DENSITY OF MAGNON STATES IN A FERROMAGNET WITH SCREW DISLOCATION*

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The influence of a single screw dislocation in the crystal on the density of magnon states in the band bottom region is considered. The physical counterpart of the situation under consideration is to be found in whiskers. The dislocation is dealt with as a deformation of the lattice in the entire volume of the crystal, its influence on the magnetic properties being described by the magneto-elastic energy. The strain tensor, in the approximation of linear elasticity theory, is modified by a function of the Yukawa potential shape ensuring finiteness of the strains on the dislocation line. The resolvent of the equation of motion is used to calculate the density of magnon states with zero wave vector component k_z along the dislocation line, as well as that of the frequencies from the magnon band bottom. The numerical analysis has shown the existence of the greater density of states at the band bottom for the modes $k_z = 0$. The density of states peak at the band bottom is separated from the other states by a large gap for whiskers having a small radius and for magnetic materials with a large magneto-elastic coupling constant.

1. Model of the physical situation

Contrary to Refs [1, 2], the dislocation is now considered as destroying the symmetry of the crystal in the entire volume of the latter, the influence of the dislocation is described in terms of strain, in the linear approximation of elasticity theory. In the region of the dislocation core the expressions for the strain tensor [3] are modified by a function of the Yukawa potential shape to remove its divergence in the linear approximation used. The modification consists in the replacement of $1/\varrho$ by $[1-\exp(-\varrho/r_0)]/\varrho$. The non-zero elements of the strain tensor are assumed in the form

$$e_{xz}(\boldsymbol{r}) = -(b/4\pi)\,[1-\exp(-\varrho/r_0)]\,(1/\varrho)\sin\varphi,$$

$$e_{yz}(\boldsymbol{r}) = (b/4\pi)\,[1-\exp(-\varrho/r_0)]\,(1/\varrho)\cos\varphi. \tag{1}$$

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with: r_0 — the radius of the dislocation core, b the length of the Burgers vector, and ϱ, φ polar coordinates ($r_x = \varrho \cos\varphi$, $r_y = \varrho \sin\varphi$). The influence of the dislocation is dealt with as that of a uniform field, described by the magneto-elastic energy

$$\mathscr{H}_{\mathrm{me}} = \sum_{\boldsymbol{l}} \sum_{i,j=x,y,z} \{(vB/S^2) S_i(\boldsymbol{l}) S_j(\boldsymbol{l}) - J' \sum_{\boldsymbol{\Delta}} [S_i(\boldsymbol{l}) S_j(\boldsymbol{l}+\boldsymbol{\Delta}) - S_i(\boldsymbol{l}) S_j(\boldsymbol{l})]\} e_{ij}(\boldsymbol{r}_l), \tag{2}$$

where B is the magneto-elastic coupling constant, $S_j(\boldsymbol{l})$ and S the j-component and length of spin in the $\boldsymbol{l}$-th lattice node, v the volume per magnetic site, whereas the constant J' is defined by the pressure-variation of the exchange integral (see, Appendix). Summation over $\boldsymbol{\Delta}$ extends over the nearest neighbours of a given site, whereas $\boldsymbol{l}$ indicates summation over all spins.

The total Hamiltonian of the system is

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_{\mathrm{me}}. \tag{3}$$

$\mathscr{H}_0$ comprises exchange, Zeeman and dipolar interaction [4]. The external magnetic field is assumed sufficiently strong for (*i*) effects of spin wave ellipticity to be negligible and (*ii*) magnetisation to be almost homogeneous and effects due to static inhomogeneity negligible. We shall refrain here from considering the case of a ferromagnetic undergoing a phase transition in the core region under the effect of stress. The Holstein-Primakoff transformation [4] brings the Hamiltonian (3) to the form

