SMALL-GAP EXCITATIONS OF A SUPERFLUID FERMI LIQUID AT ZERO TEMPERATURE AND IN THE ACOUSTIC LIMIT

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The density-density autocorrelation function for superfluid Fermi liquids with BCS and BW pairing is calculated in the acoustic limit and at $T = 0$. We assume that some of the interaction harmonics in the particle-particle channel can be suitably close to that in the pairing channel. This assumption leads to the appearance of collective excitations with a small gap. Our results allow us to establish additional stability conditions for superfluid systems. The case of only one harmonic, close to that in the pairing channel, is discussed in detail.

At vanishing absolute temperature and in the collisionless regime the physical properties of a superfluid Fermi liquid are well described by the effective interactions of two types: the effective interaction in the particle-particle (p–p) channel, connected with a given type of pairing, and the effective interaction in the particle-hole (p–h) channel, i. e. the ordinary Landau function. In all calculations in this paper the LMC method is used. This procedure, for systems with *S*-pairing, was previously given by Larkin and Migdal [1], and subsequently extended by Czerwonko [2], to systems with BW-pairing. This pairing has become more interesting after the identification of such a state with the *B* phase of superfluid ^{3}He [3]. In the case of ^{3}He we have two superfluid phases and the gap matrix changes its angular dependence and hence, it is possible that our results and the results of [4] are connected with a real physical phenomena. On the other hand Foulkes and Gyorffy recently suggested that *P*-pairing or a mixture of *P* and *S* pairing appear in very pure metals Rh, W, Pd in the mK temperature range [5]. Also in this case higher harmonics of the (p–p) interaction may be important.

In the LMC approach, which gives good results in the acoustic limit, the gap equation plays the selfconsistent role. If we know the gap matrix we will get some restrictions for the (p–p) channel [2]. For systems with BCS pairing some of the higher harmonics of the (p–p) channel may be suitably close to the zeroth or first harmonic. The appear-

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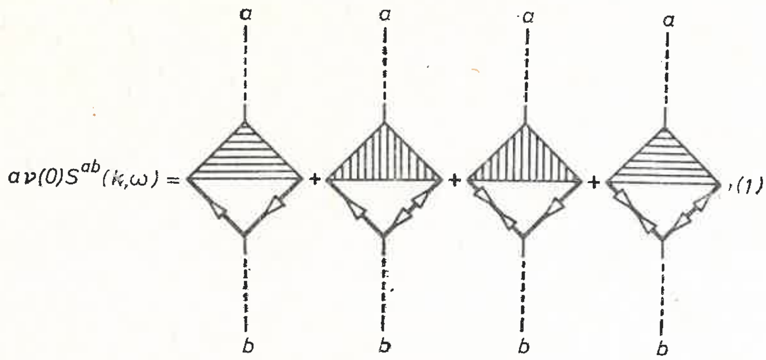
ance of higher harmonics, suitably close to the zeroth or first, leads to small-gap excitations [4].

In our paper we will discuss only the spin-independent autocorrelation function. Similar calculation for spin susceptibility has been performed by Czerwonko in papers [4, 6]. Our formulae for the autocorrelation function will be obtained at zero absolute temperature and for the acoustic limit, i. e. ω, $\boldsymbol{kv} \ll \Delta$, where v denotes the velocity of quasiparticles on the Fermi sphere, and Δ is the gap energy. We discuss only the case of a finite number of nonvanishing harmonics.

In the general case, independent of the type of pairing, we have the following graphical expression for the correlation function $a\nu(0)S^{ab}(\boldsymbol{k}, \omega)$



where the effective anomalous vertices $\mathscr{T}_{(1)}, \mathscr{T}_{(2)}$ are lined vertically, the effective normal vertices are lined horizontally and $\nu(0)$ denotes the density of states on the Fermi sphere [2].

