EFFECT OF INHOMOGENEOUS MAGNETIZATION IN VICINITY OF DISLOCATION ON FERROMAGNETIC RESONANCE LINEWIDTH*

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The two-magnon relaxation time for uniform magnons, scattered on a magnetic inhomogeneity generated by dislocation, is calculated. The main magnetic inhomogeneity is given by the stress field of dislocation and magnetostrictive effects. In the scattering amplitude of magnons the ground state of the dislocated ferromagnet is taken into account. The corrections due to the inhomogeneous magnetization are very important in cases corresponding to measurements performed at low internal field. The numerical results for the single dislocation model are of the same order as the experimental literature data.

1. Introduction

Effects of inhomogeneous magnetization in the vicinity of a dislocation line have been neglected in numerical calculations of the broadening of the ferromagnetic resonance (FMR) line and the results obtained were many times less than the experimental data. In a number of papers [2–5], the experimental measurements [6–7] were accounted for quantitatively by dislocation structures. In the present paper, the scattering amplitude of magnons is found to change strongly when taking into account the change of the ground state of the dislocated ferromagnet. The calculated linewidth is of the same order as the broadening measured [6, 7].

In the appendix of paper [1], the theory for an arbitrary magnetic field is presented. In our paper, the simpler case is discussed. We neglect the ellipticity of spin precession in the scattering amplitude.

In the present paper, the relaxation time is calculated for a single dislocation. The stress field of the dislocation has a finite range and extends over the distance r_1 from the dislocation line. For low density of dislocations the radius r_1 is given by the magnetostrictive effect. In stage II of the work-hardening curve, the radius r_1 is taken as half the distance between neighbouring dislocations. We assume that two neighbouring dislocations have

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opposite Burgers vectors and are situated in neighbouring glide planes. In a dipole configuration, the radius r_1 can be taken as

$$r_1 = x/\sqrt{2}\,, \tag{1}$$

where x is the distance between the glide planes.

2. Holstein-Primakoff transformation in dislocated ferromagnet

In order to express the magnetization in terms of spin wave creation and annihilation operators, coordinate systems: $x_1x_2x_3$ and XYZ are introduced. They are chosen so that the x_3-axis is parallel to the internal field (the applied magnetic field and the demagnetizing field), and the Z-axis in the local quantization direction of the local magnetic moment density $\boldsymbol{M}(\boldsymbol{r})$ at the point $\boldsymbol{r}$. Introducing the direction cosines $\tilde{\gamma}_i(\boldsymbol{r})$ of the quantization direction at the point $\boldsymbol{r}$, the situation of the system XYZ with respect to $x_1x_2x_3$ may be defined with accuracy to an angle of rotation about the Z-axis. We fix the angle about the Z-axis so that the transformation matrix has the simplest form. The transformation matrix calculated with accuracy to square powers of the direction cosines $\tilde{\gamma}_1(\boldsymbol{r})$ and $\tilde{\gamma}_2(\boldsymbol{r})$ is given by

$$\begin{pmatrix} M_1(\boldsymbol{r}) \\ M_2(\boldsymbol{r}) \\ M_3(\boldsymbol{r}) \end{pmatrix} = \begin{pmatrix} 1 & 0 & \tilde{\gamma}_1(\boldsymbol{r}) \\ 0 & 1 & \tilde{\gamma}_2(\boldsymbol{r}) \\ -\tilde{\gamma}_1(\boldsymbol{r}) & -\tilde{\gamma}_2(\boldsymbol{r}) & 1 \end{pmatrix} \begin{pmatrix} M_X(\boldsymbol{r}) \\ M_Y(\boldsymbol{r}) \\ M_Z(\boldsymbol{r}) \end{pmatrix}. \tag{2}$$

In the present paper, the value of the applied magnetic field is chosen so that the conditions

$$|\tilde{\gamma}_1(\boldsymbol{r})| \ll 1, \quad |\tilde{\gamma}_2(\boldsymbol{r})| \ll 1, \quad \tilde{\gamma}_3(\boldsymbol{r}) \simeq 1 \tag{3}$$

shall be fulfilled.

