Vol. A54 (1978) ACTA PHYSICA POLONICA No 4

SPIN WAVE PARAMETRIC RESONANCE IN THIN FERROMAGNETIC FILM NEAR THE PHASE TRANSITION POINT

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(Received December 30, 1977)

Processes arising under the influence of a variable magnetic field in the uniaxial ferromagnetic film are discussed. The anisotropy axis is perpendicular to the film plane and the magnetic field exists in it. Calculations are made for the case in which the constant component of the external magnetic field is near the critical value with regard to phase transition from homogeneous to inhomogeneous magnetization state i. e. to the domain structure. Possibility of spin wave parametric resonance realization for low frequencies is shown.

Let us consider a thin ferromagnetic film of thickness L. Let us assume that the film surfaces are placed in an (x, y) plane. The dimensions of the film in x and y directions are larger than thickness L. The ferromagnetic material of the film has uniaxial anisotropy. The easy axis of magnetization is perpendicular to the film plane. The anisotropy constant β is less than 4π. The film is located in an external homogeneous magnetic field $\vec{H}^e = (0, H^e, 0)$. Let us also assume that thickness L is larger than critical thickness $L_c(0)$ for which, if $H^e = 0$, the phase transition from homogeneous to inhomogeneous magnetization state occurs [1]. If $L > L_c(0)$, the phase transition appears for the critical field $H_c^e(L)$. If $H^e > H_c^e(L)$, the homogeneous magnetization lies on y axis, but if $H^e < H_c^e(L)$, the domain structure is observed.

The aim of this paper is to describe the phenomena which arise in thin magnetic film under the influence of an external magnetic field $H^e(t) = H^e + \delta H^e(t)$. We assume that:

$$H^e \gtrsim H_c^e(L); \quad \delta H^e(t) = \delta H_0 \sin \omega t; \quad \delta H_0 \ll H^e. \tag{1}$$

The equations of motion of the magnetization density vector $\vec{M}(\vec{r}, t)$ have the Landau form:

$$\frac{\partial \vec{M}}{\partial t} = g[\vec{M} \times \vec{H}^{ef}], \tag{2}$$

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where g is the gyromagnetical coefficient. The effective field $\vec{H}^{ef}$ is defined as follows:

$$\vec{H}^{ef} = -\frac{\delta F}{\delta \vec{M}}. \tag{3}$$

Energy F of the system is expressed by the functional:

$$F = \int_V \{\tfrac{1}{2}\alpha(\nabla\vec{M})^2 - \tfrac{1}{2}\beta M_z^2 - H^e M_y - \tfrac{1}{2}\vec{H}^m\vec{M}\}dV, \tag{4}$$

where $\vec{H}^m$ is the demagnetization field, α — isotropic exchange constant, β — uniaxial anisotropy constant. The equation of motion should be solved simultaneously with Maxwell's equations which have the following form in quasistatic approximation:

$$\text{rot}\,\vec{H}^i = 0$$

$$\text{div}\,(\vec{H}^m + 4\pi\vec{M}) = 0, \tag{5}$$

where $\vec{H}^i = \vec{H}^e + \vec{H}^m$. The set of equations (5) can be expressed as follows:

$$\vec{H}^i = -\text{grad}\,\phi; \quad \phi = \phi^e + \phi^m, \tag{6}$$

$$4\pi\,\text{div}\,\vec{M} - \Delta\phi = 0; \quad \Delta\phi^e = 0, \tag{7}$$

where Δ is Laplace's operator. The system of equations (2), (7) has the following form in the linear approximation with regard to m_x, m_y:

$$(M_0 g)^{-1}\frac{\partial m_x}{\partial t} + \left\{-\alpha\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) - \beta - h\right\} m_z + \frac{\partial\varphi}{\partial z} = 0,$$

$$\left\{h - \alpha\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)\right\} m_x - (M_0 g)^{-1}\frac{\partial m_z}{\partial t} + \frac{\partial\varphi}{\partial x} = 0,$$

$$4\pi\frac{\partial m_x}{\partial x} + 4\pi\frac{\partial m_z}{\partial z} - \Delta\varphi = 0,$$

where

$$\vec{m} = \frac{\delta\vec{M}}{M_0}; \quad h = \frac{H^e}{M_0}; \quad \varphi = \frac{\phi}{M_0}, \tag{8}$$

and $\delta\vec{M}$ is a small deviation of the magnetization density vector from the basic state vector $\vec{M}_0 = (0, M_0, 0)$.

