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FIELD EXCITED BY SOURCES IN ANISOTROPIC THIN-FILM OPTICAL WAVEGUIDES

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A complete system of waves excited by a line-source in layered anisotropic media is presented. A three-layer uniaxial guiding structure with an arbitrary orientation for the optic axis in each layer is considered. The complete field excited by the source in the guiding configuration consists of waves with a discrete spectrum and waves with a cintinuous spectrum. The discrete spectrum contains not only surface waves (guided modes), but also leaky waves. The waves which correspond to the continuous spectrum represent a radiation field.

1. Introduction

The knowledge of space structure and different types of electromagnetic fields propagating in a thin-film dielectric waveguide is of great importance in integrated optics. The propagation and properties of guided waves in isotropic layered media which are of interest in integrated optics are well covered in textbooks on this subject [1–4]. Various thin-film devices utilizing anisotropic guides could become important in the advancing field of integrated optics [9].

While several recent studies have been concerned with the properties of guided modes in anisotropic layered media [5–8], no solution has been obtained for the excitation of such waveguides by a source and on the conversion of the incident field into guided modes. In this paper, an analytic method is presented to determine the complete system of TM-waves excited by a magnetic line source in a three-layer uniaxial guiding structure with an arbitrary optic axis orientation in each layer. When a source is accounted for by expressing the resulting field in the form a Fourier integral, the waves appear as contributions

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produced by pole singularities of the integrand. These waves are referred to as surface waves (guiding modes) and leaky waves. Their presence represents a discrete spectrum contribution. The total field excited by a source in addition to guided modes and leaky waves corresponding to the discrete spectrum also includes radiation waves (space modes), which correspond to the continuous spectrum obtained from the Fourier integral.

Although the discrete modes can be obtained directly via a source-independent procedure [6], such as the transverse-resonance method, a field representation taking into account sources also furnishes some additional information on the discrete modes and their properties. Actually, for the reasons given below, the representation is obtained from another simpler one by a contour deformation in the appropriate complex plane.

2. *Formulation of the problem*

As shown in Fig. 1 the waveguide treated here is a two-dimensional uniaxial structure consisting of three layers; the substrate, the film and the top layer. The thickness of the film is $2l$. The optic axis orientation in each layer is fixed by a diiection at unit vector $\boldsymbol{c}_j$.

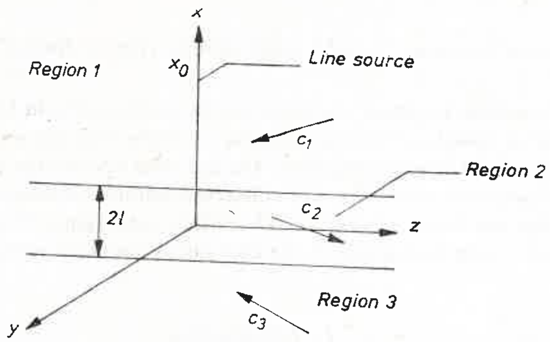

Fig. 1. Geometry for the source-excited anisotropic thin-film waveguide

The relative-permittivity tensors in the j-region ($j = 1, 2, 3$) of guide, corresponding to the optic axis in the x, z-plane (Fig. 1), are all of the form

$$ {}^{(j)}\hat{\varepsilon} = \begin{pmatrix} {}^{(j)}\varepsilon_{11} & 0 & {}^{(j)}\varepsilon_{13} \\ 0 & {}^{(j)}\varepsilon_{22} & 0 \\ {}^{(j)}\varepsilon_{31} & 0 & {}^{(j)}\varepsilon_{33} \end{pmatrix}, \tag{2.1} $$

with

$$ \varepsilon_{ij} = \varepsilon^{o}(\delta_{ij} - c_i c_j) + \varepsilon^{e} c_i c_j, \tag{2.2} $$

where ε^{o} and ε^{e} are the ordinary (o) and extraordinary (e) relative permittivities. The structure is uniform in the y-direction.

