DISCUSSION OF METHOD OF SEPARATION OF THE ELECTRONIC AND LATTICE COMPONENTS OF THERMAL CONDUCTIVITY FOR BRASS

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The separation of thermal conductivity components for eight brass samples with zinc concentration from 3 to 37% has been conducted. The separation was made by four different methods proposed in scientific literature, which use different physical assumptions. The four above mentioned methods were discussed and the one considered to be the most physically correct was chosen.

1. Introduction

Scientists occupied with the separation methods of thermal conductivity components give different points of view in the investigation of strongly doped metals and alloys.

The problem of the separation of the electronic component λ_e and lattice λ_l consists in finding λ_e and λ_l independently by using the known total thermal conductivity value λ:

$$\lambda = \lambda_e + \lambda_l \tag{1}$$

and some auxiliary measurements. Lattice component λ_l can be designated with good accuracy only in the case when λ_l constitutes a considerable part of λ. It is necessary to diminish considerably λ_e and in this case λ_e will be comparable with λ_l. This can be realised by three methods:

(*i*) diminishing of λ_e by use of a magnetic field,

(*ii*) diminishing of λ_e by transition in the superconducting state,

(*iii*) diminishing of λ_e by doping of the samples.

This paper is a continuation of research published in [1, 2]. The samples investigated in this paper have been carefully described in [2]. The separation of brass thermal conductivity components discussed in [2] has been done in this paper by four different methods proposed by different authors according to theoretical models.

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2. The methods of separation of brass thermal conductivity components

2.1. The ideal thermal resistance of an alloy and basic metal is equal

This method used by Klemens et al. [3–5] is based on the assumption that ideal electronic thermal resistance W_i for CuZn alloy is W_i for a pure basic metal in an alloy, in this case copper. This assumption is applied not only for diluted brass alloys but also for alloys containing 32% of zinc [4]. The ideal electronic thermal resistance of a metal is designated accordingly by Cezairliyan [6] as:

$$W_i = \alpha \cdot T^n, \tag{2}$$

where

$$\alpha = \alpha_0 \left(\frac{\beta}{n\alpha_0}\right)^{\frac{m-n}{m+1}} \tag{3}$$

and α_0, n, m are constants for a given metal. For copper according to Thermophysical Properties Research Center [7] data:

$$\alpha_0 = 0.0423 \cdot 10^{-4} \text{ cm K}^{(n-m-mn+1)/n+1} \text{ W}^{-1},$$

$$m = 2.63, \quad n = 2.21, \quad \beta = \frac{\varrho_0}{L_0} = 0.0237 \text{ cm K}^2\text{W}^{-1},$$

and from here

$$W_i = 1.047 \cdot 10^{-5} T^{2.21} \text{ cm K/W}.$$

The electronic component of the thermal conductivity coefficient has the form

$$\lambda_{e_1} = \frac{1}{W_e} = \frac{1}{W_0 + W_i} = \frac{1}{\varrho_0/L_0 T + 1.047 \cdot 10^{-5} T^{2.21}}. \tag{4}$$

The lattice component is

$$\lambda_{l_1} = \lambda - \lambda_{e_1}. \tag{5}$$

In figures 1 to 8 the dependences of $\lambda_e(T)$ and $\lambda_l(T)$ obtained according to the proposed method for investigated samples of different Cu concentration are presented for the temperature range 4.2–80 K. λ_e and λ_l marked by index 1 in the figures concern the case discussed in this paragraph.

The dependences of $\lambda_l(T)$ show a maximum. The λ_l maximum diminishes monotonically from 0.31 W/cm K for a sample with 97% Cu concentration to 0.23 W/cm K for 63% Cu concentration as presented in Fig. 8.

