

# ELECTRON PARAMAGNETIC RESONANCE OF $Mn^{2+}$ AND $Gd^{3+}$ IN $Nd_3Mg_3(NO_3)_{12} \cdot 24H_2O$ SINGLE CRYSTALS

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The electron paramagnetic resonance (EPR) of  $Mn^{2+}$  and  $Gd^{3+}$  doped in  $Nd_3Mg_3(NO_3)_{12} \cdot 24H_2O$  single crystals has been studied at the X-band.  $Mn^{2+}$  substitutes for two  $Mg^{2+}$  sites, while  $Gd^{3+}$  substitutes for single type of  $Nd^{3+}$  sites. The spin-Hamiltonian analysis of the EPR spectra is presented for 298 K and 77 K.

## 1. Introduction

The electron paramagnetic resonance (EPR) studies required the presence of unpaired electrons in the sample to be investigated. In general, these are not present in the pure material but can be introduced by doping with paramagnetic substances such as the transition elements or by radiation damage. Among all the ions which exhibit EPR spectrum the  $d^5$  and  $f^7$  ions (S-state ions) are particularly interesting when one is concerned with the local symmetry of the environment of the ion. In fact, all sublevels of S-state are generally observed at low magnetic fields for two reasons: (a) the zero-field splitting is zero to the first order and (b) only higher-order effects permit lifting the degeneracy of the fundamental states ( $^6S$  and  $^8S$ ). Therefore, a detailed study of the angular variation of the paramagnetic resonance absorption lines permits one to establish the point symmetry around the magnetic ion.

The hydrated double nitrates of the rare earth elements form an interesting series of salts for EPR studies. Their general formula is  $M_2'''M_3''(NO_3)_{12} \cdot 24H_2O$ , where  $M'''$  is a trivalent cation (Bi or an ion of the 4f group) and  $M''$  is a divalent cation (Zn, Mg or an ion of the 3d group). Although the trivalent lanthanide ions are generally paramagnetic, their spin-lattice relaxation times are so much shorter than those of the S-state

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ions (at sufficiently high temperature) that the magnetic interaction between the paramagnetic ions and the S-state ions are averaged essentially to zero with the consequence that magnetic resonance absorption by the S-state ion can be observed without extensive broadening. In the present paper we describe EPR of  $\text{Mn}^{2+}$  and  $\text{Gd}^{3+}$  in  $\text{Nd}_2\text{Mg}_3(\text{NO}_3)_{12} \cdot 24\text{H}_2\text{O}$  (NMN) single crystals at 298 K and at 77 K. The EPR of  $\text{Gd}^{3+}$  in NMN was reported earlier only for room temperature [1].

## 2. Crystal structure

The crystal structure of  $\text{Ce}_2\text{Mg}_3(\text{NO}_3)_{12} \cdot 24\text{H}_2\text{O}$  has been determined by Zalkin et al. [2]. The other hydrated double nitrates can be expected to have similar structure. The primitive cell containing one formula unit is rhombohedral with dimension  $a = 13.165 \text{ \AA}$  and  $\alpha = 49.37^\circ$ . The space group is  $R\bar{3}$ . The corresponding hexagonal cell containing three formula units has dimension  $a = 11.004 \text{ \AA}$ ,  $c = 34.592 \text{ \AA}$ . The rhombohedral unit cell contains three divalent ions situated at two different lattice sites. One of them occupies a site with the point symmetry  $C_{3i}$  (Y sites or site I) and other two divalent ions occupy lattice sites with the point symmetry  $C_3$  (X site or site II). The trivalent ion is found at a site of  $C_{3i}$  point symmetry and rest of the atoms are in a position of the general type. Culvahouse et al. [3] has shown that one  $\text{Zn}^{2+}$  lies on the body centre of the cell (Y site) and two  $\text{Zn}^{2+}$  at special positions  $\pm(x, x, x)$  ( $x = 0.399$ ) which are along the long diagonal of the rhombohedron and  $34.18x \text{ \AA}$  to either side of the centre of the cell (X site). There are two trivalent ions along this diagonal and  $8.54 \text{ \AA}$  to either side of the centre of the cell. The long diagonal of the rhombohedron is along the trigonal axis. The divalent ions are surrounded by six water molecules forming together with the central ion a nearly regular octahedral  $\text{M}''(\text{H}_2\text{O})_6^{2+}$  complex. Each trivalent ion is coordinated by 12 oxygens belonging to six nitrate ions, located at the corners of a somewhat irregular icosahedron.