Quantization of the system is achieved by putting [9]:

$$M_X(\boldsymbol{r}) \pm iM_Y(\boldsymbol{r}) = (2\gamma\hbar M_0)^{1/2} a^{\mp}(\boldsymbol{r}) + \ldots,$$
$$M_Z(\boldsymbol{r}) = M_0 - \gamma\hbar a^{+}(\boldsymbol{r}) a^{-}(\boldsymbol{r}), \tag{4}$$

where γ is the gyromagnetic ration, M_0 — the saturation magnetization, and $a^+(\boldsymbol{r})$ and $a^-(\boldsymbol{r})$ are creation and annihilation operators which obey the boson commutation rules.

Spin waves in the system are introduced by the Fourier transform

$$a_{\boldsymbol{k}}^{-} = V^{-1/2} \int d\boldsymbol{r} a^{-}(\boldsymbol{r}) \exp(-i\boldsymbol{k}\cdot\boldsymbol{r}) \tag{5}$$

of the operator $a^-(\boldsymbol{r})$ (consult [8, 9] for details), where V is the volume per one dislocation. On insertion of (3) and (4) into (2), the local magnetization can be expressed as

$$m^{\pm}(\boldsymbol{r}) = M_1(\boldsymbol{r}) \pm iM_2(\boldsymbol{r}) = \sum_{\boldsymbol{k}} \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) \{(2\gamma\hbar M_0/V)^{1/2} a_{\mp\boldsymbol{k}}^{\mp} + M_0[\gamma_1(\boldsymbol{k}) \pm i\gamma_2(\boldsymbol{k})]\} + \ldots, \tag{6}$$

$$m_3(\boldsymbol{r}) = M_3(\boldsymbol{r}) - M_0 = \sum_{\boldsymbol{k},\boldsymbol{k}'} \exp(i\boldsymbol{k}\cdot\boldsymbol{r}) \{-(\gamma\hbar M_0/2V)^{1/2}[[\gamma_1(\boldsymbol{k}')$$
$$+ i\gamma_2(\boldsymbol{k}')] a_{\boldsymbol{k}'-\boldsymbol{k}}^{+} + [\gamma_1(\boldsymbol{k}') - i\gamma_2(\boldsymbol{k}')] a_{\boldsymbol{k}-\boldsymbol{k}'}^{-}] - (\gamma\hbar/V) a_{\boldsymbol{k}'-\boldsymbol{k}}^{+} a_{\boldsymbol{k}'}^{-}\} + \ldots, \tag{7}$$

where $\gamma_1(\boldsymbol{k})$ and $\gamma_2(\boldsymbol{k})$ are the Fourier transforms of the direction cosines

$$\gamma_i(\boldsymbol{k}) = (1/V) \int d\boldsymbol{r} \tilde{\gamma}_i(\boldsymbol{r}) \exp(-i\boldsymbol{k} \cdot \boldsymbol{r}). \tag{8}$$

Eqs (6) and (7) represent the Holstein–Primakoff transformation for the dislocated ferromagnet.

Detailed discussions of the direction cosines have been given previously in many papers (e.g. [10]). In the case of isotropic ferromagnets the Fourier transforms $\gamma_1(\boldsymbol{k})$ and $\gamma_2(\boldsymbol{k})$ can be written as

$$\gamma_i(\boldsymbol{k}) = \frac{3\lambda\sigma_{i3}(\boldsymbol{k})}{M_0(H+\alpha k^2)} - \frac{12\pi\lambda}{(\omega_{\boldsymbol{k}}/\gamma)^2}\left[\frac{k_1 k_i}{k^2}\sigma_{13}(\boldsymbol{k}) + \frac{k_2 k_i}{k^2}\sigma_{23}(\boldsymbol{k})\right]. \tag{9}$$