$$\mathscr{H}/\hbar = \sum_{k_z} \sum_{\boldsymbol{k}_\varrho} \sum_{\boldsymbol{k}_{\varrho'}} K(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z) a^{+}_{\boldsymbol{k}_\varrho, k_z} a^{-}_{\boldsymbol{k}_{\varrho'}, k_z}, \tag{4}$$

where $a^{\pm}_{\boldsymbol{k}_\varrho, k_z}$ are boson creation and annihilation operators. The vectors $\boldsymbol{k}_\varrho$ lie in the plane perpendicular to the dislocation line and k_z is the $\boldsymbol{k}$ vector component along a dislocation line. The kernel $K(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z)$ is of the following form

$$K(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z) = \omega_k \delta_{\boldsymbol{k}_\varrho . \boldsymbol{k}_{\varrho'}} + (4\pi/r_1^2) i A L(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z). \tag{5}$$

The frequency ω_k of modes of the dislocation-free ferromagnet is taken into account in the Herring-Kittel approximation [4]

$$\omega_k = \gamma\{H + \alpha(k_\varrho^2 + k_z^2) + 2\pi M_0 \sin^2 \vartheta_k\}, \tag{6}$$

with: H the internal magnetic field, M_0 the complete magnetisation, α the spin-wave dispersion coefficient, γ the magneto-mechanical ratio, and ϑ_k the angle between the wave vector $\boldsymbol{k}$ and the magnetisation direction. A single screw dislocation is assumed within the entire volume of the crystal, as in the case of whiskers. The z-direction is that of the dislocation line, and r_1 the whisker radius. The coefficient A is of the form

$$A = (3\gamma B/8\pi^2 M_0) b \sin 2\vartheta_d, \tag{7}$$

where ϑ_d is the angle between the magnetisation direction and the dislocation line (whisker axis). The function $L(k_\varrho, k'_\varrho, k_z)$ is given by the components k_x, k_y of $\boldsymbol{k}_\varrho$ as follows:

$$L(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z) = \frac{k_y - k'_y}{(k_x-k'_y)^2+(k_y-k'_y)^2}\left\{\frac{1}{\sqrt{1+r_0^2[(k_x-k'_x)^2+(k_y-k'_y)^2]}}\right.$$

$$\left.-J_0(\gamma_1\sqrt{(k_x-k'_x)^2+(k_y-k'_y)^2})\right\}\left\{1-\frac{M_0 J'/J}{12B}[2k_z^2\right.$$

$$\left.+3(k_x^2+k_x'^2+k_y^2+k_y'^2)-4(k_xk'_x+k_yk'_y)]\right\}. \tag{8}$$

2. The equation of motion for the resolvent

The equation of motion for the mode $a^-_{k_\varrho,k_z}$

$$a^-_{k_\varrho,k_z} = -i[a^-_{k_\varrho,k_z}, \mathcal{H}/\hbar] \tag{9}$$

leads to the following "integral" equation

$$\alpha^-_{k_\varrho}(k_z, \omega) = \frac{1}{\omega}\sum_{k_\varrho} K(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z)\alpha^-_{k_\varrho}(k_z, \omega). \tag{10}$$

The mode $\alpha^-_{k_\varrho}(k_z, \omega)$ is the Fourier transform of $a^-_{k_\varrho,k_z}$. We shall henceforth concentrate on the resolvent of the homogeneous equation which, in the case of a perturbing function $f(\boldsymbol{k}_\varrho)$ equal to zero, goes over into (10)

$$\alpha^-_{k_\varrho}(k_z, \omega-i\eta) = \frac{1}{\omega-i\eta}\sum_{k'_\varrho} K(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z)\alpha^-_{k_\varrho}(k_z, \omega-i\eta)+f(\boldsymbol{k}_\varrho). \tag{11}$$