## 3. Experimental

Single crystals of NMN doped with  $\text{Mn}^{2+}$  and  $\text{Gd}^{3+}$  were grown by slow evaporation of an aqueous solution of  $\text{Nd}(\text{NO}_3)_3 \cdot 6\text{H}_2\text{O}$  and  $\text{Mg}(\text{NO}_3)_2 \cdot 6\text{H}_2\text{O}$ , mixed in stoichiometric ratios, at room temperature. The  $\text{Mn}^{2+}$  and  $\text{Gd}^{3+}$  were introduced into the lattice by adding a small amount (0.1% by weight) of manganese and gadolinium nitrates. The double nitrates grow in flat hexagonal plates, the plane of which is perpendicular to the trigonal axis. The experiments were performed on Varian V-4502 EPR spectrometer, operating at X-band, provided with 100 kHz field modulation. As a reference for magnetic field strength the resonance line of DPPH with  $g = 2.0036$  is used. The magnetic field at DPPH resonance was measured with the help of a Varian F-8A fluxmeter while the frequency of proton signal is measured by Systronics Type 701 frequency counter. The crystals were mounted on quartz rods and angular variation studies were performed using a Varian E-229 goniometer.

#### 4. Results and discussion

##### 4.1. $Mn^{2+}$

For an arbitrary orientation, the EPR spectrum consists of a number of lines corresponding to allowed and forbidden transitions. Angular variation studies of  $Mn^{2+}$  spectra reveal the presence of two inequivalent  $Mn^{2+}$  centre of unequal intensity. The  $Mn^{2+}$  substitutes for  $Mg^{2+}$  and shows the spectrum of two  $Mn^{2+}$  complexes. The spectrum

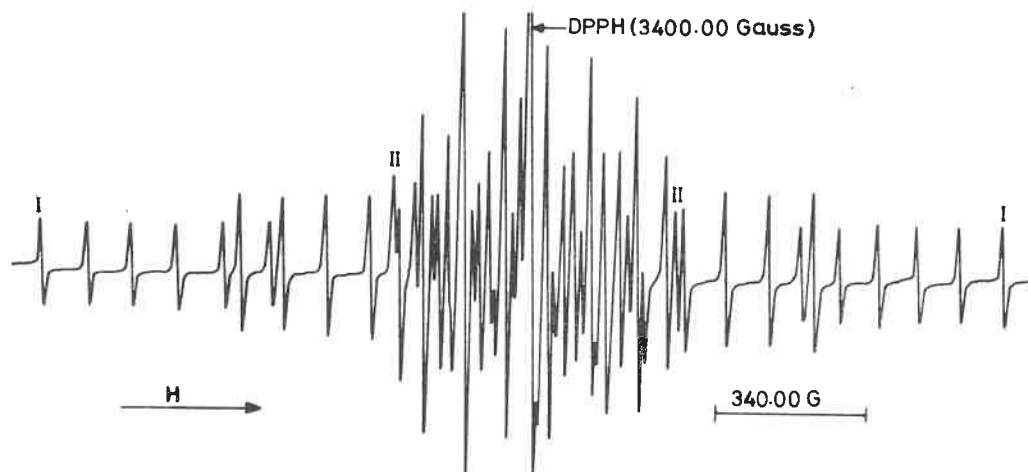


Fig. 1. EPR spectrum of  $Mn^{2+}$  in  $Nd_2Mg_3(NO_3)_{12} \cdot 24H_2O$  single crystals at 298 K. The magnetic field is along the  $c$  axis. The position of extreme hyperfine lines of  $Mn^{2+}$  complexes for two sites are designated by I and II respectively