Let us start from BCS systems. Equation (1) in its analytical form, together with the equations for the normal $\mathscr{T}_a$ vertex and the anomalous λ_a vertex, form the complete set of equations. We have

$$S^{ab}(\boldsymbol{k}, \omega) = \langle \mathscr{T}_a^{(0)}[L\mathscr{T}_b(\hat{\boldsymbol{p}}') - O\mathscr{T}_b(-\hat{\boldsymbol{p}}') + 2M\lambda_b(\hat{\boldsymbol{p}}')]\rangle_{\hat{p}'},$$

$$\mathscr{T}_b(\hat{\boldsymbol{p}}) = \mathscr{T}_b^{\omega}(\hat{\boldsymbol{p}}) + \langle A(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}')\,[L\mathscr{T}_b(\hat{\boldsymbol{p}}') - O\mathscr{T}_b(-\hat{\boldsymbol{p}}') + 2M\lambda_b(\hat{\boldsymbol{p}}')]\rangle_{\hat{p}'},$$

$$\lambda_b(\hat{\boldsymbol{p}}) = \left\langle f^{\xi}_{\mp 1}(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}')\left[\left(N + O + \ln\frac{2\xi}{\Delta}\right)\lambda_b(\hat{\boldsymbol{p}}') - 2M\mathscr{T}_b(\hat{\boldsymbol{p}}')\right]\right\rangle_{\hat{p}'}. \tag{2}$$

The circumflex over a symbol denotes a unit vector directed along a given vector and brackets $\langle ... \rangle_{\hat{p}'}$ denote the averaging over spherical angles. In addition, $A(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}')$ is the spin-direct part of the Landau interaction and $f^{\xi}_{\mp 1}(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}')$ is the spin-antisymmetric part of the (p–p) interaction (see Appendix A). We will derive only the density-density autocorrelation function $S^{00}(\boldsymbol{k}, \omega) = S(\boldsymbol{k}, \omega)$. The poles of the $S^{ab}(\boldsymbol{k}, \omega)$ for the vector vertices a and b are the same as poles of $S(\boldsymbol{k}, \omega)$ for both BCS and BW types of pairing (see Appendix C). The anomalous vertices $\hat{\mathscr{T}}_{(1)}, \hat{\mathscr{T}}_{(2)}$, with two incoming or outgoing lines, are connected with λ as:

$$\hat{\mathscr{T}}_{(1)} = -i\sigma^y\lambda, \quad \hat{\mathscr{T}}_{(2)} = +i\sigma^y\lambda,$$

as a result of time-reversal invariance.

It is obvious that the most interesting case from the physical point of view is the one with only two harmonics, suitably close to one another. This is equivalent to the case of two nonvanishing harmonics, at least in the acoustic limit. Hence, we restrict ourselves here to the detailed discussion of this case. The general discussion is given in Appendix A. For the present case, we have to solve a system of two linear equations

$$a_0\lambda_0+c_0\lambda_2 = -\omega\mathcal{T}^{\omega}, \quad b_2\lambda_0+a_2\lambda_2 = 0, \tag{3}$$

with formula for a_0, a_2, b_2, c_0 given in Appendix A. From (3) we obtain the function $S(k, \omega)$

$$S(k,\omega) = -\frac{\mathcal{T}^{\omega}}{D_0}-\frac{\omega}{D_0}\lambda_0, \quad D_j = 1+A_j,$$

$$S(k,\omega) = \frac{\mathcal{T}^{\omega}}{3}D_0k^2v^2$$

$$\times\frac{\omega^2-D_2\ln\dfrac{\Delta}{r_2}-\frac{1}{35}k^2v^2D_2(9D_3-\frac{44}{3}D_1)}{(\omega^2-\frac{1}{3}k^2v^2D_0D_1)\left[-D_2\ln\dfrac{\Delta}{r_2}+\omega^2-\frac{1}{5}k^2v^2D_2(\frac{4}{3}D_1+\frac{9}{7}D_3)\right]-\frac{8}{35}k^4v^4D_0D_1^2D_2} \tag{4}$$

where ω, kv are taken in 2Δ units. For $\left|\ln\dfrac{\Delta}{r^2}\right| \gtrsim 1$ we obtain the formula previously obtained in paper [2]

$$S(k,\omega) = \frac{1}{3}\frac{\mathcal{T}^{\omega}D_0}{\omega^2-\frac{1}{3}k^2v^2D_0D_1}k^2v^2.$$

