# RESONANT SCATTERING OF LASER PULSES BY A SYSTEM OF $N$ ATOMS

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Resonant scattering of a sequence of laser pulses by an assembly of $N$ identical harmonic oscillator atoms contained in a volume having linear dimensions much smaller than the radiation wave length is studied. It is seen that the natural frequency of the atom is also generated in the scattering. It is found that the Lamb shift and the relaxation time for damping due to spontaneous emission in this case are $N$ and $1/N$ times, respectively, of that for the case of scattering by a single atom, the total scattering cross-section for natural frequency is seen to depend linearly on the number of atoms $N$.

## 1. *Introduction*

In a previous paper [1] we have studied the resonant scattering of pulses of intense radiation by a single harmonic oscillator atom and developed the quantum theory of generation of the atomic natural frequency. It was noted that the effect is important when the frequency of the incident radiation is close to the natural frequency of electrons and that it can be observed experimentally using short laser pulses having a high pulse repetition frequency.

A realistic scatterer, however, never consists of a single atom, and hence it is interesting to investigate the interaction with an assembly of atoms. In the present paper we have studied the resonant scattering of a sequence of pulses of intense radiation by an assembly of $N$ identical harmonic oscillator atoms. The intense radiation has been treated as a classical field. It is assumed that (*i*) the population density of atoms is so low that they interact with each other only through their coupling with radiation, and (*ii*) the atoms are confined to a volume having dimensions small compared to the radiation wave length, and thus the atoms see the same radiation. It is seen that the natural frequency of the atom is also generated in the scattering. It is found that the Lamb shift and the relaxation time for

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damping due to spontaneous emission in this case are $N$ and $1/N$ times, respectively, of that for the case of scattering by a single atom. The total scattering cross-section for the natural frequency is seen to depend linearly on the number of atoms $N$. If we take $N = 1$ the result becomes identical with the classical results for single atom [2].

## 2. *Equations of motion*

To study the resonant scattering of a one-mode intense radiation field by an assembly of $N$ identical harmonic oscillator atoms we can consider a system of $N$ identical harmonic oscillator atoms interacting with a classical radiation field which represents the intense incident mode and other quantized modes of radiation which represent the scattered modes. The Hamiltonian for such a system in the dipole and rotating approximations is given by

$$H = \omega_0 \sum_i [b_i^\dagger(t)b_i(t)+\tfrac{1}{2}]+ \sum_k{}' \omega_k a_k^\dagger(t)a_k(k)$$

$$+\beta \int_{-\infty}^{\infty} d\omega \sum_i [b_i^\dagger(t)+b_i(t)]A(\omega)e^{-i\omega t}+ \sum_i \sum_k{}' \beta_k[a_k^\dagger(t)b_i(t)+a_k(t)b_i^\dagger(t)]. \tag{2.1}$$

Here $\omega_0$ is the common natural frequency of the atoms, $b_i^\dagger(t)$ and $b_i(t)$ are the excitation and de-excitation operators for the $i^{\text{th}}$ atom ($i$ takes values from 1 to $N$), $a_k^\dagger(t)$ and $a_k(t)$ are the annihilation and creation operators for the $k$-mode of radiation defined by the frequency $\omega_k$, wave vector $\vec{k}$ and polarization $\vec{\varepsilon}_k$, $\sum_k'$ denotes summation over $k$ on all modes except the incident mode, and $\beta$ and $\beta_k$ are the coupling constants given by

$$\beta_k = e(\pi\omega_0/m\omega_k V)^{1/2}(\vec{\varepsilon}\cdot\vec{\varepsilon}_k)$$

$$\beta = e(\omega_0/2m)^{1/2}(\vec{\varepsilon}\cdot\vec{\varepsilon}_I),$$

where $\vec{\varepsilon}_I$ is the polarization of the incident radiation, which is given by the vector potential $\vec{\varepsilon}_I \int_{-\infty}^{\infty} d\omega A(\omega)\, e^{-i\omega t}$ in the radiation gauge (the atoms have been taken near the origin) and $V$ is the volume used for normalization.