In the inhomogeneous equation, the frequency ω has been replaced by $\omega-i\eta$. The solution of (11) can, in general, be written in the following form

$$\alpha^-_{k_\varrho}(k_z, \omega-i\eta) = f(\boldsymbol{k}_\varrho)+\frac{1}{\omega-i\eta}\sum_{k_\varrho'} \Gamma(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, \omega-i\eta, k_z)f(\boldsymbol{k}'_\varrho), \tag{12}$$

which defines the resolvent $\Gamma(\ldots)$ of Eq. (11). By insertion of (12) into (11) one easily verifies that the resolvent fulfils the following relation

$$\Gamma(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, \omega-i\eta, k_z) = \frac{1}{\omega-i\eta}\sum_{k_\varrho''} K(\boldsymbol{k}_\varrho, \boldsymbol{k}''_\varrho, k_z)\Gamma(\boldsymbol{k}''_\varrho, \boldsymbol{k}'_\varrho, \omega-i\eta, k_z)$$

$$+K(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z). \tag{13}$$

By having recourse to the form (5) of the kernel, we transform (13) to

$$\Gamma(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, \omega - i\eta, k_z) - iA\left(\frac{4\pi}{r_1^2}\right)\sum_{\boldsymbol{k}_\varrho''} \frac{L(\boldsymbol{k}_\varrho, \boldsymbol{k}''_\varrho, k_z)}{\omega - \omega_k - i\eta}$$

$$\times \Gamma(\boldsymbol{k}''_\varrho, \boldsymbol{k}'_\varrho, \omega - i\eta, k_z) = \frac{(\omega - i\eta)K(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, k_z)}{\omega - \omega_k - i\eta}. \tag{14}$$

The kernel of Eq. (14) has the highest symmetry for $k_z = 0$. On reflection at the plane $k_x = 0$ it remains unchanged, but undergoes a change in sign on reflection at the plane $k_y = 0$. For $k_z \neq 0$, the symmetry related with reflection at the plane $k_y = 0$ is maintained.

3. Density of magnon states with vector $k_z = 0$

For simplicity, we shall be considering only the density of states of magnons with zero wave vector component along the dislocation line. The density of states is given by the imaginary part of the resolvent. That with fixed k_z is given as follows

$$N(\omega, k_z) = \frac{1}{\pi\omega^2} \operatorname{Im} \sum_{\boldsymbol{k}_\varrho} \Gamma(\boldsymbol{k}_\varrho, \boldsymbol{k}_\varrho, \omega - i\eta, k_z), \qquad \eta \to 0^+. \tag{15}$$

On taking the trace of (14), we obtain the following equation for the density of states with $k_z = 0$

$$N(\omega, 0) = \frac{4A}{\omega^2 r_1^2} \operatorname{Im} i \sum_{\boldsymbol{k}_\varrho} \sum_{\boldsymbol{k}_{\varrho'}} \frac{L(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, 0)}{\omega - \omega_k - i\eta} \Gamma(\boldsymbol{k}'_\varrho, \boldsymbol{k}_\varrho, \omega - i\eta, 0) + N_0(\omega, 0),$$

$$\eta \to 0^+, \tag{16}$$

where $N_0(\omega, 0)$ is the density of states $k_z = 0$ for a ferromagnetic free of dislocations

$$N_0(\omega, 0) = (1/\pi) \operatorname{Im} \sum_{\boldsymbol{k}_\varrho} \frac{1}{\omega - \omega_k - i\eta}, \qquad \eta \to 0^+. \tag{17}$$

For simplicity, we calculate the right-hand side of (17) for η so large that summation can be replaced by integration. For frequencies from the band bottom, the density of states $k_z = 0$ for the dislocation-free ferromagnetic is expressed as follows

$$N_0(\omega, 0) \approx \frac{r_1^2}{8\pi^2\gamma\alpha} \operatorname{Im} \int_{-\pi}^{\pi} d\Phi_k \int_{\omega_1(\Phi_k)}^{\omega_{\max}} d\omega_k \frac{1}{\omega - \omega_k - i\eta} \approx \frac{r_1^2}{2\pi\gamma\alpha} \Phi_0, \tag{18}$$

where

$$\Phi_0 = \begin{cases} 0 \text{ for } \omega \leq \omega_1(0) \\ \arcsin \dfrac{\omega-\omega_1(0)}{\omega_1(\pi/2)-\omega_1(0)} \text{ for } \omega_1(0) \leqslant \omega \leqslant \omega_1(\pi/2) \\ \pi/2 \text{ for } \omega > \omega_1(\pi/2). \end{cases} \tag{19}$$