Formula (4) yields two branches of excitations

$$2\omega^2 = \delta_2^2+k^2v^2(V_2^2+V_0^2)\pm\{[\delta_2^2+k^2v^2(V_2^2-V_0^2)]^2+4Uk^4v^4\}^{1/2}, \tag{5}$$

where

$$\delta_2^2 = D_2\ln\frac{\Delta}{r_2}, \quad V_2^2 = \tfrac{1}{5}D_2(\tfrac{4}{3}D_1+\tfrac{9}{7}D_3), \quad U = \tfrac{8}{35}D_0D_1^2D_2.$$

The nonlinear dependence of ω with respect to k in the whole acoustic regime is connected with the appearance of two units of distance if two harmonics of the effective interaction in the (p–p) channel are suitably close to one another (cf. [6]). For $k^2v^2 \ll \delta_2^2$, i. e. in the acoustic regime but with regard to the energy gap δ_2^2, formula (5) gives

$$\omega^2 = \tfrac{1}{3}k^2v^2D_0D_1 \tag{6a}$$

— the well known sound branch spectrum, and

$$\omega^2 = D_2\ln\frac{\Delta}{r_2}+\tfrac{1}{5}k^2v^2D_2(\tfrac{4}{3}D_1+\tfrac{9}{7}D_3) \tag{6b}$$

— the small-gap excitation. From the general formulas given in Appendix A it follows that for very small k each harmonic, suitably close to that in the pairing channel, leads to a small-gap excitation of type (6b).

For the systems with BW pairing the equations and results are quite similar. In this case the system (2) has the following form [2]

$$S(\boldsymbol{k},\omega) = \langle\{L\mathscr{T}(\hat{p}')-O\mathscr{T}(-\hat{p}')-M[\hat{T}(\hat{p}'),(\hat{\sigma}\hat{p}')]_+\}\rangle_{\hat{p}'},$$

$$\mathscr{T}(\hat{p}) = \mathscr{T}^{\omega}+\langle A(\hat{p}\hat{p}')\{L\mathscr{T}(\hat{p}')-O\mathscr{T}(-\hat{p}')-M[\hat{T}(\hat{p}'),(\hat{\sigma}\hat{p}')]_+\}\rangle_{\hat{p}'}, \tag{7}$$

$$\hat{T}(\hat{p}) = \left\langle f^{\xi}_{-1}(\hat{p}\hat{p}')\left\{\left(N+\ln\frac{2\xi}{\Delta}\right)\hat{T}(\hat{p}')+O(\hat{\sigma}\hat{p}')\hat{T}(\hat{p}')(\hat{\sigma}\hat{p}')+2M(\hat{\sigma}\hat{p}')\mathscr{T}(\hat{p}')\right\}\right\rangle_{\hat{p}'},$$

where $\hat{T}(\hat{p}) = \hat{\kappa}_1\sigma^y = \sigma^y\hat{\kappa}_2$ and $\hat{\kappa}_i$ is the spin symmetric part of the anomalous vertex $\mathscr{T}_{(i)}$. In analogy to the BCS case, we will solve (7) for the two harmonic model, i. e.

$$t_1(\omega^2-\tfrac{3}{5}k^2v^2D_0D_1)+t_3(-\tfrac{2}{5}k^2v^2D_0D_1)+h_0(\omega^2-\tfrac{1}{3}k^2v^2D_0D_1)$$
$$+h_2(-\tfrac{2}{3}k^2v^2D_0D_1) = \omega\mathscr{T}^{\omega},$$
$$-\tfrac{3}{5}t_1+\tfrac{3}{5}t_3+h_0(-\tfrac{2}{15}k^2v^2D_1)+h_2\left[\frac{\omega^2}{D_2}-\tfrac{1}{5}k^2v^2(\tfrac{4}{3}D_1+\tfrac{9}{7}D_3)\right] = 0,$$
$$+\tfrac{2}{5}t_1-\tfrac{2}{5}t_3+h_0(-\tfrac{2}{15}k^2v^2D_1)+h_2\left[-\ln\frac{\Delta}{r_3}+\frac{\omega^2}{D_2}-\tfrac{1}{5}k^2v^2(\tfrac{4}{3}D_1+\tfrac{9}{7}D_3)\right] = 0,$$
$$t_1[-\tfrac{6}{35}k^2v^2(\tfrac{5}{9}+\tfrac{4}{9}D_3)]+(-\tfrac{5}{9}t_3)+h_2(-\tfrac{4}{21}k^2v^2D_3) = 0, \tag{8}$$