While solving the set of equations (8) the boundary conditions should be taken into consideration on the surfaces $z = \pm L/2$. According to [2] these conditions have following form:

$$\left(\frac{\partial m_x}{\partial z} \pm \eta m_x\right)_{z=\pm\frac{L}{2}} = 0, \quad \left(\frac{\partial m_z}{\partial z} \pm \eta m_z\right)_{z=\pm\frac{L}{2}} = 0, \tag{9}$$

where η is the surface anisotropy constant. The boundary conditions put on φ follow from the continuity of the z-component of the magnetic induction vector $B_z = H_z^i + 4\pi M_z$. They are expressed by the formula:

$$\left[4\pi m_z - \left(\frac{\partial\varphi}{\partial z} - \frac{\partial\varphi^e}{\partial z}\right)\right]_{z=\pm\frac{L}{2}} = 0. \tag{10}$$

We shall express quantities m_x, m_z, φ in Fourier's representation:

$$m_j(x, z, t) = \int d\omega d\kappa dk m_j(\omega, \kappa, k) \exp\{i(\omega t + \kappa x + kz)\}; \quad j = x, z$$

$$\varphi(x, z, t) = \int d\omega d\kappa dk \varphi(\omega, \kappa, k) \exp\{i(\omega t + \kappa x + kz)\}. \tag{11}$$

By means of (11) the equations (8) have the form:

$$i\Omega m_x + (\alpha q^2 - \beta + h)m_z + ik\varphi = 0,$$

$$(\alpha q^2 + h)m_x - i\Omega m_z + i\kappa\varphi = 0,$$

$$4\pi i\kappa m_x + 4\pi i k m_z + q^2\varphi = 0, \tag{12}$$

where $\Omega = (M_0 g)^{-1}\omega$; $q^2 = \kappa^2 + k^2$. The set of equations (12) can have non-trivial solutions only when its determinant equals zero. Such condition allows one to determine Ω:

$$Y = (i\Omega)^2 = q^{-2}\{4\pi\beta\kappa^2 - q^2(\alpha q^2 + h)(\alpha q^2 + h - \beta + 4\pi)\}. \tag{13}$$

For $Y =$ const and $\kappa =$ const equation (13) has three roots with regard to k^2, or six roots with regard to k. A detailed analysis of solutions of the set of equations (12) which satisfy the boundary conditions (9), (10) has been made in [1]. Further considerations will concern the small real root

$$k_n = \frac{n\pi}{L}; \quad n = 1, 2, \ldots . \tag{14}$$

In this case ($q^2 \approx \kappa^2$) relation (13) has the form:

$$Y = \frac{4\pi^3\beta n^2}{\kappa^2 L^2} - (4\pi + h)(\alpha^2\kappa^2 - \beta + h). \tag{15}$$

The maximum of $Y(\kappa)$ exists for $\kappa = \kappa_{01}$, where

$$\kappa_{01}^2 = \frac{\pi}{L}\sqrt{\frac{4\pi h}{\alpha(4\pi + h)}}. \tag{16}$$

$Y(\kappa, h)$ for $n = 1$ is shown in Fig. 1. For $h > h_c$ we have $Y(\kappa, h) < 0$, i. e. $\Omega(\kappa, h)$ is a real quantity describing the precession frequency of spin waves. For $h < h_c$ frequency Ω is an imaginary quantity in the $\kappa_1 < \kappa < \kappa_2$ domain. In this case solutions of the set of equations (8) are proportional to exp $(\sqrt{|Y|}t)$ i. e. they grow in time. The quickest growth is observed for $\kappa = \kappa_{01}$. These solutions describe the process of inhomogeneous magnetization creation, i. e. the creation of domain structure. The point of phase transition is determined by condition $Y(\kappa_{01}, h_c) = Y_0 = 0$ allowing to determine the critical field $h_c(L)$

$$h_c(L) = \beta - \frac{4\pi}{L}\sqrt{\frac{\pi\alpha\beta}{4\pi+\beta}} \tag{17}$$