A harmonically oscillating magnetic line source excitation is located at $x = x_0$, $z = 0$ in the top layer. The solution of such a source problem is obtained from Maxwell's equations

$$\operatorname{rot} \boldsymbol{H} = -i\omega\varepsilon_0\hat{\varepsilon}E, \tag{2.3}$$

$$\operatorname{rot} \boldsymbol{E} = i\omega\mu_0\boldsymbol{H}-\boldsymbol{j}^m, \tag{2.4}$$

where ε_0 and μ_0 are the permittivity and permeability of vacuum, respectively, and j^m is the source of the magnetic field. A time dependence $\exp(-i\omega t)$ is suppressed. The problem considered is two-dimensional, having no variation along the y-coordinate, i.e. $\partial/\partial y = 0$. The lack of y-dependence plus the form of dielectric tensor (2.1) leads to the separating of the component equations of the (2.3)—(2.4) in two groups describing two independent fields in the considered uniaxial guiding structure:

TE type field

$$\frac{\partial E_y}{\partial z} = -i\omega\mu_0 H_x,$$

$$\frac{\partial E_y}{\partial x} = i\omega\mu_0 H_z,$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = -i\omega\varepsilon_0\varepsilon_{22}E_y. \tag{2.5}$$

TM type field

$$\frac{\partial H_y}{\partial z} = i\omega\varepsilon_0(\varepsilon_{11}E_x+\varepsilon_{13}E_z),$$

$$\frac{\partial H_y}{\partial x} = -i\omega\varepsilon_0(\varepsilon_{13}E_x+\varepsilon_{33}E_z),$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i\omega\mu_0 H_y - j_y^m. \tag{2.6}$$

The TE type field considered above is analogous to the TE field in an isotropic waveguide [8]; it will not be taken into consideration here.

In an unbounded, homogeneous uniaxial medium, the source-excited electromagnetic field may be derived from Green's function. In our problem, the scalar Green's function for magnetic line source excitation, satisfies the inhomogeneous wave equation

$$\left({}^{(1)}\varepsilon_{11}\frac{\partial^2}{\partial x^2} + 2\,{}^{(1)}\varepsilon_{13}\frac{\partial^2}{\partial x\partial z} + {}^{(1)}\varepsilon_{33}\frac{\partial^2}{\partial z^2} + k_0^2\alpha_1\right)G(x-x_0, z) = -\delta(z)\delta(x-x_0). \tag{2.7}$$

Eq. (2.7) is obtained from equations (2.6) for a TM type field. Here, $G(x-x_0, z)$ represents the magnetic field component $H_y(x-x_0, z)$ of the TM field generated by the source $H_y(x-x_0, z) = G(x-x_0, z)$; the remaining components E_x and E_z are given by

$$E_x(x-x_0, z) = i\omega^{-1}\alpha_1^{-1}\left({}^{(1)}\varepsilon_{13}\frac{\partial H_y(x-x_0, z)}{\partial x} + {}^{(1)}\varepsilon_{33}\frac{\partial H_y(x-x_0, z)}{\partial z}\right), \tag{2.8}$$

$$E_z(x-x_0, z) = -i\omega^{-1}\alpha_1^{-1}\left({}^{(1)}\varepsilon_{11}\frac{\partial H_y(x-x_0, z)}{\partial x} + {}^{(1)}\varepsilon_{13}\frac{\partial H_y(x-x_0, z)}{\partial z}\right). \tag{2.9}$$

The components ${}^{(j)}H_y(x, z) = G_j(x, z)$ of the reflected field in region 1 as well as fields in regions 2 and 3 satisfy the homogeneous equations

$$\left({}^{(j)}\varepsilon_{11}\frac{\partial^2}{\partial x^2}+2\,{}^{(j)}\varepsilon_{13}\frac{\partial^2}{\partial x\partial z}+{}^{(j)}\varepsilon_{33}\frac{\partial^2}{\partial z^2}+k_0^2\alpha_j\right)G_j(x, z) = 0, \tag{2.10}$$

where

$$\alpha_j = ({}^{(j)}\varepsilon_{11}\,{}^{(j)}\varepsilon_{33}-{}^{(j)}\varepsilon_{13}^2). \tag{2.11}$$

The components ${}^{(j)}E_x(x, z)$ and ${}^{(j)}E_z(x, z)$ are given by formulas analogous to (2.8) and (2.9).

