THE SUSCEPTIBILITY OF SUPERFLUID ^{3}He-B IN THE ACOUSTIC LIMIT AT NONZERO TEMPERATURES (SPIN WAVES)

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The tensor of spin susceptibility for systems in the Balian-Werthamer state is calculated using the Larkin-Migdal-Czerwonko theory at nonzero temperatures, in the acoustic limit and collisionless regime. The final results contain two first Landau parameters. The poles of the obtained expression are connected with spin waves. It checks out that this tensor corresponds to previously calculated tensors in all known limits.

1. Introduction

The purpose of this paper is to calculate the spin susceptibility tensor in the acoustic limit for the *B*-phase of the superfluid ^{3}He, for nonzero temperatures. According to [1] phase *B* of superfluid ^{3}He is identified with the BW state. The method applied here is based on the zero-temperature Larkin-Migdal theory [2] for systems with *S*-pairing extended by Czerwonko [3] for other systems. Leggett, in [4], extended the LM theory to nonzero temperatures. Hence, the Leggett theory gives the generalization of the LMC theory for nonzero temperatures. Analogously as in [2–4], we assume that the system is in the collisionless regime, and we do not include the spin-unconserving weak dipole-dipole interaction (cf. [5]).

Let us outline the contents of this paper. In Section 2 we demonstrate that the LMC theory can be used also at nonzero temperatures. For details see [3] and [4]. In Section 3, using Leggett's calculations [6], we give the form of kernels L, M, N, O (cf. A4)). It is easy to see that these kernels pass into known ones in the following limiting cases: the temperature tending to zero (cf. e.g. [3]) the static field cf. [4] and the normal state (cf. [7, 8]). In Section 4 we calculate the tensor of spin susceptibility. We cannot get the final results in closed form without imposing any restrictions on the Landau parameters. It becomes obvious if we remember that our results have to be true for the normal liquid ($T = T_c$). For those calculations carried out to completion, we restricted ourselves to the case when

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only b_0 and b_1 do not vanish, but the system of equations will be given for an even more general case (cf. Eqs. (3)).

The spin wave problems were recently investigated by several authors [6, 9–11]. Czerwonko [9] showed that the spin wave velocities at zero temperatures depend on four Landau parameters only. Combescot [10] and Maki [11] investigated these problems for nonzero temperatures, assuming that the antisymmetric Fermi-liquid interaction is constant. On the other hand we know from [12–16] that only the Landau parameter b_2 can be considered small. Also the experimental observations of the spin waves [17] indicate the necessity to take into account at least two Landau parameters b_0 and b_1. Hence the physical arguments confirm the reasonableness of the imposed restrictions, and the precision of the presented calculations seems to satisfy the needs of the present experiments.

In the last Section explanations of the symbols occurring in the text and some characteristic properties of the introduced functions are listed.

2. *Application LMC theory to systems with nonzero temperatures*

Our one-particle Green's functions differ from those considered in [2] and [3] because they are Matsubara's Green's functions. Their Fourier-transforms have the properties:

$$F(p) = -F^T(-p), \qquad F_1(p) = -F_2^+(p),$$

where $p = (\boldsymbol{p}, \varepsilon_n)$ and $\varepsilon_n = i\dfrac{2\pi}{\beta}n$. These properties are the same as (1.6) in [3]. Hence we can rewrite the whole procedure developed by Czerwonko in [3] to derive. Eqs (2.27) by recalling that

$$\frac{1}{2\pi}\int d\varepsilon \to T\sum_{\varepsilon_n}.$$

Hereafter we confine ourselves to the system with BW pairing [1] and only to the pairing channel in the particle-particle effective interaction i.e. $f^{\xi}_{-1}(\hat{p}\hat{p}') = f^{\xi}_{-1,1}\,3\,(\hat{p}\hat{p}')$. Then Eqs. (2.27) of [3] can be rewritten in the form:

$$\overset{s}{\mathcal{T}}{}^i_j = \delta_{ij} + \langle \overset{s}{B}[(L-O)\overset{s}{\mathcal{T}}{}^i_j + (L+O)\overset{a}{\mathcal{T}}{}^i_j + 2O\overset{s}{\mathcal{T}}{}^k_j\hat{p}_k\hat{p}_i$$

$$-2O\overset{a}{\mathcal{T}}{}^k_j\hat{p}_k\hat{p}_i - 2M\varepsilon_{ikn}\tau^{kr}_j\hat{p}_n\hat{p}_r]\rangle, \tag{1a}$$