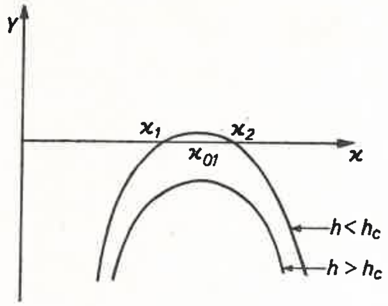

Fig. 1

By means of (16), (17) one can obtain period λ of the domain structure for $L > L_c(0)$ and $h \lesssim h_c$:

$$\lambda = \frac{2\pi}{\kappa_{01}} \cong \sqrt[4]{\frac{\alpha(4\pi+\beta)}{4\pi^2\beta}L^2}\,. \tag{18}$$

Previous considerations have concerned the static case with regard to external magnetic field h. According to (1), let us now put

$$h \to h(t) = h + \delta h_0 \sin\omega t; \qquad h \gtrsim h_c(L); \qquad \delta h_0 \ll h. \tag{19}$$

From (13) one can receive

$$Y(h+\delta h) = Y - (2\alpha q^2 - \beta + 2h + 4\pi)\delta h. \tag{20}$$

If we substitute $(M_0 g)^{-1}\dfrac{\partial}{\partial t}$ for $i\Omega$ in the last two equations of system (12) we obtain

$$m_x = \frac{1}{\Delta_0}\left\{q^2(M_0 g)^{-1}\frac{\partial m_z}{\partial t} - 4\pi\kappa k m_z\right\},$$

$$\varphi = -\frac{1}{\Delta_0}\left\{4\pi i\kappa(M_0 g)^{-1}\frac{\partial m_z}{\partial t} + 4\pi i k(\alpha q^2 + h)m_z\right\}, \tag{21}$$

where

$$\Delta_0 = q^2\left(\alpha q^2 + h + 4\pi\frac{\kappa^2}{q^2}\right). \tag{22}$$

By means of (21) $\left(\text{and } i\Omega \to (M_0 g)^{-1} \frac{\partial}{\partial t}\right)$ the first equation of set (12) has the form:

$$\frac{\partial^2 m_z}{\partial t^2} - \frac{q^2}{\Delta_0} \frac{\partial(\delta h)}{\partial t} \frac{\partial m_z}{\partial t} + \left\{ -(M_0 g)^2 Y + (2\alpha q^2 - \beta + 2h + 4\pi)\delta h + \frac{4\pi\kappa k M_0 g}{\Delta_0} \frac{\partial(\delta h)}{\partial t} \right\} m_z = 0. \tag{23}$$

After introducing the following notation:

$$m_z(t) = \psi(t) \exp\left\{ \frac{q^2 \delta h_0}{\Delta_0} \sin \omega t \right\}, \tag{24}$$

and

$$\omega_0^2 = -(M_0 g) Y, \quad a = \Delta_0^{-1} 4\pi M_0 g \kappa k \omega \delta h_0,$$
$$b = \{(2\Delta_0)^{-1} q^2 \omega^2 + (M_0 g)^2 (2\alpha q^2 + 2h - \beta + 4\pi)\}\delta h_0$$
$$\varepsilon = \omega_0^{-2} \sqrt{a^2 + b^2}; \quad \vartheta = \operatorname{arctg} \frac{a}{b}; \quad \tau = t + \frac{\vartheta}{\omega}, \tag{25}$$

equation (23) has the form

$$\frac{d^2\psi}{d\tau^2} + \omega_0^2 (1 - \varepsilon \cos \omega\tau)\psi = 0. \tag{26}$$