3. *Integral representation of Green's function for a medium with uniaxial anisotropy*

The Green's function $G(x-x_0, z)$ may be represented by

$$G(x-x_0, z) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(h, x)\exp[-ihz]\,dh, \tag{3.1}$$

where h is the propagation constant of the waves in the z-direction. Substituting (3.1) and the Fourier integral representation of the Dirac delta function $\delta(z)$ into (2.7), we obtain

$$\left({}^{(1)}\varepsilon_{11}\frac{\partial^2}{\partial x^2}-2ih\,{}^{(1)}\varepsilon_{13}\frac{\partial}{\partial x}-{}^{(1)}\varepsilon_{33}h^2+k_0^2\alpha_1\right)g(h, x) = \delta(x-x_0). \tag{3.2}$$

Multiplying in both sides of (3.2) by $(2\pi)^{-1}\exp(i\kappa x)$ and integrating throughout with respect to x from $-\infty$ to $+\infty$, we find

$$g(\kappa, h) = \frac{\exp[i\kappa x_0]}{{}^{(1)}\varepsilon_{11}\kappa^2+2h\,{}^{(1)}\varepsilon_{13}\kappa+{}^{(1)}\varepsilon_{33}h^2-k_0^2\alpha_1}, \tag{3.3}$$

where we define

$$g(\kappa, h) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(h, x)\exp[i\kappa x]\,dx, \tag{3.4}$$

and κ is the propagation constant of the waves in the x-direction. We have

$$g(h, x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(\kappa, h)\exp[-i\kappa x]\,d\kappa, \tag{3.5}$$

and therefore

$$g(h, x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\exp[-i\kappa(x-x_0)]\,d\kappa}{{}^{(1)}\varepsilon_{11}\kappa^2+2\,{}^{(1)}\varepsilon_{13}h\kappa+{}^{(1)}\varepsilon_{33}h^2-k_0^2\alpha_1}. \tag{3.6}$$

Evaluation of the last expression in the complex $\varkappa$-plane gives

$$g(x, h) = \exp\left[\mp {}^{(1)}\varepsilon_{11}^{-1}(\gamma_1 \mp i^{(1)}\varepsilon_{13}h)(x-x_0)\right]/2\gamma_1. \tag{3.7}$$

Finally, therefore, we obtain

$$G(x-x_0, z) = \frac{1}{4\pi}\int_{-\infty}^{+\infty} \frac{\exp\{\mp {}^{(1)}\varepsilon_{11}^{-1}[\gamma_1 \mp i^{(1)}\varepsilon_{13}h](x-x_0) - ihz\}}{\gamma_1}\, dh, \tag{3.8}$$

where $\gamma_1 = \sqrt{\alpha_1(h^2-k_1^2)}$; the minus sign is to be used for $x-x_0 > 0$, the plus sign for $x-x_0 < 0$, $k_1 = k_0\sqrt{{}^{(1)}\varepsilon_{11}}$.

4. *Integral representation of the fields*

The solutions of the homogeneous equations (2.10) may be obtained from a superposition of the waves of the type (3.8) with unknown spectrum functions $g_1(h)$, $g_2(h)$, $g_3(h)$ in the regions 1, 2, 3, respectively. For a reflected wave in region 1, we have

$$G_1(x, z) = \frac{1}{4\pi}\int_{-\infty}^{+\infty} g_1(h)\frac{\exp\{-{}^{(1)}\varepsilon_{11}^{-1}[\gamma_1 - i^{(1)}\varepsilon_{13}h]x - ihz\}}{\gamma_1}\, dh. \tag{4.1}$$

The considered waveguide is asymmetric and the conditions of reflection on the upper and lower surfaces of the film differ from each other. Therefore in region 2, we seek the following form for the field

$$G_2(x, z) = \frac{1}{4\pi}\int_{-\infty}^{+\infty} g_{2+}(h)\frac{\exp\{-{}^{(2)}\varepsilon_{11}^{-1}[\gamma_2 - i^{(2)}\varepsilon_{13}h]x - ihz\}}{\gamma_2}\, dh$$
$$+ \frac{1}{4\pi}\int_{-\infty}^{+\infty} g_{2-}(h)\frac{\exp\{{}^{(2)}\varepsilon_{11}^{-1}[\gamma_2 + i^{(2)}\varepsilon_{13}h]x - ihz\}}{\gamma_2}\, dh, \tag{4.2}$$

where the first part of the expression corresponds to a reflected field on a lower surface, the second to a reflected field on the upper surface of the film, $\gamma_2 = \sqrt{\alpha_2(h^2-k_2^2)}$, $k_2 = k_0\sqrt{{}^{(2)}\varepsilon_{11}}$. Likewise, in region 3 we have