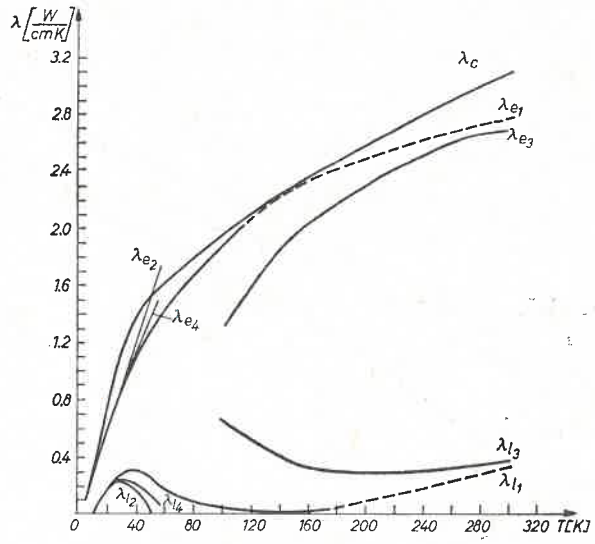

Fig. 1. Separation of thermal conductivity components for M97* sample of brass

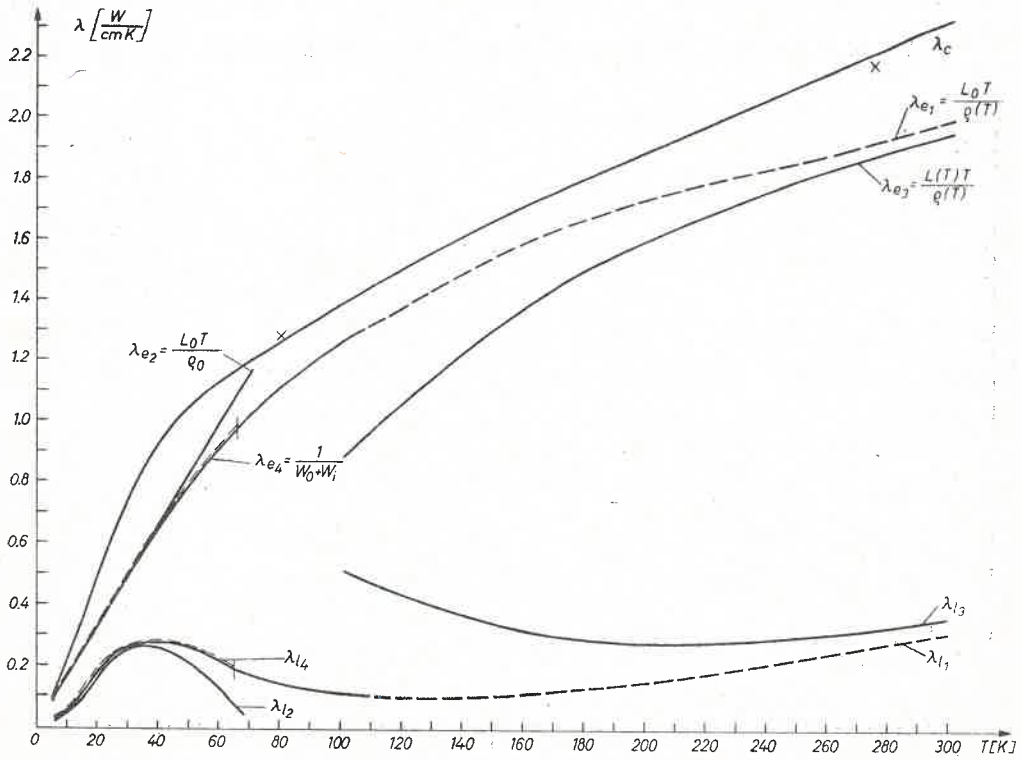

Fig. 2. Separation of thermal conductivity components for M96 sample of brass

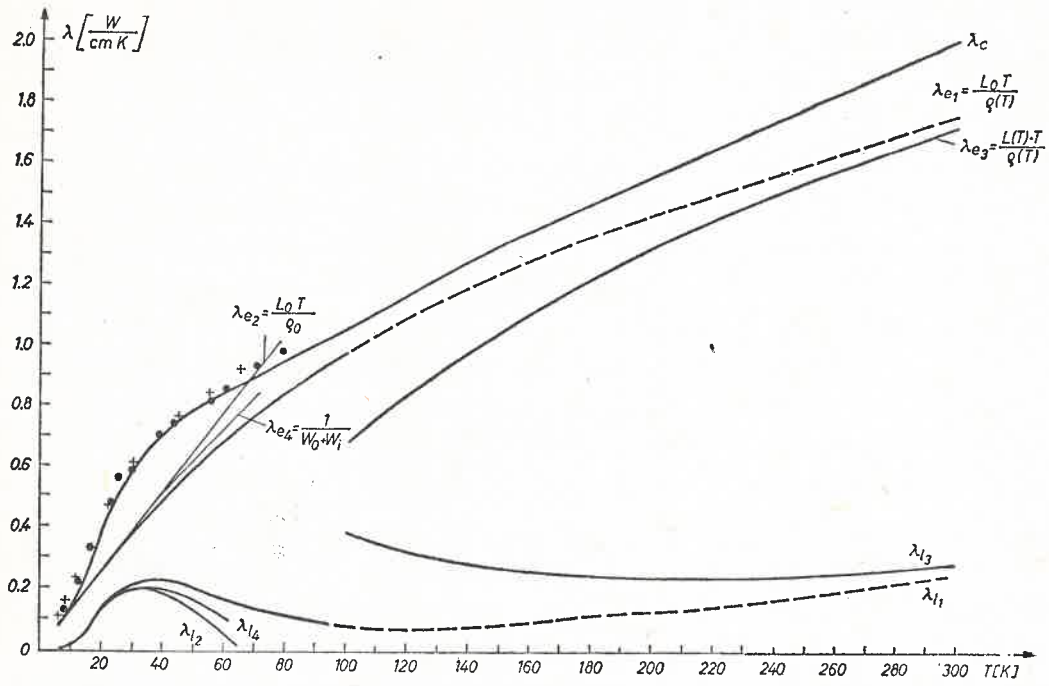

Fig. 3. Separation of thermal conductivity components for M90 sample of brass

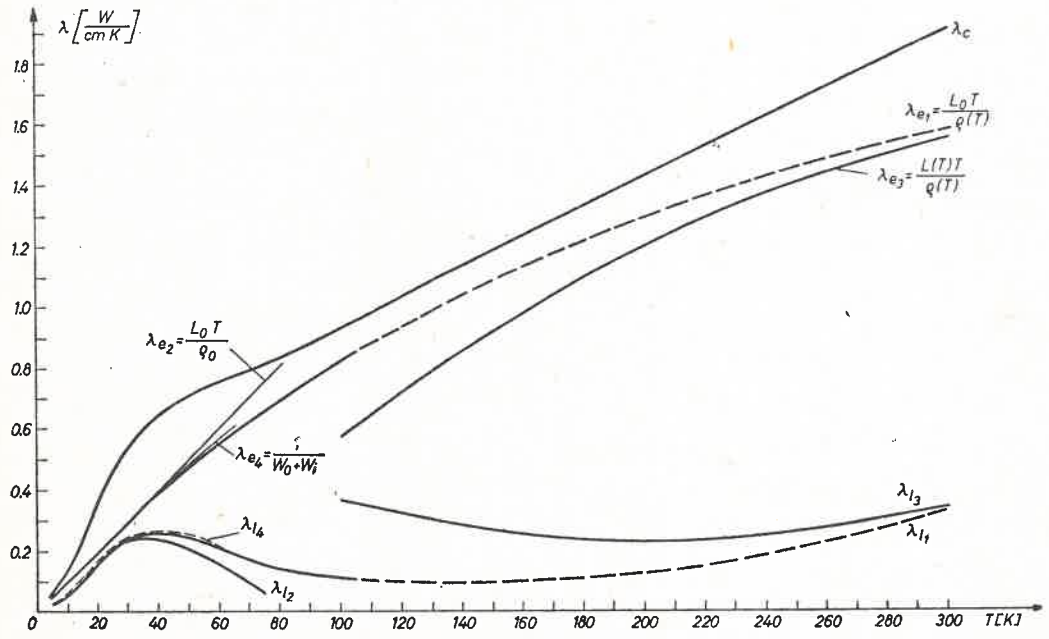

Fig. 4. Separation of thermal conductivity components for M86* sample of brass

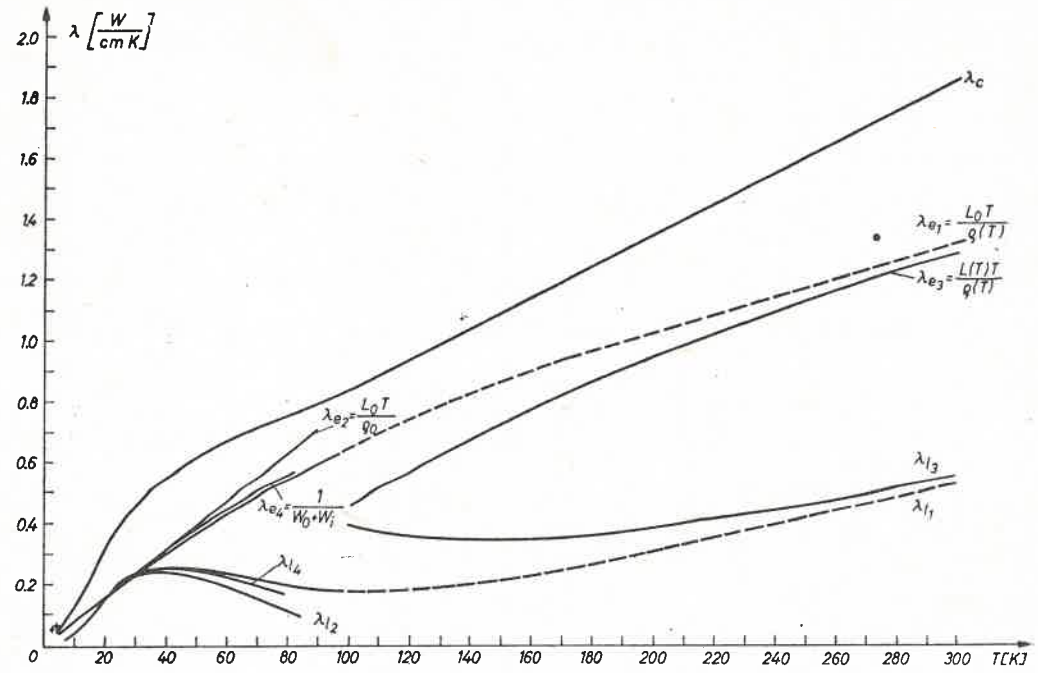

Fig. 5. Separation of thermal conductivity components for M85 sample of brass

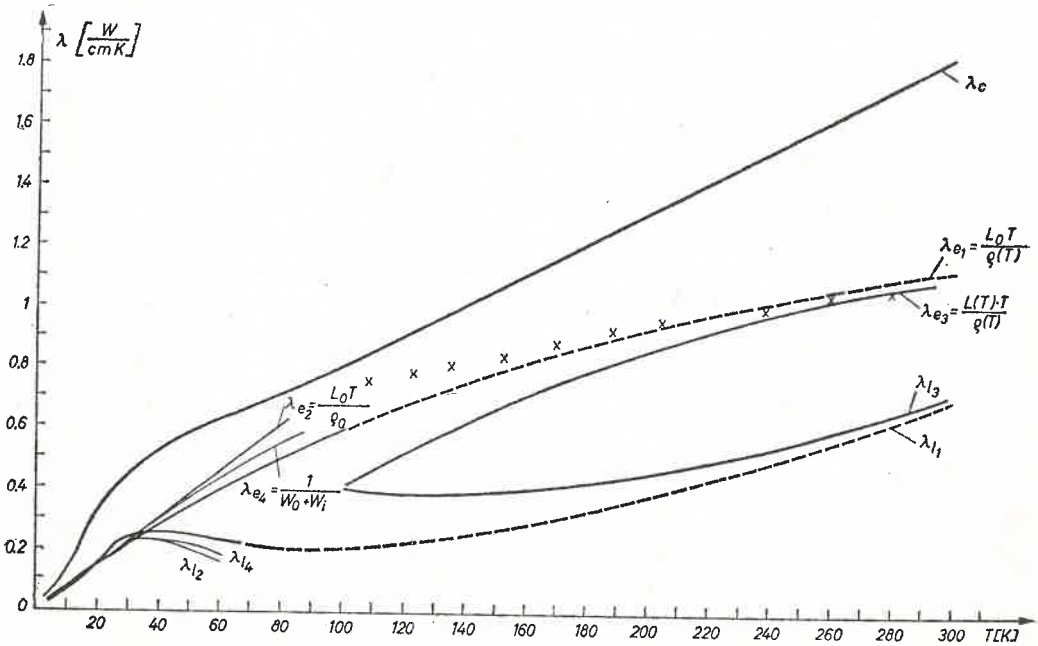

Fig. 6. Separation of thermal conductivity components for M70 sample of brass

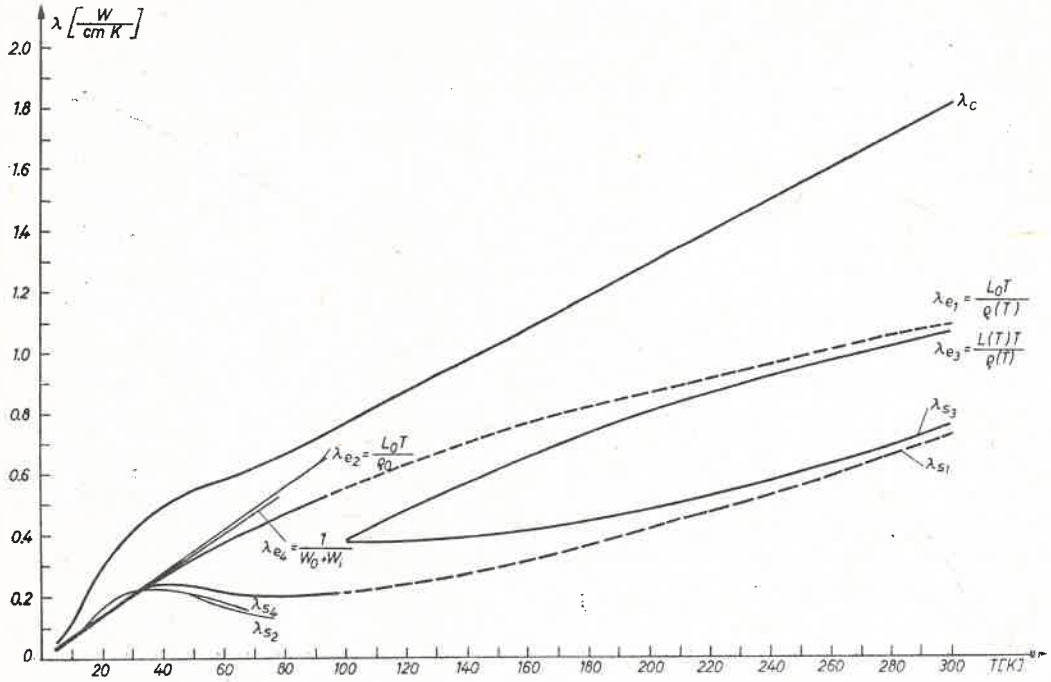

Fig. 7. Separation of thermal conductivity components for M65* samples of brass

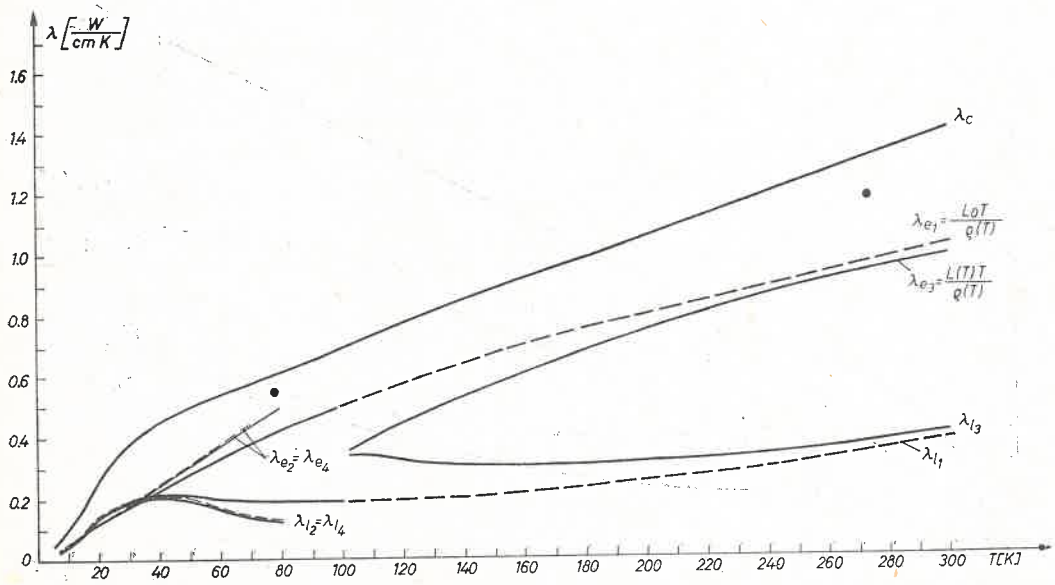

Fig. 8. Separation of thermal conductivity components for M63 sample of brass

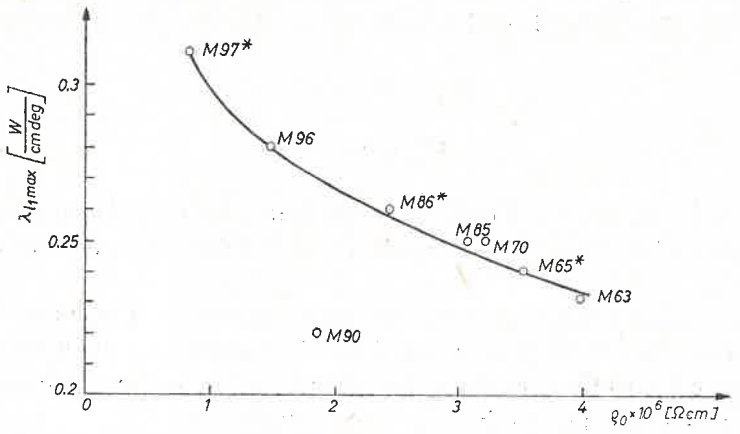