The function $\omega_1(\Phi_k)$ represents the value of ω_k for $k = 0$

$$\omega_1(\Phi_k) = \gamma\{H+2\pi M_0(1-\sin^2 \vartheta_d \cos^2 \Phi_k)\}. \tag{20}$$

To calculate $N(\omega, 0)$ from (16), we approximate the function $L(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, 0)$ in Eqs. (14) and (16), applying Lagrange's method [5]. In our calculations, we shall make use of coordinates ω_k and Φ_k, defined as follows

$$k_x = k_\varrho \cos \Phi_k, \quad k_y = k_\varrho \sin \Phi_k, \tag{21a}$$

where k_ϱ is a function of ω_k and Φ_k defined by (6) for $k_z = 0$

$$\omega_k = \gamma\{H+\alpha k_\varrho^2+2\pi M_0(1-\sin^2 \vartheta_d \cos^2 \Phi_k)\}. \tag{21b}$$

With regard to the factor $1/(\omega-\omega_k-i\eta)$, when expanding in ω_k we have restricted ourselves to a Lagrange expansion based on a single interpolation point $\tilde{\omega}$ and equal to ω if $\omega > \omega_1(\Phi_j^\sigma)$ but equal to $\omega_1(\Phi_j^\sigma)$ if $\omega \leqslant \omega_1(\Phi_j^\sigma)$. The basis of functions $l_j^\sigma(\Phi_k)$ wherein we expand $L(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, 0)$ in the coordinate Φ_k is chosen in a manner that the above-mentioned symmetry of the kernel of Eq. (14) shall be conserved. We assume $l_j^\sigma(\Phi_k)$ in the following form

$$l_j^\sigma(\Phi_k) = \begin{cases} \dfrac{\pi(1-x_j^2)}{2nP_{n-1}(x_j)} \dfrac{P_n\left(\dfrac{2}{\pi}\Phi_k-\sigma\right)}{\Phi_k-\Phi_j^\sigma} & \text{for } \left|\dfrac{2}{\sigma}\Phi_k-\sigma\right| \leqslant 1 \\ 0 & \text{for } \left|\dfrac{2}{\pi}\Phi_k-\sigma\right| > 1 \end{cases} \tag{22}$$

where $\sigma = \pm 1, j = 1, \ldots, n$. We expand the function $L(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, 0)$ in the interval $(-\pi, 0)$ in the basis $l_j^-(\Phi_k)$ and, separately, in the interval $(0, \pi)$ in the basis $l_j^+(\Phi_k)$. The interpolation points Φ_j^σ of the Lagrange approximation are chosen as follows

$$\Phi_j^\sigma = (\pi/2)(\sigma+x_j); \quad j = 1, \ldots, n; \quad \sigma = \pm 1, \tag{23}$$

where x_j is the j-th root of the Legendre polynomial $P_n(x)$. We have chosen the interpolation points Φ_j^σ and determined the basis of the functions $l_j^\sigma(\Phi_k)$ as in the Nyström's integral equation method. Henceforth, we shall be assuming the n as even.