Herein, $\omega_{\boldsymbol{k}}$ has the form of the spin wave frequency

$$\omega_{\boldsymbol{k}} = \gamma\{[H+\alpha k^2]\,[H+\alpha k^2+4\pi M_0(1-k_3^2/k^2)]\}^{1/2}, \tag{10}$$

λ is the magnetostrictive constant, α — the spin-wave dispersion coefficient, H — the internal field, and $\sigma_{ij}(\boldsymbol{k})$ are the Fourier transforms of the stress components. In the special case of the single dislocation these transforms in a coordinate system xyz are given by (see [2])

$$\sigma_{ij}(\boldsymbol{k}) = -\frac{ibG}{V}\,\frac{\sin k_z L/2}{k_z}\,\frac{1-J_0(r_1 k_\varrho)}{k_\varrho} f_{ij}(\varphi_k), \quad i, j = x, y, z. \tag{11}$$

In the case of the edge dislocation

$$\begin{aligned}
f_{xx}(\varphi_k) &= (1-\nu)^{-1}(3\sin\varphi_k - \sin 3\varphi_k),\\
f_{yy}(\varphi_k) &= (1-\nu)^{-1}(\sin\varphi_k + \sin 3\varphi_k),\\
f_{zz}(\varphi_k) &= (1-\nu)^{-1} 4\nu \sin\varphi_k,\\
f_{xy}(\varphi_k) &= (1-\nu)^{-1}(\cos 3\varphi_k - \cos\varphi_k),\\
f_{xz}(\varphi_k) &= f_{yz}(\varphi_k) = 0,
\end{aligned} \tag{12}$$

and for the screw dislocation

$$\begin{aligned}
f_{xx}(\varphi_k) &= f_{yy}(\varphi_k) = f_{zz}(\varphi_k) = f_{xy}(\varphi_k) = 0,\\
f_{xz}(\varphi_k) &= 2\sin\varphi_k,\\
f_{yz}(\varphi_k) &= -2\cos\varphi_k.
\end{aligned} \tag{13}$$

Above, we use the notation: ν — Poisson's constant, L — the length of the dislocation, b — the length of the Burgers vector, G — shear modulus, and k_z, k_ϱ, φ_k — the cylindrical coordinates of the vector $\boldsymbol{k}$: $k_x = k_\varrho \cos\varphi_k$, $k_y = k_\varrho \sin\varphi_k$. The coordinate system xyz is chosen so that the z-axis is parallel to the dislocation line and the x-axis is taken parallel to the Burgers vector in the case of the edge dislocation. The transformation from xyz to $x_1x_2x_3$ is given by

$$r_i = \sum_{j=x,y,z} \beta_{ij} r_j, \quad i = 1, 2, 3, \tag{14}$$

where

$$\begin{aligned} \beta_{1x} &= \cos\varphi, \quad \beta_{1y} = -\sin\varphi, \quad \beta_{1z} = 0,\\ \beta_{2x} &= \cos\vartheta \sin\varphi, \quad \beta_{2y} = \cos\vartheta\cos\varphi, \quad \beta_{2z} = -\sin\vartheta,\\ \beta_{3x} &= \sin\vartheta\sin\varphi, \quad \beta_{3y} = \sin\vartheta\cos\varphi, \quad \beta_{3z} = \cos\vartheta. \end{aligned} \tag{15}$$

The angles ϑ and φ are defined in Fig. 1.

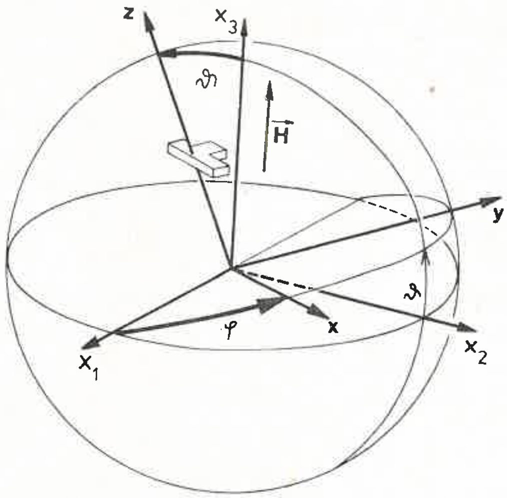

Fig. 1. Conversion of the coordinate system $x_1x_2x_3$ into xyz. The system $x_1x_2x_3$ can be brought into xyz by rotating about the x_1-axis through an angle ϑ and than about the z-axis through an angle φ