Fig. 9. The dependence of the maximum thermal conductivity coefficient of the lattice component on residual resistivity of brass samples

2.2. The ideal thermal resistivity of an alloy can be neglected

The assumption $W_i \ll W_0$ enables one to omit W_i. This method of separation was used by Klemens [8], White and Woods [5, 9], Lindenfeld and Pennebacker [10] and others. With the mentioned assumption we obtain

$$\lambda_{e_2} = \frac{1}{W_0} = \frac{L_0 T}{\varrho_0}, \tag{6}$$

$$\lambda_{l_2} = \lambda - \lambda_{e_2}. \tag{7}$$

In the two discussed methods the $\lambda_e(T)$ plot is linear but the straight line with index 2 has a larger slope angle than the straight line with index 1. For the two methods with temperature increasing λ_{e_1} and λ_{e_2} exceed λ and this is the upper temperature limit for using these two separation methods.

2.3. Ideal thermal resistivity $W_i = \varrho_i / L_0 T$

The third separation method is, according to the present authors, the most physically correct and uses the following dependence

$$\lambda_{e_3} = \frac{L_0 T}{\varrho_0 + \varrho_i(\varrho_0, T)}. \tag{8}$$

In Eq. (8) the deviation from the Matthiessen Rule is considered. Writing the dependence (8) in the form of thermal resistivity we have

$$W_{e_3} = W_0 + \frac{\varrho_i}{L_0 T}. \tag{9}$$

We combine with Eq. (9) the dependences describing the two former separation methods

$$W_{e_1} = W_0 + \alpha T^n, \tag{10}$$

$$W_{e_2} = W_0. \tag{11}$$

We suppose, that formula ϱ_i/L_0T is the best approximation of real changes occurring in the band structure of the alloy. The third method has advantages in comparison to $W_i = \alpha T^n$ and $W_{e_2} = W_0$.

The former gives the ideal electronic thermal resistivity for a very pure basic metal, the latter does not consider the ideal thermal resistivity component at all. In the temperature range from 4.2 K to 10–30 K, where the equivalence $\varrho = \varrho_0$ is valid, the third method is equivalent to the two previously discussed methods.