Applying the Lagrange interpolations for $L(\boldsymbol{k}_\varrho, \boldsymbol{k}'_\varrho, 0)$ with respect to the coordinates Φ_k and ω_k of the vector $\boldsymbol{k}_\varrho$, we arrive at the following expression for the density of states

$$N(\omega, 0) = N_0(\omega, 0)+(A/\pi\omega^2) \sum_{j=1} \sum_{\sigma=\pm 1} \operatorname{Re} g_{j,j}^{\sigma,\sigma}. \tag{24}$$

The functionals $g^{\sigma,\sigma}_{j,j}$ are to be calculated from the following set of $2n$ equations

$$g^{\sigma,\sigma'}_{j,j'} - iA \sum_{l=1}^{n} \sum_{\sigma''=\pm 1} G^{\sigma,\sigma''}_{j,l}\, g^{\sigma'',\sigma'}_{l,j} = F^{\sigma,\sigma'}_{j,j'},$$

$$j, j' = 1, \ldots, n; \quad \sigma, \sigma' = \pm 1. \tag{25}$$

We have obtained the set (25) from equation of motion (14). Above, we have introduced the following notation

$$G^{\sigma,\sigma'}_{j,j'} = (4\pi/r_1^2) \sum_{k_{\varrho'}} \frac{L(\boldsymbol{k}_\varrho(\tilde{\omega}, \Phi^\sigma_j), \boldsymbol{k}'_\varrho, 0) l^{\sigma'}_{j'}(\Phi_{k'})}{\omega - \omega_{k'} - i\eta}, \tag{26}$$

$$F^{\sigma,\sigma'}_{j,j'} = (4\pi/r_1^2)(\omega - i\eta) \sum_{k_{\varrho'}} \sum_{k_{\varrho''}} \frac{L(\boldsymbol{k}_\varrho(\tilde{\omega}, \Phi^\sigma_j), \boldsymbol{k}''_\varrho, 0) l^{\sigma'}_{j'}(\Phi_{k'}) K(\boldsymbol{k}''_\varrho, \boldsymbol{k}'_\varrho, 0)}{(\omega - \omega_{k''} - i\eta)(\omega - \omega_{k'} - i\eta)}$$

$$\simeq (4\pi/r_1^2)(\omega - i\eta) \sum_{k_{\varrho'}} \frac{\omega_{k'} L(\boldsymbol{k}_\varrho(\tilde{\omega}, \Phi^\sigma_j), \boldsymbol{k}'_\varrho, 0) l^{\sigma'}_{j'}(\Phi_{k'})}{(\omega - \omega_{k'} - i\eta)^2}. \tag{27}$$

The functions $G^{\sigma,\sigma'}_{j,j'}$, $G^{\sigma,\sigma'}_{j,j'}$ possess the following symmetry

$$G^{\sigma,\sigma'}_{j,j'} = -G^{-\sigma,-\sigma'}_{j,j'} = -G^{-\sigma,-\sigma'}_{n-j+1,n-j'+1} = G^{\sigma,\sigma'}_{n-j+1,n-j'+1}, \tag{28}$$

$$F^{\sigma,\sigma'}_{j,j'} = -F^{-\sigma,-\sigma'}_{j,j'} = -F^{-\sigma,-\sigma'}_{n-j+1,n-j'+1} = F^{\sigma,\sigma'}_{n-j+1,n-j'+1}. \tag{29}$$

Eqs (28) and (29) are the result of our assumption of a basis conserving the symmetry of the kernel of Eq. (14). The identities (28), applied when solving Eqs (25), lead to a partial factorisation of the set of $2n$ equations. In place of the set of $2n$ equations (25), we obtain four separate sets of $n/2$ equations each.