3. *Scattering amplitude of spin waves*

The energy of the system under consideration comprises Zeeman, dipolar, exchange and magnetoelastic contributions. The Zeeman energy (including the contribution due to the demagnetizing field) is given by

$$\mathscr{H}_z = -\int d\boldsymbol{r} \boldsymbol{M} \cdot \boldsymbol{H}. \tag{16}$$

The exchange energy is

$$\mathscr{H}_{ex} = \frac{\alpha}{2M_0} \int d\boldsymbol{r} \left\{ \left(\frac{\partial \boldsymbol{M}}{\partial x_1}\right)^2 + \left(\frac{\partial \boldsymbol{M}}{\partial x_2}\right)^2 + \left(\frac{\partial \boldsymbol{M}}{\partial x_3}\right)^2 \right\}. \tag{17}$$

The dipolar energy of inhomogeneous magnetization can be expressed as

$$\mathscr{H}_{dip} = -\tfrac{1}{2} \int d\boldsymbol{r} \boldsymbol{m} \cdot \boldsymbol{h}_{dip}, \tag{18}$$

where

$$\boldsymbol{h}_{\text{dip}} = -4\pi \sum_{k\neq 0} e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \frac{\boldsymbol{k}\cdot\boldsymbol{m}_k}{k^2}\boldsymbol{k}, \tag{19}$$

and $\boldsymbol{m}_k$ is the Fourier coefficient of the magnetization $\boldsymbol{m}(\boldsymbol{r}) = \sum_k \boldsymbol{m}_k \exp(i\boldsymbol{k}\cdot\boldsymbol{r})$. The magnetoelastic coupling energy in isotropic materials has the form

$$\mathscr{H}_\lambda = -\tfrac{3}{2}\lambda \sum_{i,j=1,2,3} \int d\boldsymbol{r}\, \frac{M_i M_j}{M^2}\tilde{\sigma}_{ij}(\boldsymbol{r}), \tag{20}$$

where $\tilde{\sigma}_{ij}(\boldsymbol{r})$ are the components of the stress tensor.

We express the total energy of the ferromagnetic medium in terms of magnon operators. In the lowest order approximation, corresponding to the theory of non-interacting magnons, we have

$$\mathscr{H} = \sum_k \hbar\omega_k \overset{+}{a}_k \overset{-}{a}_k + \sum_{k,k'} \{W_{k,k'} \overset{+}{a}_k \overset{-}{a}_{k'} + V_{k,k'} \overset{+}{a}_k \overset{-}{a}_{k'} + \text{c.c.}\}. \tag{21}$$

The terms in the dipolar energy due to ellipticity of spin precession were transformed away by a canonical Bogolyubov transformation [1, 9]. The latter does not change essentially the amplitudes of the non-diagonal terms in Eq. (21) describing the effect of dislocations on magnons; therefore, at this point, the transformation is neglected.

In the present paper we intend to calculate the relaxation time of uniform magnons scattered into the degenerate spectrum of $k \neq 0$. In (21), the terms

$$\mathscr{H}_i = \sum_k W_{k,0} \overset{+}{a}_k \overset{-}{a}_0 + \text{c.c.} \tag{22}$$

are responsible for these transitions, where $W_{k,0}$ is given by

$$\begin{aligned} W_{k,0} &= \gamma h\{(-3\lambda/2M_0)\,[\sigma_{11}(\boldsymbol{k})+\sigma_{22}(\boldsymbol{k})-2\sigma_{33}(\boldsymbol{k})] \\ &-6\pi M_0(k_3/k^2)\,[k_1\gamma_1(\boldsymbol{k})+k_2\gamma_2(\boldsymbol{k})]-2\pi i M_0(k_3/k^2)\,[k_2\gamma_1(\boldsymbol{k})-k_1\gamma_2(\boldsymbol{k})]\}. \end{aligned} \tag{23}$$