$$G_3(x, z) = \frac{1}{4\pi}\int_{-\infty}^{+\infty} g_3(h)\frac{\exp\{{}^{(3)}\varepsilon_{11}^{-1}[\gamma_3 + i^{(3)}\varepsilon_{13}h]x - ihz\}}{\gamma_3}\, dh, \tag{4.3}$$

where $\gamma_3 = \sqrt{\alpha_3(h^2-k_3^2)}$, $k_3 = k_0\sqrt{{}^{(3)}\varepsilon_{11}}$. We shall integrate the last expressions along the entire real axis of the complex h-plane. The wave fields (4.1) and (4.3) satisfy the radiation conditions since the $\gamma_{1,3}$ has a positive real value when $|h| > k_{1,3}$ and a positive imaginary value when $|h| < k_{1,3}$.

The spectral densities $g_1(h), g_{2+}(h), g_{2-}(h), g_3(h)$ can be obtained from requirement of continuity of the tangential-field at $x = \pm l$. For a TM type of field they may be expressed as follows

$$g_1(h) = \frac{a+b}{c+d} \exp\{{}^{(1)}\varepsilon_{11}^{-1}[\gamma_1 + i^{(1)}\varepsilon_{13}h](l-x_0) + {}^{(1)}\varepsilon_{11}^{-1}[\gamma_1 - i^{(1)}\varepsilon_{13}h]l\}, \tag{4.4}$$

$$g_{2+}(h) = \frac{q}{c+d} \exp\{{}^{(1)}\varepsilon_{11}^{-1}[\gamma_1 + i^{(1)}\varepsilon_{13}h](l-x_0) - {}^{(2)}\varepsilon_{11}^{-1}[\gamma_2 + i^{(2)}\varepsilon_{13}h]l\}, \tag{4.5}$$

$$g_{2-}(h) = \frac{f}{c+d} \exp\{{}^{(1)}\varepsilon_{11}^{-1}[\gamma_1 + i^{(1)}\varepsilon_{13}h](l-x_0) + {}^{(2)}\varepsilon_{11}^{-1}[\gamma_2 - i^{(2)}\varepsilon_{13}h]l\}, \tag{4.6}$$

$$g_3(h) = \frac{m}{c+d} \exp\{{}^{(1)}\varepsilon_{11}^{-1}[\gamma_1 + i^{(1)}\varepsilon_{13}h](l-x_0) + {}^{(3)}\varepsilon_{11}^{-1}[\gamma_3 + i^{(3)}\varepsilon_{13}h]l - 2i^{(2)}\varepsilon_{13}hl\} \tag{4.7}$$

$$a = (\alpha_2^2\gamma_1\gamma_3 - \alpha_1\alpha_3\gamma_2^2)\,\mathrm{sh}\,(2^{(2)}\varepsilon_{11}^{-1}\gamma_2 l), \tag{4.8}$$

$$b = \alpha_2\gamma_2(\alpha_3\gamma_1 - \alpha_1\gamma_3)\,\mathrm{ch}\,(2^{(2)}\varepsilon_{11}^{-1}\gamma_2 l), \tag{4.9}$$

$$c = (\alpha_2^2\gamma_1\gamma_3 + \alpha_1\alpha_3\gamma_2^2)\,\mathrm{sh}\,(2^{(2)}\varepsilon_{11}^{-1}\gamma_2 l), \tag{4.10}$$

$$d = \alpha_2\gamma_2(\alpha_1\gamma_3 + \alpha_3\gamma_1)\,\mathrm{ch}\,(2^{(2)}\varepsilon_{11}^{-1}\gamma_2 l), \tag{4.11}$$

$$q = -\alpha_2\gamma_2(\alpha_2\gamma_3 - \alpha_3\gamma_2), \tag{4.12}$$

$$f = \alpha_2\gamma_2(\alpha_2\gamma_3 + \alpha_3\gamma_2), \tag{4.13}$$

$$m = 2\alpha_2\alpha_3\gamma_2\gamma_3. \tag{4.14}$$

The spectral representations (4.1), (4.2) and (4.3) may be interpreted as waves with a continuous spectrum of eigenfunctions $\exp(-ihz)$ and the real eigenvalues h. Notice that the direction of propagation of the waves of the spectrum is determined by the direction of the x coordinate. The waves will be propagating in region $x > l$ when $|h| < k_{1,3}$ and vanishing when $|h| > k_{1,3}$.