where ω, $\boldsymbol{kv}$ are taken in 2Δ units. The meaning of the symbols and a more general discussion of this case is given in Appendix B. According to (4), the autocorrelation function has the form

$$S(\boldsymbol{k},\omega) = \frac{\mathscr{T}^{\omega}}{3}k^2v^2D_1$$
$$\times\frac{\omega^2-\tfrac{3}{5}\ln\dfrac{\Delta}{r_3}+\tfrac{9}{35}k^2v^2D_2D_3}{(\omega^2-\tfrac{1}{3}k^2v^2D_0D_1)\left[\omega^2-\tfrac{3}{5}\ln\dfrac{\Delta}{r_3}-\tfrac{1}{5}D_2k^2v^2(\tfrac{4}{3}D_1+\tfrac{9}{7}D_3)\right]-\tfrac{4}{45}k^4v^4D_0D_1^2D_2}. \tag{9}$$

Formula (9) yields the two branches of excitations

$$2\omega^2 = \delta_3^2+k^2v^2(V_2^2+V_0^2)\pm\{[\delta_3^2+k^2v^2(V_2^2-V_0^2)]^2+4U'k^4v^4\}^{1/2}, \tag{10}$$

where $\delta_3^2 = \tfrac{3}{5}D_2\ln\dfrac{\Delta}{r_3}$, $U' = \tfrac{4}{45}D_0D_1^2D_2$. In the acoustic regime, with regard to the energy gap δ_3, i. e. for $\boldsymbol{kv} \ll \delta_3$, we find

$$\omega^2 = \tfrac{1}{3}k^2v^2D_0D_1, \tag{11a}$$

i. e. the same result as (6a), and

$$\omega^2 = \tfrac{3}{5} D_2 \ln \frac{\Delta}{r_3} + \tfrac{1}{5} k^2 v^2 D_2 (\tfrac{4}{3} D_1 + \tfrac{9}{7} D_3), \tag{11b}$$

— the small-gap excitation branch. Also in this case formula (9) for $\left|\ln \frac{\Delta}{r_3}\right| \gtrsim 1$ gives the well known result derived in [2].

In addition we can prove that all terms on the right-hand sides of Eqs. (6a, b), (11a,b) are positive. From the Pomeranchuk inequality ($D_j > 0$, [7]) we find that all terms proportional to k^2v^2 are positive. In order to obtain the suitable inequalities for δ_2^2, δ_3^2, we use the spectral representation of the autocorrelation function $S(\boldsymbol{k}, \omega)$, following from [6]. We have

$$S(\boldsymbol{k}, \omega) = \frac{1}{av(0)} \sum_n \frac{2\omega_{n0} |\varrho_{kn0}|^2}{(\omega + i\delta)^2 - \omega_{n0}^2}, \tag{12}$$

where ω_{n0} is the excitation energy of the n-th excited state and ϱ_{kn0} is the transition element between the ground (0) and the n-th excited state of the $\boldsymbol{k}$-th Fourier transform of the density operator ϱ, $\delta = 0^+$. From (12) one can see that all terms near ω^{-2m} ($m = 1, 2, \ldots$) in the series expansion of (12) are positive. This leads to the inequalities

$$\ln \frac{\Delta}{r_2} > 0, \quad \ln \frac{\Delta}{r_3} > 0. \tag{13}$$

The meaning of these inequalities is very simple. They mean that the interaction in the pairing channel is always stronger than in the remaining channels, cf. [6]. Conditions (13) play the role of additional stability conditions for superfluid Fermi liquids, in comparison to $D_l > 0$.

The spin-independent collective excitations with a small gap for BCS were considered in paper [8], but without the Fermi liquid interaction. The discussion for spin vertices is given in paper [4] for BW pairing and in paper [6] for BCS systems.