having large intensity is due to  $Mn^{2+}$  substituting for  $Mg^{2+}$  at site II while spectrum having low intensity is due to  $Mn^{2+}$  at site I (Fig. 1). The  $Mn^{2+}$  centre occupying the site II is more intense because there are twice as many X sites as Y sites. The principal axes of the spectra were located by searching the directions of extrema in the spread of the spectrum. It was found that the principal  $z$  axis of two  $Mn^{2+}$  complexes are along the trigonal axis ( $c$  axis) and  $x$  axis is perpendicular to the trigonal axis. For a field applied along the trigonal axis, each type of  $Mn^{2+}$  complexes produces an EPR spectrum consisting of five nearly equally spaced sets of six nearly equal spaced hyperfine components (Fig. 1). The spectra of  $Mn^{2+}$  showing large zero-field splitting (site I) were measured for various angles of the magnetic field relative to the  $c$  axis. Fig. 2 shows the angular variation of the allowed fine structure transitions ( $\Delta M = \pm 1$ ) in the  $zx$  plane for site I. It is seen from the angular variation plot in the  $zx$  plane that fine structure transition lines move in rapidly from the  $z$  axis as  $\theta$  changes. The lines collapse to a very small spread at an orientation  $\theta = 52.7^\circ$  away from the  $z$  axis. As  $\theta$  increases further, the lines cross each other and spread out to a second maximum at  $\theta = 90^\circ$  ( $x$  axis). A  $\pi/3$  rotational symmetry of the spectrum was observed when the crystal was rotated about the trigonal axis. The  $g$  value, however, shows no angular variation in this plane. The observed behaviour of the spectrum is consistent with the trigonal symmetry of the  $Mg^{2+}$  sites. Experiments were also performed

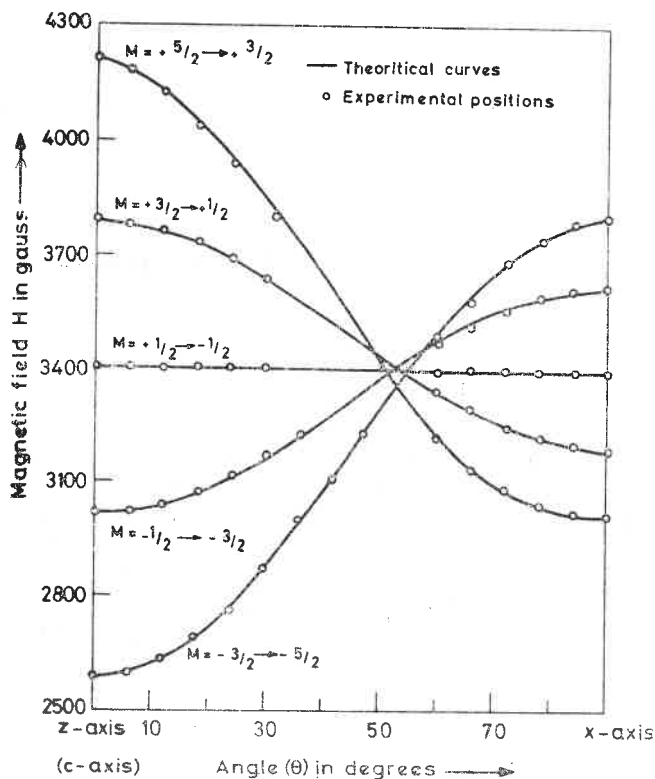


Fig. 2. Angular dependence of allowed fine structure transitions ( $\Delta M = \pm 1$ ) of  $Mn^{2+}$  for site I in the  $zx$  plane at 298 K

at 77 K in order to observe the effect of paramagnetic  $Nd^{3+}$  ions on  $Mn^{2+}$  spectra. However, at this temperature no effect on the spectrum was observed and experiments at 4.2 K were not possible.

The EPR spectrum of  $Mn^{2+}$  for both sites can be described by the spin-Hamiltonian of the form [4]

$$\begin{aligned} \mathcal{H} = & \beta g_{\parallel} H_z S_z + \beta g_{\perp} (H_x S_x + H_y S_y) + D(S_z^2 - 35/12) - (7/36)(a - F) \\ & \times (S_z^4 - (95/14)S_x^2 + 81/16) + A I_z S_z + B(S_x I_x + S_y I_y) + (a \sqrt{2}/36) \\ & \times \{S_z [S_+^3 \exp(-3i\Phi) + S_-^3 \exp(3i\Phi)] + [S_+^3 \exp(-3i\Phi) + S_-^3 \exp(3i\Phi)] S_z\}, \end{aligned}$$

where the  $z$  axis is parallel to the trigonal axis of the crystal. The symbols have their usual meaning and  $S = I = 5/2$  for  $Mn^{2+}$ . Using the above spin-Hamiltonian the  $Mn^{2+}$  EPR spectra are analysed and the best-fit parameters, thus obtained, are listed in Table I for both sites at 298 K and 77 K. The signs of the parameters (given in Table I) are relative. Since the hyperfine splitting constant is always found to be negative for a manganese ion

TABLE I

Spin-Hamiltonian parameters for  $Mn^{2+}$  in  $Nd_2Mg_3(NO_3)_{12} \cdot 24H_2O$  single crystals at 298 K and 77 K. All crystal field and h. f. parameters are in units of  $10^{-4} cm^{-1}$