4. *Magnon relaxation by two-magnon processes*

The relaxation time of uniform magnons due to two-magnon scattering processes is given by the formula (see e.g. [9])

$$\frac{1}{\tau} = \frac{2\pi}{\hbar}\sum_k |W_{k,0}|^2 \delta(\hbar\omega_k - \hbar\omega_0). \tag{24}$$

The frequency ω_0 of the uniform mode is equal to the microwave frequency ω.

In order to calculate the relaxation time from (23) and (24) it is convenient to replace summation over $\boldsymbol{k}$ by integration with respect to polar coordinates k_ϱ, φ_k, k_z. The integrations over k_z and φ_k are simplified by the approximation $\{\sin(k_z L/2)/k_z\}^2 \simeq (\pi L/2)\delta(k_z)$ (cf. 1) and the factor $\delta(\hbar\omega_k - \hbar\omega_0)$. We obtain the following expression for the lifetime:

$$1/\tau = (3/8\pi)^2 (bg\lambda)^2 \gamma NS/M_0^3, \tag{25}$$

where N is the dislocation density ($N = L/V$),

$$S = \Omega \int_{k_{\min}}^{k_{\max}} \frac{k_\varrho dk_\varrho}{\sqrt{[\Omega_k^2(0) - \Omega^2][\Omega^2 - \Omega_k^2(\pi/2)]}} \left(\frac{1 - J_0(r_1 k_\varrho)}{k_\varrho}\right)^2$$
$$\times \{F(\psi(k_\varrho) - \varphi, k_\varrho) + F(\pi - \psi(k_\varrho) - \varphi, k_\varrho)\}, \quad \text{for} \quad \Omega > \Omega_0(\pi/2) \tag{26}$$

and $S = 0$ for $\Omega < \Omega_0(\pi/2)$.

The integration over k_ϱ in Eq. (26) will be performed in the next Section for a special case. Above, we use the notation: Ω — normalized frequency ($\Omega = \omega/4\pi\gamma M_0$) and $\Omega_k(\varphi_k)$ — normalized frequency of the spin wave in the case $k_z = 0$

$$\Omega_k(\varphi_k) = \sqrt{[\Omega_H + Dk_\varrho^2][\Omega_H + Dk_\varrho^2 + 1 - \sin^2\vartheta \sin^2\varphi_k]}, \tag{27}$$

where $D = \alpha/4\pi M_0$ is the normalized exchange constant, and Ω_H — normalized internal field ($\Omega_H = H/4\pi M_0$). The limits of integration are:

$$k_{\max} = \{[(\Omega_H + 0.5\cos^2\vartheta)^2 + \Omega^2 - \Omega_0^2(\pi/2)]^{1/2} - \Omega_H - 0.5\cos^2\vartheta\}^{1/2}, \tag{28}$$

$$k_{\min} = \begin{cases} 0 \quad \text{for} \quad \Omega \leqslant \Omega_0(0), \\ \{\{[(\Omega_H + 1/2)^2 + \Omega^2 - \Omega_0^2(\pi/2)]^{1/2} - \Omega_H - 1/2\}D^{-1}\}^{1/2} \quad \text{for} \quad \Omega > \Omega_0(0). \end{cases} \tag{29}$$