$$\overset{a}{\mathcal{T}}{}^i_j = \langle \overset{a}{B}[(L-O)\overset{s}{\mathcal{T}}{}^i_j + (L+O)\overset{a}{\mathcal{T}}{}^i_j + 2O\overset{s}{\mathcal{T}}{}^k_j\hat{p}_k\hat{p}_i$$

$$-2O\overset{a}{\mathcal{T}}{}^k_j\hat{p}_k\hat{p}_i - 2M\varepsilon_{ikn}\tau^{kr}_j\hat{p}_n\hat{p}_r]\rangle, \tag{1b}$$

$$\tau^i_j = \langle f^{\xi}_{-1}\{[N+O+G_sG^-(1-\theta_\xi)]\tau^i_j$$

$$-2O\tau^n_j\hat{p}_n\hat{p}_i - 2M\varepsilon_{ikn}\mathcal{T}^k_j\hat{p}_n\}\rangle, \tag{1c}$$

and expression (3.30) of [3] — in the form

$$\chi_{ij} = -\mu_B^2 \nu\langle (L-O)\overset{s}{\mathscr{T}}{}^i_j + (L+O)\overset{a}{\mathscr{T}}{}^i_j + 2O\overset{s}{\mathscr{T}}{}^k_j \hat{p}_k \hat{p}_i$$

$$-2O\overset{a}{\mathscr{T}}{}^k_j \hat{p}_k \hat{p}_i - 2M\varepsilon_{ikn}\tau^{kr}_j \hat{p}_n \hat{p}_r\rangle. \tag{1d}$$

Since the gap equation $\hat{d} = \langle f^{\xi}_{-1} G_s G^-(1-\theta_\xi)\hat{d}\rangle$ is valid if $\hat{d} = (\sigma\hat{p})\sigma^y$ then according to our assumptions we can write $\hat{\tau}_j = \langle f^{\xi}_{-1} G_s G^-(1-\theta_\xi)\hat{\tau}_j\rangle$. Hence (1c) can be rewritten in the form

$$\langle \hat{p}_r[(N+O)\tau^{in}_j \hat{p}_n - 2O\tau^{kn}_j \hat{p}_k \hat{p}_n \hat{p}_i]\rangle$$

$$= 2\varepsilon_{ikn}\langle M(\overset{s}{\mathscr{T}}{}^k_j + \overset{a}{\mathscr{T}}{}^k_j)\hat{p}_n \hat{p}_r\rangle. \tag{1e}$$

3. *The kernels L, M, N, O*

To compute the kernels L, M, N, O we should apply the method given by Eliashberg [18]. Since this was done by Leggett [6] in the acoustic limit, we confine ourselves to some elementary transformations of Leggett's results to rewrite them in the form (cf. (A4) and (A6))

$$L-O = -1+\left[1+\frac{kv}{\omega}(\hat{k}\hat{p})\right](F_0-F_1)\,[1-\beta^2(\hat{k}\hat{p})^2]^{-1}$$

$$L+O = -f_0+F_0[1-\beta^2(\hat{k}\hat{p})^2]^{-1}+\frac{kv}{\omega}(\hat{k}\hat{p})(F_0-F_1)\,[1-\beta^2(\hat{k}\hat{p})^2]^{-1}$$

$$2O = 1-f_0+F_1[1-\beta^2(\hat{k}\hat{p})^2]^{-1},$$

$$2M = -[\omega+kv(\hat{k}\hat{p})]\,\{1-f_0+F_1[1-\beta^2(\hat{k}\hat{p})^2]^{-1}\}$$

$$N+O = [\omega^2-k^2v^2(\hat{k}\hat{p})^2]\,\{1-f_0+F_1[1-\beta^2(\hat{k}\hat{p})^2]^{-1}\}, \tag{2}$$

where

$$\beta = \frac{kv}{\omega}\,\frac{\varepsilon}{E}.$$

As we see, Eqs. (2) are unanalytic functions of the variables kv and ω. This causes that the final results contain an infinite number of Landau parameters if we do not impose any restrictions on them. On the other hand these kernels in the limiting cases where $T = 0$ or $\omega = 0$ become an analytic function of kv and ω or kv, respectively. This alone allows one to obtain the final results in the closed form without any restrictions imposed on Landau parameters (cf. [9] and [19]).