Spin-Hamiltonian parameters	Site I		Site II	
	298 K	77 K	298K	77 K
$D$	$-187.2 \pm 1$	$220.0 \pm 1$	$19.4 \pm 2$	$-54.6 \pm 2$
$a-F$	$8.5 \pm 1$	$9.4 \pm 1$	$9.3 \pm 1$	$9.4 \pm 1$
$g_{\parallel}$	$2.0031 \pm 0.0005$	$2.003 \pm 0.0005$	$2.002 \pm 0.001$	$2.001 \pm 0.001$
$g_{\perp}$	$2.0066 \pm 0.0005$	$2.006 \pm 0.0005$	$2.009 \pm 0.002$	$2.009 \pm 0.002$
$A$	$-89.9 \pm 0.5$	$-90.0 \pm 0.5$	$-98.0 \pm 1$	$-89.3 \pm 1$
$B$	$-89.8 \pm 0.5$	$-89.8 \pm 0.5$	$-89.2 \pm 1$	$-89.5 \pm 1$

[5, 6] this sign is taken for  $A$  in this case as well. The examination of the magnitude of the splitting of the hyperfine sextets appearing at high fields and low fields when the magnetic field is parallel to  $z$  axis attributes a sign to the axial parameter  $D$  [7].

The  $g$ -factor for both the sites is found to be isotropic and independent of temperature, any anisotropy being within the experimental error. The small deviation from free spin value may be attributed to the effects of covalent bonding [8] although it is difficult to extract any meaningful covalency parameter out of it.

The value of the hyperfine splitting constant for both the sites is found to be almost isotropic and independent of temperature, any anisotropy being within the experimental error. It was recognized that nearest neighbour interactions are dominating for hyperfine splitting in contrast to other EPR parameters like  $D$ ,  $a$  and  $g$  values where long range interactions can be important [9]. The determination of the hyperfine splitting constant in the EPR spectra of  $Mn^{2+}$  makes it possible to evaluate the nature of the bond between the manganese ion and its immediate surrounding. Van Wieringen [10] has shown that the hyperfine splitting of  $Mn^{2+}$  decreases with increasing covalency of the magnetic complex. The relation between hyperfine parameter (average hyperfine splitting) and the local bonding of the  $Mn^{2+}$  and the nearest neighbour ligands was put phenomenologically on a more quantitative basis by plotting hyperfine parameters versus Pauling's covalency parameter [11-14]. The plots are almost linear. Here, the covalency parameters are derived from the electronegativity of  $Mn^{2+}$  and ligands and the number of ligands, using the expression developed by Hannay and Smyth [15]. The hyperfine parameter in these plots were obtained experimentally from a series of crystals in which  $Mn^{2+}$  is surrounded by the same type of ligands (it is interesting to note that hyperfine splitting is almost independent of the host cation in a series of isomorphous crystals). It is well established that  $Mn^{2+}$  in Tutton salts [16], fluosilicate [17], struvite [18], GASH [19] and perchlorates [20] where it is surrounded by six water molecules, the average value of the hyperfine splitting is observed  $90.0 \times 10^{-4} cm^{-1}$  which corresponds to a covalency of 7% from the curve (hyperfine parameter vs covalency) given by Simanek and Müller [14]. The observed value of the hyperfine splitting in  $NMN: Mn^{2+}$  indicates that  $Mn^{2+}$

at both sites is surrounded by six water molecules since the average value of hyperfine splitting corresponds to a covalency parameter of 7% which is characteristic of  $\text{Mn}(\text{H}_2\text{O})_6^{2+}$  complex. Moreover, the linewidth of  $\text{Mn}^{2+}$  in NMN is of the order of 11 gauss. In hydrated crystals an appreciable degree of line broadening of the order of 10 gauss can originate due to local magnetic fields of the proton moments in the water molecules [8].