5. *The longitudinal representation of the fields*

From the point of view of physical interpretation, it is convenient to consider the spectrum of waves propagating in the z direction. To obtain a representation which would exhibit such waves explicitly it is necessary to employ a formulation corresponding to transmission in the z direction. However, the modes appropriate to this transmission direction are not so simple as those corresponding to propagation along x because of the more complicated boundary conditions. We shall avoid this difficulty by a suitable deformation of the path of integration in the complex h-plane; then we shall obtain the representation in the z-direction of propagation directly from (4.1)—(4.3).

It is then necessary to take into account that the integrand in (4.1)—(4.3) is not unique since the square-roots $\gamma_1, \gamma_2, \gamma_3$ appear. To make the double-valued functions $\gamma_1, \gamma_2, \gamma_3$

unique, it is necessary to introduce a complex h-plane, with branch cuts providing a means of passing from one Riemann sheet to another. On every sheet the functions $\gamma_1, \gamma_2, \gamma_3$ are uniquely specified. From the radiation conditions which separate the waves running out from those running in, we may choose the proper sheet of the Riemann surface. In our case the radiation conditions require that

$$\operatorname{Re} \gamma_{1,3} \geqslant 0, \tag{5.1}$$

$$\operatorname{Im} \gamma_{1,3} \leqslant 0. \tag{5.2}$$

We take the branch cuts from the branch points $h = \pm k_1$, $h = \pm k_2$, $h = \pm k_3$ in complex h-plane, assuming that the medium has small losses tending to zero (Fig. 2). Each branch cut is drawn along contours on which $\operatorname{Re} \gamma_{1,2,3} = 0$.

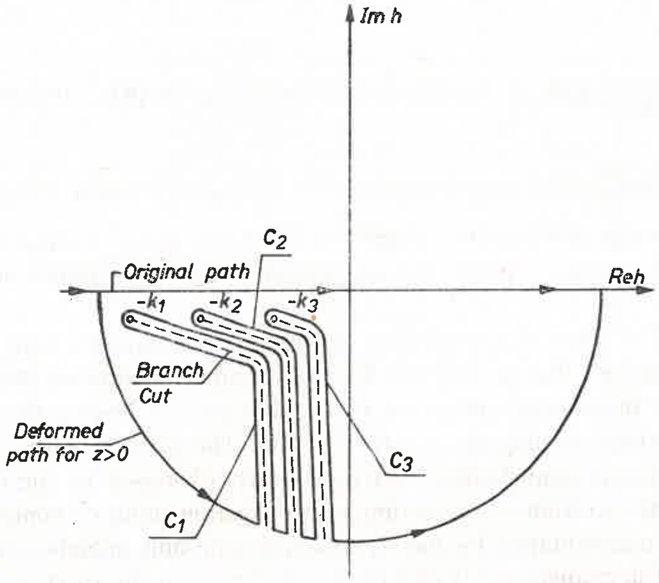

Fig. 2. Path of integration and branch cuts

In such a deformation we must take into account all the singularities of the integrands in (4.1)—(4.3), due to zeros of the denominator in the expressions $g_1(h), g_2(h), g_3(h)$

$$(\alpha_2^2\gamma_1\gamma_3+\alpha_1\alpha_3\gamma_2^2) \operatorname{sh} (2^{(2)}\varepsilon_{11}^{-1}\gamma_2 l)$$
$$+\alpha_2\gamma_2(\alpha_1\gamma_3+\alpha_3\gamma_1) \operatorname{ch} (2^{(2)}\varepsilon_{11}^{-1}\gamma_2 l). \tag{5.3}$$

After localizing all singularities, it is possible to deform the path of integration to obtain the spectral representation of Green's function corresponding to propagation along the z-direction. Referring again to (4.1)—(4.3), it is evident that any deformation of the path of integration must lay entirely in the lower half of the h-plane for $z < 0$ and in the upper half, for $z > 0$. This guarantees the regularity of (4.1)—(4.3) at infinity for arbitrary values of z and x.

In particular, the path may be deformed to a semicircle of the radius tending to infinity. The integral along the semicircle vanishes since the integrand becomes infinitely attenuated. By Cauchy's theorem, the contribution to the integral is then due to the singularities lying between the original and the deformed paths. In this case there will be a contribution due to the integration around a branch cut and residue contributions arising from the presence of the poles.

