solid line represents the electron, wavy line — the magnon and the helical line represents the plasmon (LA-phonon). The arrows indicate absorption (incoming line) and creation (outgoing line) of the appropriate excitation. No arrow denotes that both directions are possible

what is equivalent to the neglect of the damping of the collective excitations (plasmons and phonons). In Eq. (16) ω_q^c is defined as a solution of the equation:

$$\varepsilon(\boldsymbol{q}, \omega_q^c) = 0, \tag{17}$$

i.e., as the frequency of the above-mentioned collective excitations. There are two different solutions of this equation for the sufficiently small wave vector $\boldsymbol{q}$. One is the longitudinal acoustical phonon and second is the plasmon.

The contributions to the magnon energy and to the magnon lifetime given by Eqs. (15, 16) are due to the processes shown in Fig. 1. It is visible that in the FS do not exist the processes responsible for the hybridization, i.e., the processes with one magnon line incoming (outgoing) and one plasmon or phonon line outgoing (incoming).

3. *Conclusions*

After calculating the contributions to the magnon energy and to the magnon lifetime we have concluded that the *s-d* interaction cannot lead to the magnon-plasmon and to the magnon — LA-phonon hybridizations.

A similar result for the contributions of the first two processes in Fig. 1 is given in [6] where, however, the diagonal part of the *s-d* interaction was not included into the unperturbed Hamiltonian and the screening was not taken into account. We are mainly interested in those contributions which are due to the interaction between magnons and plasmons and between magnons and LA-phonons. Such contributions were calculated for the processes of the lowest order in the magnon-plasmon and in the magnon — LA-phonon systems, i.e., for the scattering of the magnon related to the creation (absorption) of the plasmon or LA-phonon. A lack of the processes responsible for the hybridization seems to be reasonable because of the form of the Hamiltonian $H_{s\text{-}d}$. This Hamiltonian conserves the total spin of the magnon-electron system and thus it cannot lead to any process in which the total spin is changed. It is easy to notice that the processes responsible for the hybridization are processes changing the total spin. Thus to obtain the hybridization, it is necessarily to include other interactions such as the spin-orbit coupling.

A similar situation exists in antiferromagnetic semiconductors. The *s-d* interaction also cannot lead to the hybridization, but taking into account the form of this interaction, one can conclude that now it is possible a plasmon (LA-phonon) — two magnon hybridization which is not possible in the case of the ferromagnetic semiconductors.

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