From Table I it can be seen that the value of the zero-field splitting parameter  $D$  is very different for the two sites and is sensitive to temperature. The relative insensitivity of the parameter  $D$  of site I to temperature illustrated by the value in Table I has also been noted for  $\text{Ni}^{2+}$  in  $\text{La}_2\text{Mg}_3(\text{NO}_3)_{12} \cdot 24\text{H}_2\text{O}$  (LMN) and  $\text{La}_2\text{Zn}_3(\text{NO}_3)_{12} \cdot 24\text{H}_2\text{O}$  (LZN) as well [21]. The value of  $D$  increases as temperature decreases. An increase in the value of  $D$  at low temperature can probably be explained as due to thermal contraction and vibrational mechanism of the lattice [22, 23]. The differing values of  $D$  for  $\text{Mn}^{2+}$  at two sites indicate that the two sites differ greatly in the static crystal field seen by the ions. The crystal field is presumed to be due to an octahedron of water molecules oriented with the [111] axis along the trigonal axis of the crystal, and it appears that the octahedron is nearly perfect for site II and have appreciable trigonal distortion for the site I. The large difference in the  $D$  value is also observed for  $\text{Ni}^{2+}$  doped LMN, LZN single crystals [21] and for  $\text{Mn}^{2+}$  in  $\text{MgSeO}_4 \cdot 6\text{H}_2\text{O}$  single crystals [24]. In a theoretical analysis of the ground state splitting of the S-state ion it is generally recognized that more than one mechanism is required to account for the magnitude of the parameter  $D$ . Brandt et al. [25] have determined the zero-field splitting parameter  $D$  of  $\text{Mn}^{2+}$  in LMN for both sites. It is interesting to note the value of  $D$  for site I and site II obtained in this experiment for NMN:  $\text{Mn}^{2+}$  are nearly the same as the parameter  $D$  of  $\text{Mn}^{2+}$  in LMN. A theoretical interpretation of the latter for site I was given by Chatterjee et al. [26]. They concluded that the relativistic second-order effect of Wybourne [27] yields the major contribution to the value of the zero-field splitting parameter  $D$ . Their calculated values are in good agreement with the observed values for site I.

#### 4.2. $\text{Gd}^{3+}$

An angular variation study of the EPR spectrum of  $\text{Gd}^{3+}$  in NMN reveal the presence of a single set of a seven line spectrum at 298 K. The presence of only one set of spectra down to liquid nitrogen temperature, corresponding to  $\Delta M = \pm 1$  transition shows that  $\text{Gd}^{3+}$  substitutes only a single type of site. Figure 3 shows the angular variation of the fine structure transition ( $\Delta M = \pm 1$ ) with the magnetic field in the  $zx$  plane. From Fig. 3 it can be seen that the maximum fine structure separation of the absorption lines occurs when the field is parallel to the trigonal axis of the crystal. The subsidiary maxima occur when the magnetic field is perpendicular to the  $c$  axis. The angular variation of the spectrum in the  $xy$  plane (plane perpendicular to the  $c$  axis) shows a periodicity of  $\pi/3$ . This is consistent with  $C_{3i}$  symmetry as well as  $C_{3h}$  symmetry [28]. Measurements at 77 K were made to observe the effect of paramagnetic  $\text{Nd}^{3+}$  ions on the  $\text{Gd}^{3+}$  spectra. However, at this temperature no effect on the spectrum was observed except an increase in the spread of the spectrum. This increase is probably due to the thermal contraction and vibrational mechanism of the crystal. The experiments at 4.2 K were not possible.

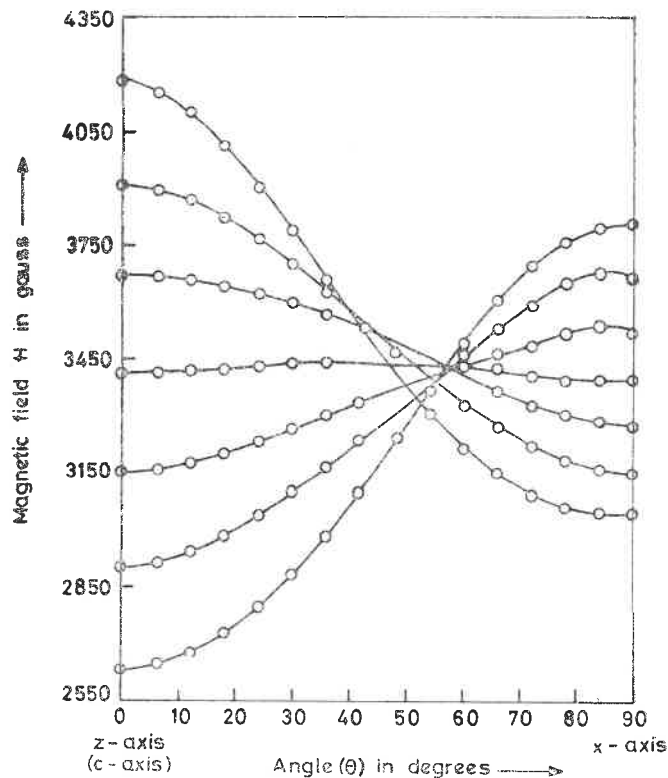


Fig. 3. Angular variation of EPR spectrum in the  $zx$  plane for  $Gd^{3+}$  in  $Nd_2Mg_3(NO_3)_{12} \cdot 24H_2O$  at 298 K. The solid lines are smooth curves that connect data points from the same transition