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APPENDIX A

We assume that the p–p interaction conserves independently the total spin and the angular momentum. In such a case, this interaction is divided into two parts: first, f^{ξ}_{+1}, which describes the interaction with vanishing total spin, acting in the BCS case, and the part f^{ξ}_{-1}, which is connected with the BW state, with the total spin equal to unity (cf. [2]). Because of the Pauli restriction, the function $f^{\xi}_{+1}(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}')$ contains only even Legendre polynomials

$$f^{\xi}_{+1}(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}') = \sum_{l=0}^{n} (4l+1) f^{\xi}_{+1,2l} P_{2l}(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}'), \tag{A1}$$

where

$$f^{\xi}_{+1,2l} = \left(\ln \frac{2\xi}{r_{2l}}\right)^{-1}, \quad r_0 = \Delta,$$

and ξ is the cut-off parameter [2]. The vertices should be of the form

$$\mathcal{T}(\hat{p}) = \sum_l (2l+1)\mathcal{T}_l P_l(\hat{\boldsymbol{k}}\hat{\boldsymbol{p}}), \quad \lambda(\hat{p}) = \sum_l (4l+1)\lambda_{2l} P_{2l}(\hat{\boldsymbol{k}}\hat{\boldsymbol{p}}). \tag{A2}$$

In the acoustic limit one can use the following expansions for the L, M, N, O functions [2]

$$O = -L = \tfrac{1}{2}, \quad 2M = -\omega - \boldsymbol{kv}, \quad N = \omega^2 - (\boldsymbol{kv})^2 - \tfrac{1}{2},$$

where ω and $\boldsymbol{kv}$ are taken in 2Δ units. After a simple integration over spherical angles we find the following system of ordinary linear equations

$$S(\boldsymbol{k}, \omega) = -\frac{\mathcal{T}^{\omega}}{D_0} - \frac{\omega}{D_0}\lambda_0, \quad \text{where } \mathcal{T}^{\omega} = \frac{1}{a}\ [2],$$

$$\mathcal{T}_0 = \frac{\mathcal{T}^{\omega}}{D_0} - \frac{A_0}{D_0}\omega\lambda_0,$$

$$\mathcal{T}_{2l} = -\frac{A_{2l}}{D_{2l}}\omega\lambda_{2l}, \quad \text{for } l = 1, 2, \ldots, n,$$

$$(4l+3)\mathcal{T}_{2l+1} = -kv(2l+1)A_{2l+1}\lambda_{2l} - kv2(l+1)A_{2l+1}\lambda_{2l+2},$$

$$\text{for } l = 0, 1, 2, \ldots, n, \tag{A3}$$

$$a_0\lambda_0 + c_0\lambda_2 = -\omega\mathcal{T}^{\omega},$$

$$b_{2l}\lambda_{2l-2} + a_{2l}\lambda_{2l} + c_{2l}\lambda_{2l+2} = 0, \quad \text{for } l = 1, 2, \ldots, n,$$

where

$$a_{2j} = -D_{2j}\ln\frac{\Delta}{r_{2j}} + \omega^2 - k^2v^2V_{2j}^2,$$

$$b_{2j} = -k^2v^2\frac{2(j+1)(2j+1)}{4j+3}\frac{D_{2j}D_{2j-1}}{4j+1},$$

$$c_{2j} = -k^2v^2\frac{2(j+1)(2j+1)}{4j+3}\frac{D_{2j}D_{2j+1}}{4j+1},$$

$$V_{2j}^2 = \frac{D_{2j}}{4j+1}\left[\frac{4j^2}{4j-1}D_{2j-1} + \frac{(2j+1)^2}{4j+3}D_{2j+1}\right],$$

$D_j = 1 + A_j$, A_j denotes the Landau amplitudes

$$A(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}') = \sum_j (2j+1)A_j P_j(\hat{\boldsymbol{p}}\hat{\boldsymbol{p}}'), \quad r_{2j} \neq 0, \text{ for } j = 0, 1, \ldots, n.$$