This is Mathieu's equation. Its solution and m_x, φ given by (21) should satisfy the boundary conditions (9), (10). Near the phase transition point, i. e. for $h \gtrsim h_c(L)$, $\kappa \approx \kappa_{01}$, $L \gg L_c(0)$ the boundary conditions have the form [1] $(m_z = 0)_{z=\pm L/2}$ i. e. are identical to Kittel's conditions [4] for the pinning case. According to (14) they are fulfilled for $k_n = n\pi/L$. By means of (25) and after taking into account these assumptions, we shall obtain approximate expressions for ω_0^2 and ε

$$\omega_0^2 = -(M_0 g) Y; \quad \varepsilon = \frac{4\pi + \beta}{|Y|} \delta h_0. \tag{27}$$

Condition $\varepsilon \ll 1$ is a criterion of the smallness of the variable magnetic field amplitude

$$\delta h_0 \ll \frac{|Y|}{4\pi + \beta}. \tag{28}$$

The condition $\varepsilon \ll 1$ is fulfilled for very small quantities $\delta h_0 \approx 0$ near the phase transition point $|Y| \approx 0$.

The solutions of equation (26) can be found with the aid of the asymptotic method [5]. We shall only present the solutions for first parametric resonance frequency, i. e. for $\omega \approx 2\omega_0$. From (27) it follows that $\omega_0 \sim \sqrt{|Y|}$ and resonance frequency $\omega_r = 2M_0 g \sqrt{|Y|} \ll 2M_0 g$ for $h \gtrsim h_c$. In this case the parametric resonance can appear for very low frequencies with regard to $M_0 g$. The solution of equation (26) in second order, with regard to ε approximation, has the form

$$\psi(\tau) = A\cos\left(\frac{\omega}{2}\tau+\theta\right) - \frac{A\varepsilon\omega_0}{8\left(\omega_0+\dfrac{\omega}{2}\right)}\cos\left(\tfrac{3}{2}\omega\tau+\theta\right) \tag{29}$$

where $A = A(\tau)$, $\theta = \theta(\tau)$. Let us introduce the following notation:

$$u = A\cos\theta; \quad v = A\sin\theta; \quad \Delta\omega = \omega_0 - \frac{\omega}{2},$$

$$A = \sqrt{u^2+v^2}; \quad \theta = \operatorname{arctg}\frac{v}{u}. \tag{30}$$

Magnitudes $u(\tau)$, $v(\tau)$ defined by (30) satisfy the following autonomic system of equations:

$$\frac{du}{d\tau} = -\left\{\frac{\varepsilon\omega_0^2}{2\omega}+\Delta\omega+\frac{\varepsilon^2(\omega_0+\omega)\omega_0}{32\left(\omega_0+\dfrac{\omega}{2}\right)}\right\}v,$$

$$\frac{dv}{dt} = \left\{-\frac{\varepsilon\omega_0^2}{2\omega}+\Delta\omega+\frac{\varepsilon^2(\omega_0+\omega)\omega_0}{32\left(\omega_0+\dfrac{\omega}{2}\right)}\right\}u. \tag{31}$$

The solutions of the set (31) have the form:

$$u(\tau) = C_1 e^{S\tau}+C_2 e^{-S\tau},$$

$$v(\tau) = \frac{1}{S}\left\{\frac{\varepsilon^2(\omega_0+\omega)\omega_0}{32\left(\omega_0+\dfrac{\omega}{2}\right)}+\Delta\omega-\frac{\varepsilon\omega_0^2}{2\omega}\right\}(C_1 e^{S\tau}-C_2 e^{-S\tau}), \tag{32}$$

where S is the root of the characteristic equation of system (31)

$$S = \left\{\frac{\varepsilon^2\omega_0^4}{2\omega^2}-\left[\Delta\omega+\frac{\varepsilon^2(\omega_0+\omega)\omega_0}{32\left(\omega_0+\dfrac{\omega}{2}\right)}\right]^2\right\}^{1/2}. \tag{33}$$

The stability domain of solutions $\psi(\tau)$ of equation (26) is defined by condition $S = i|S|$, i. e. by

$$\frac{\omega}{2\omega_0} < 1-\frac{\varepsilon}{4}-\frac{\varepsilon^2}{64}; \quad \frac{\omega}{2\omega_0} > 1+\frac{\varepsilon}{4}-\frac{\varepsilon^2}{64}. \tag{34}$$