On insertion of the Cramer solution of Eqs (25) into (24) we have the following expression for the density of states $k_z = 0$

$$N(\omega, 0) = N_0(\omega, 0) - (1/\pi)\,\mathrm{Im} \sum_{j,l=1}^{n/2} \sum_{h=1}^{4} R^{-1}_{j,l}(h) T_{l,j}(h). \tag{30}$$

The matrix $R^{-1}_{j,l}(h)$ is the inverse of the matrix $R_{j,l}(h)$, defined as follows

$$R_{j,j'}(h) = \delta_{j,j'} + \sum_{l=1}^{n/2} \{B_{j,l}A_{l,j'}\delta_{1,h} + A_{j,l}B_{l,j'}\delta_{2,h}$$

$$+ D_{j,l}C_{l,j'}\delta_{3,h} + C_{j,l}D_{l,j'}\delta_{4,h}\}, \quad j, j' = 1, \ldots, n/2, \quad h = 1, \ldots 4, \tag{31}$$

where

$$A_{j,l} = A\{G^{+,+}_{j,l} + G^{+,-}_{j,l} + G^{+,+}_{j,n-l+1} + G^{+,-}_{j,n-l+1}\},$$

$$B_{j,l} = A\{G^{+,+}_{j,l} - G^{+,-}_{j,l} + G^{+,+}_{j,n-l+1} - G^{+,-}_{j,n-l+1}\},$$

$$C_{j,l} = A\{G^{+,+}_{j,l} + G^{+,-}_{j,l} - G^{+,+}_{j,n-l+1} - G^{+,-}_{j,n-l+1}\},$$

$$D_{j,l} = A\{G^{+,+}_{j,l} - G^{+,-}_{j,l} - G^{+,+}_{j,n-l+1} + G^{+,-}_{j,n-l+1}\}, \tag{32}$$

whereas the matrix $T_{j,j'}(h)$ is of the form

$$T_{j,j'}(h) = \sum_{l=1}^{n/2} \{B_{j,l}E_{l,j'}\delta_{1,h} + A_{j,l}K_{l,j'}\delta_{2,h} + D_{j,l}N_{l,j'}\delta_{3,h} + C_{j,l}M_{l,j'}\delta_{4,h}\}, \tag{33}$$

with

$$\begin{aligned} E_{l,j} &= A\omega^{-2}\{F^{+,+}_{l,j} + F^{+,-}_{l,j} + F^{+,+}_{l,n-j+1} + F^{+,-}_{l,n-j+1}\}, \\ K_{l,j} &= A\omega^{-2}\{F^{+,+}_{l,j} - F^{+,-}_{l,j} + F^{+,+}_{l,n-j+1} - F^{+,-}_{l,n-j+1}\}, \\ N_{l,j} &= A\omega^{-2}\{F^{+,+}_{l,j} + F^{+,-}_{l,j} - F^{+,+}_{l,n-j+1} - F^{+,-}_{l,n-j+1}\}, \\ M_{l,j} &= A\omega^{-2}\{F^{+,+}_{l,j} - F^{+,-}_{l,j} - F^{+,+}_{l,n-j+1} + F^{+,-}_{l,n-j+1}\}. \end{aligned} \tag{34}$$

4. Numerical analysis

The functions $G^{\sigma,\sigma'}_{j,j'}$ and $F^{\sigma,\sigma'}_{j,j'}$ depend in a highly complicated manner on ω for $\eta \to 0^+$, and summation over ω_k cannot be replaced by integration. Leaving unconsidered the discrete structure of the density of states and similarly as when calculating $N_0(\omega, 0)$ we calculate the density of states for η larger than the distance between consecutive ω_k's. For η larger than the difference between consecutive value of ω_k, we obtain the mean value of the density of states in the neighbourhood of the point ω. The calculations simplify considerably, and summation over the ω_k can be replaced by integration. Finally, we obtain the functionals $G^{\sigma,\sigma'}_{j,j'}$ and $F^{\sigma,\sigma'}_{j,j'}$ in the form of the following expressions