If r_{2j} vanishes for some index j_0, then harmonics with indices higher than j_0 will not appear in the autocorrelation function. From (A3) we derive the general formula for the autocorrelation function $S(k, \omega)$

$$S(k, \omega) = -\frac{\mathscr{T}^{\omega}}{D_0}\left(1-\frac{3}{2}\frac{\omega^2}{D_0 D_1^2}\frac{R_n}{Q_n}\right),$$

where

$$R_n = \gamma_{2n}\dots\gamma_2 - k^4v^4\sum_{j=1}^{n-1}\frac{\gamma_{2n}\dots\gamma_2}{\gamma_{2j+2}\gamma_{2j}} + \dots + (-k^4v^4)^{\tilde{n}}\begin{cases}\gamma_2+\gamma_6+\dots+\gamma_{2n} & n \text{ odd}\\ 1 & n \text{ even}\end{cases}$$

$$Q_n = \gamma_{2n}\dots\gamma_2\gamma_0 - k^4v^4\sum_{j=0}^{n-1}\frac{\gamma_{2n}\dots\gamma_2\gamma_0}{\gamma_{2j+2}\gamma_{2j}} + \dots + (-k^4v^4)^{\approx n}\begin{cases}1 & n \text{ odd}\\ \gamma_0+\gamma_2+\dots+\gamma_{2n} & n \text{ even}\end{cases}$$

$$\tilde{n} = \begin{cases}\frac{1}{2}n-\frac{1}{2} & n \text{ odd}\\ \frac{1}{2}n & n \text{ even,}\end{cases}$$

$$\approx n = \begin{cases}\frac{1}{2}n+\frac{1}{2} & n \text{ odd}\\ \frac{1}{2}n & n \text{ even,}\end{cases}$$

$$\gamma_0 = \frac{3}{2}\frac{a_0}{D_0D_1^2}, \quad \gamma_{2j} = a_{2j}\frac{4j+1}{2j+1}\frac{4j+3}{2j+2}\frac{1}{D_{2j}D_{2j-1}^2}\frac{D_{2j-3}^2}{\dfrac{D_{2j-5}^2}{\dfrac{\dots}{\dfrac{\dots}{D_3^2}}}},$$

for $j = 1, 2, \dots, n$. The singularities of $S(k, \omega)$ depend on the n parameters $\ln\dfrac{\Delta}{r_{2j}}$ $(j = 0, 1, 2, \dots, n)$ on the energy scale. For very small k i. e. $k^2v^2 \ll \left|D_{2j}\ln\dfrac{\Delta}{r_{2j}}\right|$, we obtain excitations with a small energy gap:

$$\omega^2 = D_{2j}\ln\frac{\Delta}{r_{2j}} + k^2v^2V_{2j}^2, \quad \text{for } \left|\ln\frac{\Delta}{r_{2j}}\right| \ll 1. \tag{A5}$$

APPENDIX B

Let us solve the system of Eqs. (7), corresponding to the BW case. The vertices should be of the form

$$\mathscr{T}(\hat{p}) = \sum_l (2l+1)\mathscr{T}_l P_l(\hat{k}\hat{p}),$$

$$\hat{T}(\hat{p}) = \hat{k}\hat{\sigma}\sum_l (4l+3)t_{2l+1}P_{2l+1}(\hat{k}\hat{p}) + \hat{p}\hat{\sigma}\sum_l (4l+1)h_{2l}P_{2l}(\hat{k}\hat{p}).$$

Moreover, the p–p channel should contain only odd Legendre polynomials

$$f^{\xi}_{-1}(\hat{p}\hat{p}') = \sum_{l=0}^{n} (4l+3) f^{\xi}_{-1,2l+1} P_{2l+1}(\hat{p}\hat{p}'),$$

$$f^{\xi}_{-1,2l+1} = \left(\ln \frac{2\xi}{r_{2l+1}}\right)^{-1} \quad \text{with } r_1 = \Delta, \text{ cf. paper [2].}$$

We discuss only the case where

$$f^{\xi}_{-1,2j+1} \neq 0 \quad \text{for } j = 0, 1, \ldots, n, \quad f^{\xi}_{-1,2j+1} = 0 \quad \text{for } j > n.$$