The Heisenberg equations of motion are

$$i\dot{a}_k(t) = \omega_k a_k(t)+\beta_k \sum_i b_i(t), \tag{2.2}$$

$$i\dot{b}_i(t) = \omega_0 b_i(t)+\beta \int_{-\infty}^{\infty} d\omega A(\omega)e^{-i\omega t}+ \sum_k{}' \beta_k a_k(t). \tag{2.3}$$

These sets of coupled differential equations can be solved using the Laplace transform

$$\begin{pmatrix} A_k(p) \\ B_i(p) \end{pmatrix} = \int_{-\infty}^{\infty} dt \# e^{-pt} \begin{pmatrix} a_k(t) \\ b_i(t) \end{pmatrix}.$$

Then the solution for the transformed operator comes out to be

$$\sum_i B_i(p) = \left[p+i\omega_0+N \sum_k{}' \frac{\beta_k^2}{p+i\omega_k}\right]^{-1} \left[\sum_i b_i(0) - iN\beta \int_{-\infty}^{\infty} \frac{d\omega A(\omega)}{p+i\omega} - \sum_k{}' \frac{N\beta_k a_k(0)}{p+i\omega_k}\right]. \tag{2.4}$$

It is well known that [3]

$$\sum_k{}' \frac{\beta_k^2}{p+i\omega_k} = \mu + i\eta, \tag{2.5}$$

is independent of $p$. Here $\eta$ is the Lamb shift and $(2\mu)^{-1}$ is the relaxation time for damping due to spontaneous emission for a single atom. Thus, it is clear from Eq. (2.4) that for the system of $N$ atoms the Lamb shift and the relaxation time due to spontaneous emission are $N$ and $1/N$ times, respectively, of that for the case of scattering by a single atom. $A_k(p)$ is seen to have a $c$-number term equal to

$$A_k^{\text{class}}(p) = -N\beta\beta_k \int_{-\infty}^{\infty} \frac{d\omega A(\omega)}{(p+i\omega_k)(p+N\mu+i\Omega)(p+i\omega)}, \tag{2.6}$$

where $\Omega = \omega_0 + N\eta$ is the emission frequency.

### *3. Scattering cross-section*

Let us consider the scattering of a semi-infinite pulse of radiation which takes an appreciable time to grow, given by the vector potential

$$\vec{A}_i(\vec{x}, t) = \vec{e}_0 A F(T) \cos(\varpi T + x), \tag{3.1}$$

where $\vec{e}_0$ is unit vector along the direction of polarisation, $\varpi$ is the mean frequency, $T = t - \vec{n} \cdot \vec{x}$, $\vec{n}$ is the unit vector along the direction of propagation and

$$F(T) = \begin{cases} 0 & \text{for } T < 0 \\ T/T_G & \text{for } 0 < T < T_G \\ 1 & \text{for } T > T_G. \end{cases} \tag{3.2}$$

Calculations similar to those in Ref. [1] lead to the following expression for the number of photons of natural frequency generated per unit solid angle in the direction of unit vector $\vec{n}$

$$\frac{dN_0}{d\Omega_{\vec{n}}} = \frac{Ne^4\Omega^3 A^2}{64\pi m^2 \mu T_G^2} [1-(\vec{\varepsilon} \cdot \vec{n})^2] [(\Omega+\varpi)^{-4} + (\Omega-\varpi)^{-4}] \times [\theta(t)(1-e^{-2N\mu t}) + \theta(t-T_G)(1-e^{-2N\mu(t-T_G)})]. \tag{3.3}$$

Here we note that the differential photon counting rate $(d/dt)(dN_0/d\Omega)$ decreases exponentially with time. The natural frequency is thus not generated under stabilized conditions. It can be shown that the natural frequency is also generated during the switching off of the finite pulse. Thus, if we consider a succession of pulses having pulse repetition frequency $n$, the total scattering cross-section is seen to be

$$\sigma(\Omega) = N\sigma_T(n/2\mu)(\Omega T_G)^{-2}\{[\Omega/(\Omega-\bar{\omega})]^4+[\Omega/(\Omega+\bar{\omega})]^4\}, \tag{3.4}$$

where $\sigma_T = (8\pi/3)(e^2/m)^2$ is the Thomson scattering cross-section.

We note that $\sigma(\Omega)$ is proportional to $N$, the number of atoms. This effect is important when the mean frequency of the radiation is close to $\Omega$. The effect is detectable experimentally using short laser pulses having high pulse repetition frequency.

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