The periodic solutions having the period $T = 4\pi/\omega$ are a particular case of solutions of Eq. (26). It occurs for $S = 0$. From this condition the dependence of frequency ω on parameter ε follows. In the second order with regard to the ε approximation this dependence is as follows:

$$\omega = 2\omega_0 \left(1 \pm \frac{\varepsilon}{4} + \frac{5\varepsilon^2}{64}\right)^{-1}. \tag{35}$$

For the same case by means of (24), (25), (29) we can obtain the following dependence of m_z on τ:

$$m_z(\tau) = A_0 \exp\left\{\frac{\delta h_0}{4\pi+\beta} \sin(\omega\tau - \vartheta)\right\} \left[\cos\left(\frac{\omega}{2}\tau + \theta_0\right) - \frac{\varepsilon}{16}\cos\left(\frac{3\omega}{2}\tau + \theta_0\right)\right], \tag{36}$$

where A_0, θ_0 are constants. The nonstability domain of solutions $\psi(\tau)$ of equation (26) is defined by condition $S = S^*$ i. e.

$$1 - \frac{\varepsilon}{4} + \frac{\varepsilon^2}{64} < \frac{\omega}{2\omega_0} < 1 + \frac{\varepsilon}{4} - \frac{\varepsilon^2}{64}. \tag{37}$$

Expression (37) defines the range of frequencies of the external magnetic field for which the parametric resonance phenomenon appears. The phenomenon appears for fixed quantity $\Delta\omega$, if the external magnetic field amplitude δh_0 satisfies (in first order approximation) the following condition:

$$\delta h_0 > \frac{4\sqrt{|Y|}}{M_0 g(4\pi+\beta)} |\Delta\omega|. \tag{38}$$

If we take into account the linear approximation of the equations of motion only, we shall not be able to describe the behavior of the solutions Eq. (26) in the nonstability domain. The growth of the quantities of functions $\psi(\tau)$ or $m_z(\tau)$ is limited by nonlinear terms in the equations of motion. The main nonlinear term appears [1, 3] in the first equation of system (8) or (12). This equation with regard to m_z^3 approximation has the form:

$$(M_0 g)^{-1} \frac{\partial m_x}{\partial t} + \left\{-\alpha\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) - \beta + h\right\} m_z + \tfrac{1}{2}\beta m_z^3 + \frac{\partial \varphi}{\partial z} = 0. \tag{39}$$

In our considerations we have not taken into account the dissipative terms in the equations of motion (2). An additional condition for the variable magnetic field amplitude is obtained after taking into account the weak damping. For $\omega = 2\omega_0$ the parametric resonance appearance condition is as follows:

$$\delta h_0 > h_t = \frac{4\Lambda}{\omega_0}, \tag{40}$$

where Λ is a damping decrement and h_t is a threshold amplitude quantity of the resonance excitation. For $\omega = 2\omega_0/k$ $(k = 1, 2, \ldots)$ the threshold amplitude is proportional to $\sqrt[k]{\Lambda}$, i. e. it grows with the growth of number k. In Fig. 2 the domains of Mathieu's equation nonstability are schematically presented as shaded areas in the $(\omega_0^2, \varepsilon)$ system of coordinates. From (27) the following relation is implied:

$$\varepsilon\omega_0^2 = (4\pi+\beta)(M_0 g)^2 \delta h_0. \tag{41}$$

The relation (41) is shown by the dashed curve in Fig. 2.

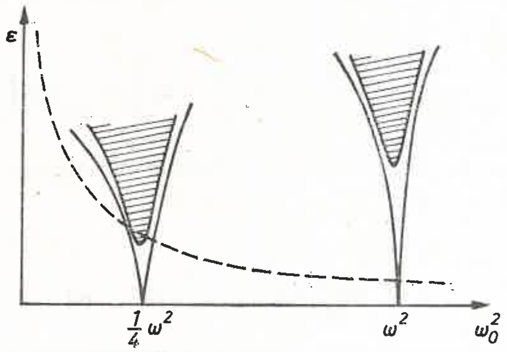

Fig. 2

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