Performing all integrations over spherical angles we obtain the following system of linear equations:

$$\mathcal{T}_0 = \frac{\mathcal{T}^{\omega}}{D_0} + \frac{A_0}{D_0}\omega(t_1+h_0),$$

$$\mathcal{T}_{2l} = \frac{A_{2l}}{D_{2l}}\omega\left(\frac{2l+1}{4l+1}t_{2l+1} + \frac{2l}{4l+1}t_{2l-1} + h_{2l}\right), \quad \text{for } l = 1, 2, \ldots, n,$$

$$\mathcal{T}_{2l+1} = A_{2l+1}kv\left[\frac{8l^2+12l+3}{(4l+1)(4l+5)}t_{2l+1} + \frac{2l}{4l+1}\,\frac{2l+1}{4l+3}t_{2l-1}\right.$$
$$\left. + \frac{2(l+1)}{4l+3}\,\frac{2l+3}{4l+5}t_{2l+3} + \frac{2l+1}{4l+3}h_{2l} + \frac{2(l+1)}{4l+3}h_{2(l+1)}\right], \quad \text{for } l = 0, 1, \ldots, n,$$

$$\omega\mathcal{T}^{\omega} = t_1(\omega^2 - \tfrac{3}{5}k^2v^2D_0D_1) + t_3(-\tfrac{2}{5}k^2v^2D_0D_1) + h_0(\omega^2 - \tfrac{1}{3}k^2v^2D_0D_1)$$
$$+ h_2(-\tfrac{2}{3}k^2v^2D_0D_1),$$

$$0 = t_{2l-3}\left[-k^2v^2\frac{2(l-1)}{4l-3}\,\frac{2l-1}{4l-1}\,\frac{2l}{4l+1}A_{2l-1}\right] + \frac{2l}{4l+1}t_{2l-1} - \frac{2l}{4l+1}t_{2l+1}$$
$$+ t_{2l+3}\left(-k^2v^2\frac{2l+2}{4l+3}\,\frac{2l+3}{4l+5}\right)\left(1 + \frac{2l+1}{4l+1}A_{2l+1}\right) + h_{2l-2}\left(-k^2v^2\frac{2l-1}{4l-1}\,\frac{2l}{4l+1}D_{2l-1}\right)$$
$$+ h_{2l}\left[-\ln\frac{\Delta}{r_{2l+1}} + \frac{\omega^2}{D_{2l}} - \frac{k^2v^2}{4l+1}\left(\frac{4l^2}{4l-1}D_{2l-1} + \frac{(2l+1)^2}{4l+3}D_{2l+1}\right)\right]$$
$$+ h_{2l+2}\left(-k^2v^2\frac{2l+1}{4l+1}\,\frac{2l+2}{4l+3}D_{2l+1}\right), \quad \text{for } l = 1, 2, \ldots, n,$$

$$0 = t_{2l-1}\left[-k^2v^2\frac{2l}{4l+1}\,\frac{2l+1}{4l+3}\left(1 + \frac{2l+2}{4l+5}A_{2l+1}\right)\right] - \frac{2l+3}{4l+5}t_{2l+1} + \frac{2l+3}{4l+5}t_{2l+3}$$
$$+ t_{2l+5}\left(-k^2v^2\frac{2l+3}{4l+5}\,\frac{2l+5}{4l+7}\,\frac{2(l+3)}{4l+9}A_{2l+3}\right) + h_{2l}\left(-k^2v^2\frac{2l+1}{4l+3}\,\frac{2l+2}{4l+5}D_{2l+1}\right)$$

$$+h_{2l+2}\left[-\ln\frac{\Delta}{r_{2l+1}}+\frac{\omega^2}{D_{2l+2}}-\frac{k^2v^2}{4l+5}\left(\frac{4(l+1)^2}{4l+3}D_{2l+1}+\frac{(2l+3)^2}{4l+7}D_{2l+3}\right)\right]$$

$$+h_{2(l+2)}\left(-k^2v^2\frac{2l+3}{4l+5}\frac{2l+4}{4l+7}D_{2l+3}\right),\quad \text{for } l=0,1,2,\ldots,n. \tag{B1}$$

$$S(k,\omega)=-\frac{\mathcal{T}^{\omega}}{D_0}+\frac{\omega}{D_0}(t_1+h_0). \tag{B2}$$