The functions $\psi(k_\varrho)$ and $F(\varphi_k, k_\varrho)$ are given by

$$\psi(k_\varrho) = \arcsin\sqrt{\frac{\Omega_k^2(0) - \Omega^2}{(\Omega_H + Dk_\varrho^2)\sin^2\vartheta}}, \tag{30}$$

$$F(\varphi_k, k_\varrho) = \{A_{11}(\varphi_k) + A_{22}(\varphi_k) - 2A_{33}(\varphi_k) + \tfrac{3}{2}\sin\vartheta\sin 2(\varphi_k + \varphi)\Gamma_1(\varphi_k, k_\varrho)$$
$$+ \tfrac{3}{2}\sin 2\vartheta \sin^2(\varphi_k + \varphi)\Gamma_2(\varphi_k, k_\varrho)\}^2 + \tfrac{9}{4}\{-\sin\vartheta\sin 2(\varphi_k + \varphi)\Gamma_2(\varphi_k, k_\varrho)$$
$$+ \sin 2\vartheta \sin^2(\varphi_k + \varphi)\Gamma_1(\varphi_k, k_\varrho)\}^2, \tag{31}$$

where

$$A_{ij}(\varphi_k) = \sum_{n,m=x,y,z} \beta_{in}\beta_{jm} f_{nm}(\varphi_k), \tag{32}$$

$$\Gamma_1(\varphi_k, k_\varrho) = \frac{A_{13}(\varphi_k)}{\Omega_H + Dk_\varrho^2} - \Omega^{-2}[A_{13}(\varphi_k)\cos^2(\varphi_k + \varphi)$$
$$+ 0.5A_{23}(\varphi_k)\cos\vartheta\sin^2(\varphi_k + \varphi)], \tag{33}$$

$$\Gamma_2(\varphi_k, k_\varrho) = A_{23}(\varphi_k)/(\Omega_H + Dk_\varrho^2) - \Omega^{-2}[0.5A_{13}(\varphi_k)\cos\vartheta\sin^2(\varphi_k + \varphi)$$
$$+ A_{23}(\varphi_k)\cos^2\vartheta\sin^2(\varphi_k + \varphi). \tag{34}$$

5. Numerical results

The results are obtained for material constants corresponding to nickel ($M_0 = 485\ G$, $D = 5.69 \times 10^{-13}\ \text{cm}^2$, $\nu = 0.32$, $\lambda = \lambda_{100} = -55 \times 10^{-6}$, $G = 7.4 \times 10^{11}\ \text{dyn/cm}^2$, $b = 2.49 \times 10^{-8}\ \text{cm}$), the normalized microwave frequency $\Omega = 0.459$ and the normalized internal field $\Omega_H = 0.1787$. Measurements for these values of the frequency and internal field have been reported in paper [6].

The integration over k_ϱ in Eq. (26) can be performed in the case of large radius ($r_1 k_{\max} > 6$). In order to integrate Eq. (26) the function $[1 - J_0(r_1 k_\varrho)]^2/k_\varrho^2$ is replaced by $0.7\, r_1 \delta(k_\varrho - 3.3/r_1)$. The point $3.3/r_1$ is chosen so that the line $k_\varrho = 3.3/r_1$ divides the area below the curve $[1 - J_0(r_1 k_\varrho)]^2/k_\varrho^2$ in two equal parts. This approximate integration is reasonable for disk sample which is magnetized parallel to the surface. In this case, the resonance equation is given by

$$\Omega^2 = \Omega_H(\Omega_H + 1). \tag{35}$$

On insertion of this resonance equation into Eq. (26) and approximating the factor $[1 - J_0(r_1 k_\varrho)]^2 k_\varrho^{-2}$ by $0.7\, r_1 \delta(k_\varrho - 3.3/r_1)$ the function S can be expressed as

$$\frac{S}{r_1} = \frac{0.7\Omega\{F(\psi(3.3/r_1) - \varphi,\ 3.3/r_1) + F(\pi - \psi(3.3/r_1) - \varphi,\ 3.3/r_1)\}}{\sqrt{D[1 + 2\Omega_H + D(3.3/r_1)^2]\,[\Omega^2 - \Omega^2_{3.3/r_1}(\pi/2)]}}. \tag{36}$$

In this calculation, the pole of the factor $\{\Omega^2 - \Omega_k^2(\pi/2)\}^{-1/2}$ at $k_\varrho = k_{\max}$ is neglected.