By using the fact that the Lorenz constant for the electronic component has a constant value for temperatures $T \ll \theta$ [11], one may use the method mentioned for component separation in the temperature range $0 \ll T \ll \theta$. Trials of component separation for temperatures above 100 K using the third method have been presented in figures 1 to 8 by a broken line. This method was used by the authors of [12] in the temperature range 50–800°C for NiNb alloys.

For the intermetallic component Mg_2Pb similar investigations were made by the authors of [13] in the temperature range 40–200 K.

Klemens et al. [14] use this method for the separation of thermal conductivity components in a Cu_3Au alloy.

The three discussed methods of component separation were based on the assumption of Wiedemann–Franz of the low validity for alloys in the temperature range 4.2–90 K.

2.4. The using of the $L(T)$ dependence for a basic metal in an alloy

The fourth separation method of thermal conductivity components for the temperature range 100–300 K has been applied by the authors of [15] for different Cu, Al and In alloys. According to the assumptions of this method the present authors have accomplished the separation of thermal conductivity components for investigated brass alloys. In this method one uses the assumption of Lorenz number dependence on temperature for a basic metal in an alloy. According to the theoretical Wilson equation the Lorenz number for pure metals is described by the equation

$$L = \frac{L_0}{1 + \frac{3}{2\pi^2}\left(\frac{\xi}{d}\right)(\theta_D/T)^2 - \frac{1}{2\pi^2}\frac{g_7(\theta_D/T)}{g_5(\theta_D/T)}}, \tag{12}$$

where $\frac{d}{\xi} = 2^{-1/3} N_e^{-2/3}$, N_e is an effective number of free electrons associated with one atom, θ_D — Debye temperature, g_7 and g_5 — Debye integrals of the seventh and fifth range. Using the tabelarized values of the integrals g_5 and g_7 one may present L/L_0 as

a dependence of T/θ_D for the effective number of free electrons of a Cu — 0.25/atom

$$\frac{L}{L_0} = 1 - \exp(-T/\theta + 0.214), \tag{13}$$

where $\theta = f(\theta_D, N_e)$ and $\theta/\theta_D = 0.225$ for Cu.