4. The linear response of the system in the acoustic limit

Substituting kernels L, M, N, O defined by Eqs. (2) into Eqs. (1) we find that

$$\begin{aligned}\overset{s}{\mathscr{T}}{}^i_j = \delta_{ij} + \Big\langle \overset{s}{B}\Big\{&[-1+(F_0-F_1)(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\overset{s}{\mathscr{T}}{}^i_j \\ &+\frac{kv}{\omega}(F_0-F_1)[(\hat{k}\hat{p})(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\overset{a}{\mathscr{T}}{}^i_j \\ &+[1-f_0+F_1(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\overset{s}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i \\ &+\omega[1-f_0+F_1(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\varepsilon_{ikn}\tau^{kr}_j\hat{p}_n\hat{p}_r\Big\}\Big\rangle,\end{aligned} \tag{3a}$$

$$\begin{aligned}\overset{a}{\mathscr{T}}{}^i_j = \Big\langle \overset{a}{B}\Big\{&\frac{kv}{\omega}(F_0-F_1)[(\hat{k}\hat{p})(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\overset{s}{\mathscr{T}}{}^i_j \\ &+[-f_0+F_0(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\overset{a}{\mathscr{T}}{}^i_j \\ &-[1-f_0+F_1(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\overset{a}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i \\ &+kv[1-f_0+F_1(1-\beta^2(\hat{k}\hat{p})^2)^{-1}](\hat{k}\hat{p})\varepsilon_{ikn}\tau^{kr}_j\hat{p}_n\hat{p}_r\Big\}\Big\rangle,\end{aligned} \tag{3b}$$

$$\begin{aligned}&\langle\hat{p}_r\{[\omega^2-k^2v^2(\hat{k}\hat{p})^2][1-f_0+F_1(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\tau^{in}_j\hat{p}_n \\ &\quad-2[1-f_0+F_1(1-\beta^2(\hat{k}\hat{p})^2)^{-1}]\tau^{kn}_j\hat{p}_k\hat{p}_n\hat{p}_i\}\rangle \\ &\quad= -\omega\{(1-f_0)\varepsilon_{ikn}\langle\overset{s}{\mathscr{T}}{}^k_j\hat{p}_n\hat{p}_r\rangle \\ &\qquad+\varepsilon_{ikn}F_1\langle[1-\beta^2(\hat{k}\hat{p})^2]^{-1}\overset{s}{\mathscr{T}}{}^k_j\hat{p}_n\hat{p}_r\rangle\} \\ &\qquad-kv\{(1-f_0)\varepsilon_{ikn}\langle\overset{a}{\mathscr{T}}{}^k_j\hat{p}_n\hat{p}_r\hat{p}_s\rangle\hat{k}_s \\ &\qquad+\varepsilon_{ikn}F_1\langle[1-\beta^2(\hat{k}\hat{p})^2]^{-1}\overset{a}{\mathscr{T}}{}^k_j\hat{p}_n\hat{p}_r\hat{p}_s\rangle\hat{k}_s\},\end{aligned} \tag{3c}$$

$$\begin{aligned}\chi_{ij} = -\mu_B^2\nu\Big\langle&\{-1+(F_0-F_1)[1-\beta^2(\hat{k}\hat{p})^2]^{-1}\}\overset{s}{\mathscr{T}}{}^i_j \\ &+\frac{kv}{\omega}(F_0-F_1)\{(\hat{k}\hat{p})[1-\beta^2(\hat{k}\hat{p})^2]^{-1}\}\overset{a}{\mathscr{T}}{}^i_j \\ &+\{1-f_0+F_1[1-\beta^2(\hat{k}\hat{p})^2]^{-1}\}\overset{s}{\mathscr{T}}{}^k_j\hat{p}_k\hat{p}_i \\ &+\omega\{1-f_0+F_1[1-\beta^2(\hat{k}\hat{p})^2]^{-1}\}\varepsilon_{ikn}\tau^{kr}_j\hat{p}_n\hat{p}_r\Big\rangle.\end{aligned} \tag{3d}$$

In order to solve the formulated problem, we should assume that all b_l for $l > l_0$ are equal to zero. Due to the increasing algebraic difficulties and taking real needs into consideration, we confine ourselves to the case, when $l_0 = 1$. Now, analysing Eqs. (3a) and (3b) we see that the vertex functions $\overset{s}{\mathscr{T}}{}^i_j$ and $\overset{a}{\mathscr{T}}{}^i_j$ are of the form (cf. (A1)).

$$\overset{s}{\mathscr{T}}{}^i_j = t_0\delta_{ij}+t_2\hat{k}_i\hat{k}_j, \tag{4a}$$

$$\overset{a}{\mathscr{T}}{}^i_j = t_1(\hat{k}\hat{p})\delta_{ij}+t_3\hat{k}_i\hat{p}_j+t_5\hat{k}_j\hat{p}_i+t_7(\hat{k}\hat{p})\hat{k}_i\hat{k}_j. \tag{4b}$$

Since our calculations are performed in the acoustic limit, so that only spin waves appear as the object of our interest. If the total angular momentum J is equal to unity then the vertex function τ^{in}_j ought to fulfil the condition $\tau^{in}_j = -\tau^{ni}_j$ (cf. [11, 20]) and (A2)).