The EPR spectra of  $Gd^{3+}$  in NMN can be described by the spin-Hamiltonian [29]

$$\mathcal{H} = \beta g H \cdot S + B_2^0 O_2^0 + B_4^0 O_4^0 + B_6^0 O_6^0 + B_6^6 O_6^6,$$

with  $b_2^0 = 3B_2^0$ ,  $b_4^0 = 60B_4^0$ ,  $b_6^0 = 1260B_6^0$ ,  $b_6^6 = 1260B_6^6$ , where the symbols have their usual meaning and  $S = 7/2$  for  $Gd^{3+}$ . Using the above spin-Hamiltonian (neglecting the small term in  $b_6^6$  since they are very small and have matrix elements only between

TABLE II

Spin-Hamiltonian parameters for  $Gd^{3+}$  in  $Nd_2Mg_3(NO_3)_{12} \cdot 24H_2O$  single crystals at 298 K and at 77 K. All crystal field parameters are in units of  $10^{-4} \text{ cm}^{-1}$

Spin-Hamiltonian parameters	298 K	77 K
$b_2^0$	$116.6 \pm 1$	$128.4 \pm 1$
$b_4^0$	$0.276 \pm 0.001$	$0.471 \pm 0.001$
$b_6^0$	$0.4 \pm 0.002$	$0.84 \pm 0.002$
$g_{\parallel}$	$1.9915 \pm 0.0005$	$1.9910 \pm 0.0005$
$g_{\perp}$	$1.9918 \pm 0.0005$	$1.9916 \pm 0.0005$

states for which  $\Delta M = \pm 6$ ),  $\text{Gd}^{3+}$  EPR spectra are analysed and the best-fit parameters thus obtained are listed in Table II. The absolute sign of  $b_2^0$  could not be determined since the measurements made here are only down to 77 K. Assuming  $b_2^0$  to be positive the relative sign of  $b_m^n$  parameters are found.

Following the calculations of Lacroix [30] explaining the deviation of the  $g$  value from the free electron  $g$  value, the parameter  $\alpha$ , which determines the mixture of the excited  ${}^6\text{P}_{7/2}$  with the ground  ${}^8\text{S}_{7/2}$ , is given by

$$g = (1 - \alpha^2)g_s + \alpha^2 g_p,$$

where  $g_s = 2.0023$  and  $g_p = 1.716$  are the  $g$  values for the  ${}^8\text{S}_{7/2}$  and  ${}^6\text{P}_{7/2}$  states respectively. Using the  $g_{\parallel}$  values, the  $\alpha$  values are computed for both the temperatures and are found to be  $0.194 \pm 0.004$  at 298 K and  $0.198 \pm 0.004$  at 77 K. The value of  $\alpha$  was found to be the same at both temperatures (within the experimental error), indicating constancy of the admixture of the  ${}^6\text{P}_{7/2}$  state with the  ${}^8\text{S}_{7/2}$  state down to 77 K.

### 5. Conclusions

The electron paramagnetic resonance of  $\text{Mn}^{2+}$  and  $\text{Gd}^{3+}$  in  $\text{Nd}_2\text{Mg}_3(\text{NO}_3)_{12} \cdot 24\text{H}_2\text{O}$  single crystals is studied. The data presented permit one to assert that in NMN,  $\text{Mn}^{2+}$  ions substitutes for two inequivalent  $\text{Mg}^{2+}$  sites and Mn atoms (and consequently Mg atoms) are surrounded by water molecules. The Gd atoms (and consequently the Nd atoms) in NMN crystals are localized in a position of single type with a magnetic multiplicity factor equal to one and structure multiplicity factor ( $K_x$ ) equal to two ( $K_x = 1$  is excluded by the chemical formula of the double nitrate, and  $K_x > 2$  does not conform with magnetic multiplicity factor). These results are in agreement with the crystallographic data on double nitrates.

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**Editorial note.** This article was proofread by the editors only, not by the author.

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