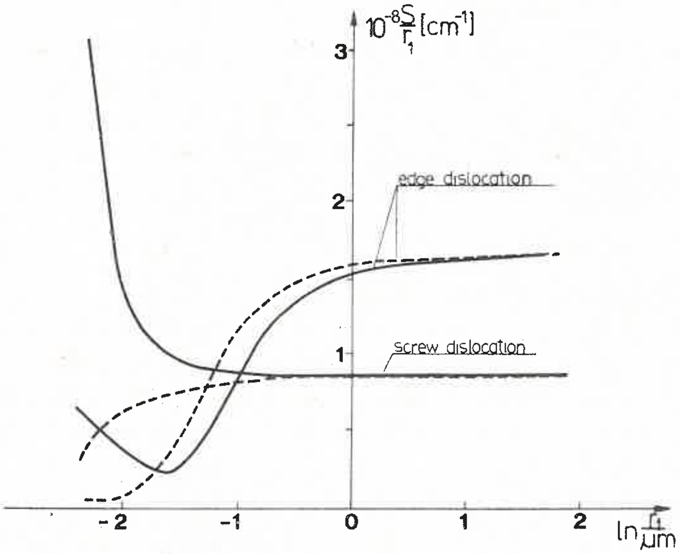

Fig. 2. Behaviour of the function S vs r_1. All other parameters are described in the text

Fig. 2 shows the graph of the function S versus r_1 for edge and screw dislocations. Solid and broken lines correspond to the case when the inhomogeneous magnetization is or, respectively, is not taken into account. The angle between the internal magnetic field direction and the screw dislocation line is equal to 45°. In the case of the edge dislocation the field direction is parallel to the glide plane and perpendicular to the dislocation line ($\vartheta = \varphi = 90°$).

Fig. 3 presents the numerical results for FMR broadening

$$\Delta H = 1/(\gamma\tau\sqrt{3}) = \frac{1}{16\pi^2\sqrt{3}}(\tfrac{3}{2}G\lambda b)^2 M^{-3}NS \tag{37}$$

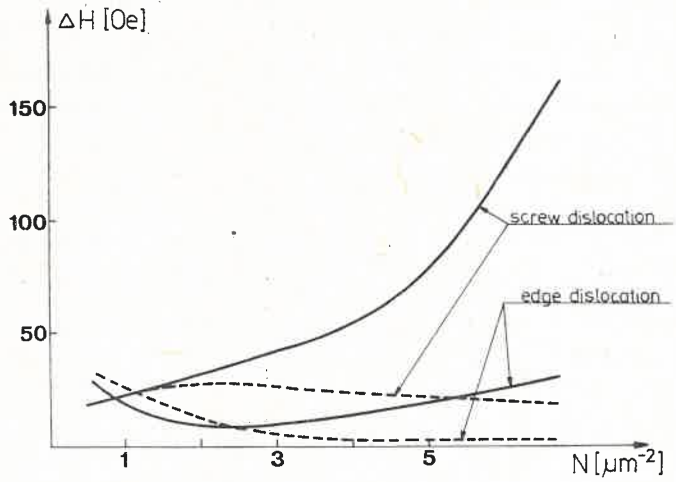

Fig. 3. Ferromagnetic resonance linewidth ΔH vs dislocation density, other parameters as in Fig. 2

versus dislocation density at the same orientation of the dislocations as in Fig. 2. The radius r_1 is given by Eq. (1). The dislocation density N and the distance x between the glide planes are calculated by (see [10, 11])

$$N = [(\sigma-\sigma_0)/0.3Gb]^2, \tag{38}$$

$$x = 0.0013\theta_2/(\sigma-\sigma_2), \tag{39}$$

with σ — shear stress, σ_0 — critical stress ($\sigma_0 = 2.5\times10^7$ dyne/cm^2), θ_2 — the work-hardening coefficient ($\theta_2 = 2.26\times10^7$ dyne/cm^2), σ_2 — a parameter of the order of shear stress in the lower limit of stage II ($\sigma_2 = 4.3\times10^9$ dyne/cm^2). The calculated broadening in Fig. 3 is less than the measured value [6, 7] but of the same order.

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