Such τ^{in}_j can be chosen in the following most general form (cf. (A7))

$$\tau^{in}_j = \tau_0\varepsilon_{ipn}\hat{k}_p\hat{k}_j+\tau_1(\varepsilon_{ijp}\hat{k}_p\hat{k}_n+\varepsilon_{pjn}\hat{k}_p\hat{k}_i). \tag{4c}$$

Substituting vertex functions in the form (4) to Eq. (1e) and taking into account that we confine ourselves to $J = 1$ we obtain

$$\begin{aligned}\tau_0[\omega^2(\tfrac{1}{3}-\tfrac{1}{3}f_0+A^1_2)-k^2v^2(\tfrac{1}{15}-\tfrac{1}{15}f_0+A^1_4)]\\ = -(t_0+t_2)\omega(\tfrac{1}{3}-\tfrac{1}{3}f_0+A^1_2)\\ -(t_1+t_3+t_7)kv(\tfrac{1}{15}-\tfrac{1}{15}f_0+A^1_4),\end{aligned} \tag{5a}$$

$$\begin{aligned}\tau_1[\omega^2(\tfrac{2}{3}-\tfrac{2}{3}f_0+C^1_2+A^1_2)-k^2v^2(\tfrac{4}{15}-\tfrac{4}{15}f_0+C^1_4+A^1_4)]\\ = -t_0\omega(\tfrac{2}{3}-\tfrac{2}{3}f_0+C^1_2+A^1_2)-t_1kv(\tfrac{4}{15}-\tfrac{4}{15}f_0+C^1_4+A^1_4)\\ -t_3kv(\tfrac{1}{15}-\tfrac{1}{15}f_0+A^1_4).\end{aligned} \tag{5b}$$

In such a way a system of two linear independent equations is obtained.

After computing the averages $\langle\ldots\rangle$ (cf. (A8), (A9) and (A10)) and verifying the linear independence of tensors, we substitute vertex functions of the form (4) into Eqs. (1a) and (1b) and obtain a system of six linear equations.

$$\begin{aligned}t_0(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C^0_0+b_0C^1_0-b_0A^1_2)\\ -t_1\frac{kv}{\omega}b_0(C^0_2-C^1_2) = 1-\tau_1\omega b_0(\tfrac{2}{3}-\tfrac{2}{3}f_0+C^1_2+A^1_2),\end{aligned} \tag{6a}$$

$$\begin{aligned}t_0b_0(-2A^1_2-B^1_2)\\ +t_2(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C^0_0+b_0C^1_0-b_0C^1_2)\\ -(t_3+t_5+t_7)\frac{kv}{\omega}b_0(C^0_2-C^1_2)\\ = -2\tau_0\omega b_0(\tfrac{1}{3}-\tfrac{1}{3}f_0+A^1_2)+\tau_1\omega b_0(\tfrac{2}{3}-\tfrac{2}{3}f_0+A^1_2+C^1_2)\end{aligned} \tag{6b}$$

$$-t_0 \frac{kv}{\omega} 3b_1(C_2^0 - C_2^1)$$

$$+t_1(1 - \tfrac{1}{5} b_1 + \tfrac{4}{5} b_1 f_0 - 3b_1 C_2^0 + 3b_1 A_4^1)$$

$$+t_3 b_1(\tfrac{1}{5} - \tfrac{1}{5} f_0 + 3A_4^1) = -\tau_1 kv 3b_1(\tfrac{4}{15} - \tfrac{4}{15} f_0 + C_4^1 + A_4^1) \tag{6c}$$