The plot of the $L(T)$ dependence for Cu is presented in Fig. 10. The Lorenz constant reaches its theoretical L_0 value at about 700 K. As one may think on the basis of graphs 1 to 8, probably in this temperature the $\lambda_{e_3} = L_0 T/\varrho(T)$ and $\lambda_{e_4} = L(T)T/\varrho(T)$ obtain the

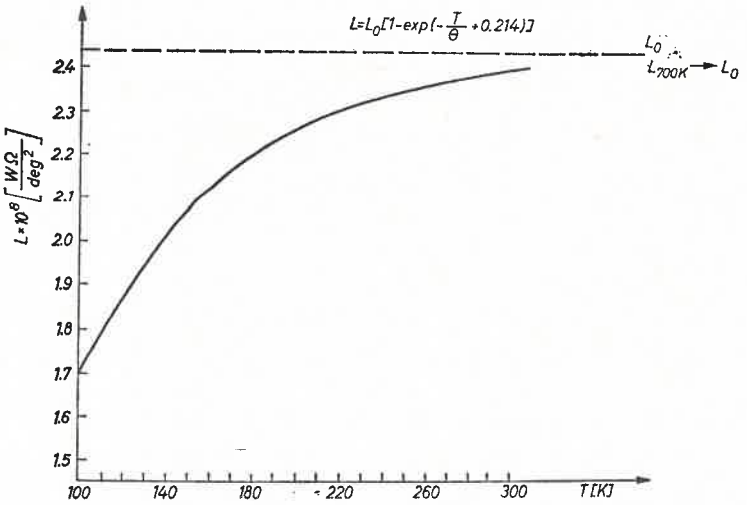

Fig. 10. Lorenz constant dependence on temperature for Cu after [15]

same values. For temperature about 100 K the λ_{e_3} and λ_{e_4} values differ considerably from each other. For heavily defected samples the λ_{e_4} and λ_{l_4} values are equal at about 100 K. Here arises the question about the plot of $\lambda_{e_4}(T)$ and $\lambda_{l_4}(T)$ functions for temperatures below 100 K.

The above considerations prove that the $L(T)$ function for a pure metal cannot be used without reservation, referring to strongly doped alloys of this metal, especially for the temperature range where $L < L_0$ for a pure metal. One can see some inconsistency in the described method. Although the $\varrho(T)$ is measured for alloy samples and this takes into consideration the changes of ϱ_0 and ϱ_i with the sample alloying (doping) this $L(T)$ function is characteristic for a pure basic metal.

3. *Conclusions*

The first separation method of brass thermal conductivity components is based on the assumption that ideal alloy and basic metal thermal resistivity is equal. The range of the application of this method has been experimentally found to be in the range 4.2–80 K (Figs. 1–8).

The second separation method uses the assumption that the ideal thermal resistivity component W_i of an alloy is much smaller than the thermal resistivity W_0. The temperature range where λ_{e_2} exceeds λ is the upper limit of application of this separation method (Figs 1–8).

The third separation method uses the assumption that ideal thermal resistivity W_i of an alloy is ϱ_i/L_0T. The expression ϱ_i/L_0T is the best approximation of real changes occurring in the band structure of an alloy. In the temperature range 4.2–30 K the third method is equivalent to the first and second ones (Figs 1–8).

The three discussed separation methods of components are based on the Wiedemann–Franz law assumption in the temperature range 4.2–90 K.

The fourth separation method assumes that the Lorenz function $L(T)$ for a pure metal can be applied for heavily doped (admixtured) alloys of this metal. It has been applied in [15] for component separation in the range 100–300 K (Figs 1–8). The authors of this paper present the point of view that the method of separation of thermal conductivity components marked as third and using Eq. (8) is the most physically correct one among the four presented.

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