$$\begin{aligned} G^{\sigma,\sigma'}_{j,j'} &\simeq \frac{1}{2\gamma\alpha} \int_{-\pi}^{\pi} d\Phi_k l^{\sigma'}_{j'}(\Phi_k) \\ &\times \left\{ \int_{\omega_1(\Phi_k)}^{\omega_{\max}} d\omega_k \frac{(\omega-\omega_k)L(\boldsymbol{k}_\varrho(\tilde{\omega}, \Phi^\sigma_j), \boldsymbol{k}_\varrho(\omega_k, \Phi_k), 0)}{(\omega-\omega_k)^2+\eta^2} \right. \\ &\left. + iL(\boldsymbol{k}_\varrho(\tilde{\omega}, \Phi^\sigma_j), \boldsymbol{k}_\varrho(\tilde{\omega}, \Phi_k), 0)\left[\frac{\pi}{2} - \operatorname{arc\,tg}\frac{\omega_1(\Phi_k)-\omega}{\eta}\right]\right\}, \end{aligned} \tag{35}$$

$$\begin{aligned} \omega^{-2}F^{\sigma,\sigma'}_{j,j'} &\simeq \frac{-i}{2\gamma\alpha}\left\{\sum_{s=1}^{4} \frac{L(\boldsymbol{k}_\varrho(\tilde{\omega}, \Phi^\sigma_j), \boldsymbol{k}_\varrho(\tilde{\omega}, \Phi_s), 0)}{2\pi\gamma M_0 \sin^2\vartheta_d \sin 2\Phi_s}\right. \\ &\times l^{\sigma'}_{j'}(\Phi_s)\left[\operatorname{arc\,tg}\frac{\omega_1(\pi/2)-\omega}{\eta} - \operatorname{arc\,tg}\frac{\omega_1(0)-\omega}{\eta}\right] \\ &+ \int_{-\pi}^{\pi} d\Phi_k l^{\sigma}_{j'}(\Phi_k)\left[\frac{\partial}{\partial\omega_k} L(\boldsymbol{k}_\varrho(\tilde{\omega}, \Phi^\sigma_j), \boldsymbol{k}_\varrho(\omega_k, \Phi_k), 0)\right]_{\omega_k=\tilde{\omega}} \\ &\left.\times\left[\frac{\pi}{2} - \operatorname{arc\,tg}\frac{\omega_1(\Phi_k)-\omega}{\eta}\right]\right\}, \end{aligned} \tag{36}$$

where

$$\Phi_1 = \arccos \sqrt{[\omega_1(\pi/2)-\omega]/[\omega_1(\pi/2)-\omega_1(0)]}\,,$$

$$\Phi_2 = \pi-\Phi_1, \quad \Phi_3 = -\Phi_1, \quad \Phi_4 = -\Phi_2. \tag{37}$$

For a frequency ω not belonging to the interval $(\omega_1(0)+\eta/2, \omega_1(\pi/2)-\eta/2)$, we calculate Φ_1 replacing ω in Eq. (37) by the respective value of the edge of the interval.

We carried out numerical calculations of the density of states $k_z = 0$ for various angles between the dislocation line and the quantisation direction for the following material constants [3, 4, 6], corresponding to nickel: $\alpha = 3.47\times10^{-9}$ Oe cm^2, $B = 6.2\times10^7$ erg/cm^3, $\gamma = 1.9\times10^7$ 1/Oe s, $J'/J = -3$, $M_0 = 510$ Gs. We assumed an internal field