Accordingly in any j-region of 1, 2, 3 we have

$$G_j(x, z) = \frac{1}{4\pi}\int_{C_1} g_j(h, k_1, k_2, k_3) \exp\{\mp{}^{(j)}\varepsilon_{11}[\gamma_j \mp i^{(j)}\varepsilon_{13}h]x - ihz\}dh$$

$$+\frac{1}{4\pi}\int_{C_2} g_j(h, k_1, k_2, k_3) \exp\{\mp{}^{(j)}\varepsilon_{11}^{-1}[\gamma_j \mp i^{(j)}\varepsilon_{13}h]x - ihz\}dh$$

$$+\frac{1}{4\pi}\int_{C_3} g_j(h, k_1, k_2, k_3) \exp\{\mp{}^{(j)}\varepsilon_{11}^{-1}[\gamma_j \mp i^{(j)}\varepsilon_{13}h]x - ihz\}dh$$

$$+2\pi i\sum_n g_j(h_n, k_1, k_2, k_3) \exp\{\mp{}^{(j)}\varepsilon_{11}^{-1}[\gamma_{nj} \mp i^{(j)}\varepsilon_{13}h_n]x - ih_n z\} \tag{5.4}$$

where the upper sign concerns $x > 0$ and the lower $x < 0$, C_1, C_2 and C_3 are the paths of integration in complex h-plane and correspond to the three branch cuts of the plane (Fig. 2).

Equation (5.4) represents a spectral decomposition corresponding to propagation along the z-direction. This is different from the purely continuous spectrum which is characteristic for propagation along the x-axis. It is easy to observe that the continuous and discrete spectrum propagates in this direction. The surface waves, as well as any other discrete spectral contributions, are now clearly expressed by the residue term. In addition, since the spectrum corresponding to propagation along x is complete (determined by the complete representation for the $\delta(z)$ function) and only analytic continuation arguments were used in changing (3.1) into (5.4), it follows that the spectrum for a representation with respect to propagation along z must also be complete.

6. *Conclusion*

The complete TM field, excited by a source in the considered guiding structure, consists of wave with discrete spectrum and waves with a continuous spectrum. From (5.3) we determine eigenvalues representing the discrete spectrum. For this it will be convenient to rewrite (5.3) in the form

$$\operatorname{tgh}(2^{(2)}\varepsilon_{11}^{-1}\gamma_2 l) = \frac{-\alpha_2\gamma_2(\alpha_1\gamma_3+\alpha_3\gamma_1)}{\alpha_2^2\gamma_1\gamma_3+\alpha_1\alpha_3\gamma_2^2} \tag{6.1}$$

The surface waves correspond to the poles, which are located on the real axis. If h_n is real, then γ_2 is real when $h_n^2 > k_2^2$ and imaginary when $h_n^2 < k_2^2$. This corresponds to the attenuated and the propagated wave in the x-direction inside a waveguide, respectively. Since we desire that the wave field propagates in waveguide, we accept the condition $h_n^2 < k_2^2$.

Then equation (6.1) takes the form

$$\operatorname{tg}(2^{(2)}\varepsilon_{11}^{-1}\gamma_2' l) = \frac{-\alpha_2\gamma_2'(\alpha_1\gamma_3+\alpha_3\gamma_1)}{\alpha_2^2\gamma_1\gamma_3+\alpha_1\alpha_3\gamma_2'^2}, \tag{6.2}$$

where $\gamma_2' = \sqrt{\alpha_2(k_2^2-h_n^2)}$. This is known as the eigenmode equation for an anisotropic waveguide [8]. It gives the absolute condition of existence of the surface waves. Namely, k_1 and $k_3 \leqslant h_n < k_2$.

The discrete spectrum of the surface waves corresponding to real values h_n is only a part of the solutions of the transverse resonance problem. In general, there are also complex roots of (6.1). These roots correspond to the spectral points for which $\mathrm{Re}\,\gamma_{1,2,3} > 0$, $\mathrm{Im}\,\gamma_{1,2,3} < 0$, or nonspectral points i.e. those which lie in the nonphysical sheet of the two-sheeted Riemann surface. The first poles are responsible for the complex waves and the second ones for leaky waves.

The continuous spectrum in (5.4) describes the radiation field, which in the far zone of the regions 1, 3 possesses the form of a divergent wave. The equiphase surface of this wave is an elliptic cylinder. In paper [10], the properties of the waves of the continuous spectrum are examined in detail.

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