From the system of equations (B1), (B2) one finds that for $\left|\ln\frac{\Delta}{r_{2l+1}}\right| \ll 1$, the excitations with a gap should be of the form

$$\omega^2 = D_{2(l+1)}\ln\frac{\Delta}{r_{2l+1}}+\frac{2l+3}{4l+5}\ln\frac{r_{2l+1}}{r_{2l+3}}+k^2v^2W^2_{2(l+1)}. \tag{B3}$$

For the velocity of these excitations we find

$$W^2_{2(l+1)} = V^2_{2(l+1)}-V'^2_{2(l+1)},\quad \text{for } l=0,1,2,\ldots,n.$$

Note that the term $V^2_{2(l+1)}$ was defined in the BCS case and for the term $V'^2_{2(l+1)}$ we have:

$$V'^2_{2(l+1)}=\left\{D_{2(l+1)}\frac{2l(2l+1)}{(4l+1)(4l+3)}\ln\frac{\Delta}{r_{2l+1}}+\frac{2l+2}{4l+5}\frac{2l+3}{4l+7}\left(1+\frac{2l+3}{4l+9}A_{2l+3}\right)\right\}$$
$$+\frac{2l+2}{4l+5}\frac{2l+3}{4l+7}D_{2(l+1)}\ln\frac{r_{2l+3}}{r_{2l+1}}\ln\frac{\Delta}{r_{2l+3}}\left(\ln\frac{r_{2l+5}}{r_{2l+3}}\right)^{-1}$$
$$+\frac{2l}{4l+1}\left(1+\frac{2l+1}{4l+5}A_{2l+1}\right)\ln\frac{r_{2l+3}}{r_{2l+1}}\left(\ln\frac{r_{2l+3}}{r_{2l-1}}\right)^{-1}$$
$$+\frac{2l+1}{4l+3}\frac{2l+2}{4l+5}D_{2l+1}D_{2l+2}. \tag{B4}$$

The term in curly brackets in (B4) does not appear for $l=n$. As we have shown there appear also the term $\ln\frac{r_{2l+3}}{r_{2l+1}}$, not only $\ln\frac{\Delta}{r_{2l+1}}$.

APPENDIX C

In this appendix we check the gauge-invariance conditions for vertex functions in the acoustic limit for both BCS and BW pairing. Such conditions were given for BW pairing in paper [2]

$$-k_a\mathcal{T}_a+\omega\mathcal{T}=\frac{\omega-kv}{a},\quad \text{(on the Fermi sphere)}$$

$$-k_a\hat{T}_a+\omega\hat{T}=\frac{\Delta}{a}\hat{\sigma}\hat{p}, \tag{C1}$$

and in paper [1] for BCS systems

$$-k_a\mathscr{T}_a+\omega\mathscr{T} = \frac{\omega-\boldsymbol{kv}}{a}, \quad -k_a\hat{T}_a+\omega\hat{T} = \frac{\Delta}{a}, \tag{C2}$$

where $\mathscr{T}$, T are scalar vertices, whereas $\mathscr{T}_a$, T_a are vector vertices. The charge of quasi-particles with respect to an external field is equal to $a\mathscr{T}^\omega = 1$. The equations for the quantities

$$X = -k_a\mathscr{T}_a+\omega\mathscr{T}, \quad \hat{Y} = -k_a\hat{T}_a+\omega\hat{T},$$

are the same as systems (7), (2) (without formula for $S(\boldsymbol{k}, \omega)$), if we substitute X for $\mathscr{T}$, Y for $\hat{T}$, and $\mathscr{T}^\omega(\hat{p}) = (\omega-\boldsymbol{kp})/a$. Using the technique described in appendices A, B and the Landau formula for the effective mass: $m^* = mD_1$, one can easily prove that

$$X = \frac{\omega-\boldsymbol{kv}}{a}, \quad \hat{Y} = \frac{\Delta}{a}\,(\text{BCS}), \quad \hat{Y} = \frac{\Delta}{a}\,\hat{\sigma}\hat{p}\,(\text{BW})$$

are unique solutions of such equations. From (C1), (C2) we find that vector and scalar vertices have the same poles. This is the reason, why we can discuss only density-density autocorrelation functions.

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