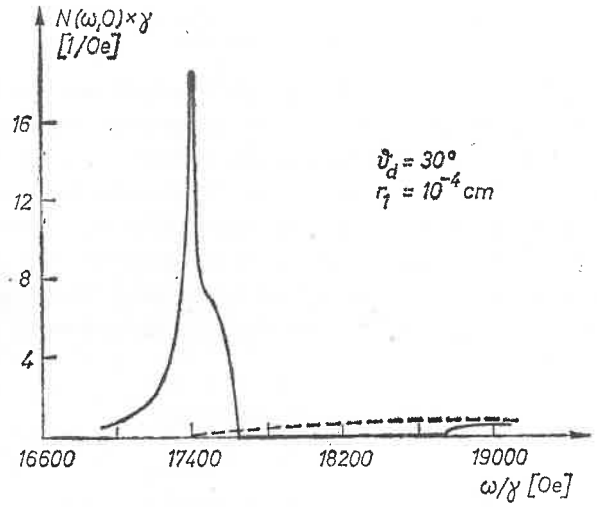

Fig. 1

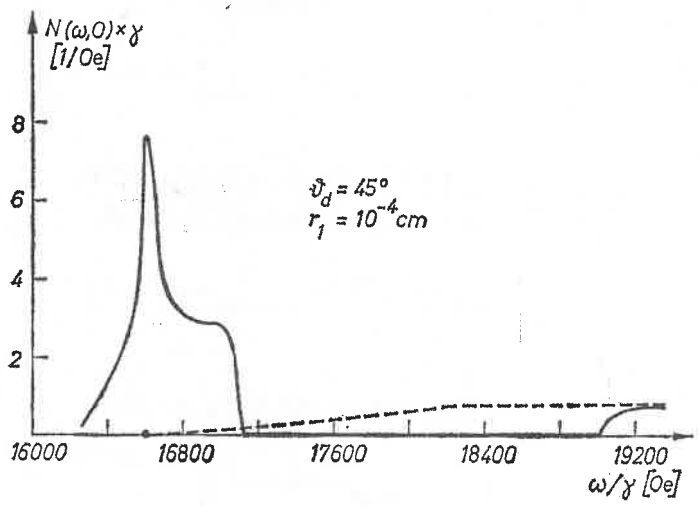

Fig. 2

of $H = 15000$ Oe, moreover $\eta/\gamma = 30$ Oe, $r_1 = 10^{-4}$ cm, $r_0 = 1.2\times 10^{-7}$ cm, $b = 2.49\times$ $\times 10^{-8}$ cm, and $n = 2$.

In the figures the continuous and the dashed lines represent the density of states with $k_z = 0$ for a dislocated and non-dislocated ferromagnets, respectively. Where the density of states of the dislocated ferromagnet is equal to zero in the figures, we have obtained negative values of the density of states. In this region for either greater dislocation radii or smaller magneto-elastic coupling only an anti-resonant state appears, therefore the negative density of states should be treated as vanishing. The numerical analysis has shown the increase of the density of states at the band bottom. This density of states peak is separated from the other states with $k_z = 0$ by a gap.

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APPENDIX

Determination of the ratio J'/J

In the Hamiltonian, we take into account the exchange interaction of the rigid lattice and a term with the magneto-elastic energy due to a change of the exchange integral caused by the strain

$$\mathcal{H} = -J\sum_{l}\sum_{\Delta} S(l)\cdot S(l+\Delta)$$

$$-J'\sum_{l}\sum_{\Delta}\sum_{i,j=x,y,z} S_i(l)S_j(l+\Delta)e_{ij}(r_l). \tag{38}$$

Assuming that the strain is caused by a pressure p which acts on a crystal, we can write Eq. (38) as follows

$$\mathcal{H} = -J\{1+J'(1-2\nu)p/[2J\mu(1+\nu)]\}\sum_{l}\sum_{\Delta} S(l)\cdot S(l+\Delta). \tag{39}$$

The expression $J\{1-J'(1-2\nu)p/[2J\mu(1+\nu)]\}$ represents the effective exchange integral for a ferromagnet subjected to the pressure. Here, ν is Poisson's constant and μ the shear modulus. Assuming the Curie temperature $T_c(p)$ in degrees Kelvin as proportional to the effective exchange integral, we derive from the ratio of Curie temperatures of the ferromagnet under pressure and free of external pressure the following expression for J'/J

$$\frac{J'}{J} = \frac{2(1+\nu)\mu}{(1-2\nu)p}\,\frac{T_c(p)-T_c(0)}{T_c(0)}. \tag{40}$$

In the case of nickel, we obtain $J'/J = -3$.

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