$$t_1 3b_1(\tfrac{1}{15} - \tfrac{1}{15} f_0 + A_4^1)$$

$$+t_3(1 + \tfrac{1}{5} b_1 + \tfrac{4}{5} b_1 f_0 - 3b_1 A_2^0 + 3b_1 A_4^1)$$

$$= \tau_1 kv 3b_1(\tfrac{1}{15} - \tfrac{1}{15} f_0 + A_4^1) \tag{6d}$$

$$(t_1 + t_3 + t_7) 3b_1(\tfrac{1}{15} - \tfrac{1}{15} f_0 + A_4^1)$$

$$+t_5(1 + b_1 + 3b_1 A_2^1 - 3b_1 A_2^0)$$

$$= \tau_0 kv 3b_1(\tfrac{1}{15} - \tfrac{1}{15} f_0 + A_4^1), \tag{6e}$$

$$t_1 3b_1 B_4^1 + t_2 \frac{kv}{\omega} 3b_1(C_2^1 - C_2^0)$$

$$+t_3 3b_1(-2A_2^0 - B_2^0 + B_4^1)$$

$$+t_5 3b_1(-2A_2^0 - B_2^0 + 2A_2^1 + B_2^1)$$

$$+t_7(1 + \tfrac{2}{5} b_1 + \tfrac{3}{5} b_1 f_0 - 3b_1 C_2^0 + 6b_1 A_4^1 + 3b_1 B_4^1)$$

$$= -\tau_0 kv 3b_1(\tfrac{1}{5} - \tfrac{1}{5} f_0 + 3A_4^1)$$

$$+\tau_1 kv 3b_1(\tfrac{1}{5} - \tfrac{1}{5} f_0 + C_4^1), \tag{6f}$$

Eqs. (6a, c, d) contain t_0, t_1, t_3 and one parameter τ_1 as the unknown quantities. Hence, they can be treated as a system of three equations for three unknown variables.

Adding Eqs. (6a) and (6b) we get

$$(t_0 + t_2)(1 + \tfrac{2}{3} b_0 + \tfrac{1}{3} b_0 f_0 - b_0 C_0^0 + b_0 C_0^1 - b_0 C_2^1)$$

$$+(t_1 + t_3 + t_5 + t_7) \frac{kv}{\omega} b_0(C_2^1 - C_2^0)$$

$$= 1 - 2\tau_0 \omega b_0(\tfrac{1}{3} - \tfrac{1}{3} f_0 + A_2^1), \tag{7a}$$

Adding Eqs. (6c, d, f) we get

$$(t_0 + t_2) \frac{kv}{\omega} 3b_1(C_2^1 - C_2^0)$$

$$+t_5 3b_1(-2A_2^0 - B_2^0 + 2A_2^1 + B_2^1)$$

$$+(t_1 + t_3 + t_7)(1 + \tfrac{2}{5} b_1 + \tfrac{3}{5} b_1 f_0 - 3b_1 C_2^0 + 6b_1 A_4^1 + 3b_1 B_4^1)$$

$$= -\tau_0 kv 3b_1(\tfrac{1}{5} - \tfrac{1}{5} f_0 + 3A_4^1), \tag{7b}$$

From Eq. (6e) we find t_5. Substituting t_5 into Eqs. (7) we obtain the system of two equations for two unknown variables (t_0+t_2) and $(t_1+t_3+t_7)$, and with one parameter τ_0.

Let us now consider expression (1d). Substituting (5) into (1d) we obtain

$$\chi_{ij} = -\mu_B^2 v\Bigg\{\Bigg[t_0(-\tfrac{2}{3}-\tfrac{1}{3}f_0+C_0^0-C_0^1+A_2^1)$$

$$+t_1\frac{kv}{\omega}(C_2^0-C_2^1)-\tau_1\omega(\tfrac{2}{3}-\tfrac{2}{3}f_0+C_2^1+A_2^1)\Bigg](\delta_{ij}-\hat{k}_i\hat{k}_j)$$

$$+\Bigg[(t_0+t_2)(-\tfrac{2}{3}-\tfrac{1}{3}f_0+C_0^0-C_0^1+C_2^1)$$

$$+(t_1+t_3+t_5+t_7)\frac{kv}{\omega}(C_2^0-C_2^1)-2\tau_0\omega(\tfrac{1}{3}-\tfrac{1}{3}f_0+A_2^1)\Bigg]\hat{k}_i\hat{k}_j\Bigg\}. \tag{8}$$

Solving the systems of equations given above and using Eq. (8) we obtain the spin susceptibility tensor in the following form:

$$\chi_{ij} = \mu_B^2 v[S^{-1}(T-U^{-1}V)(\delta_{ij}-\hat{k}_i\hat{k}_j)+W^{-1}(X-Y^{-1}Z)\hat{k}_i\hat{k}_j], \tag{9}$$

where

$$S = 1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C_0^0+\tfrac{1}{2}b_0C_0^1+\tfrac{1}{2}b_0C_2^1,$$

$$T = \tfrac{2}{3}+\tfrac{1}{3}f_0-C_0^0+\tfrac{1}{2}C_0^1+\tfrac{1}{2}C_2^1,$$

$$U = \omega^2(\tfrac{2}{3}-\tfrac{2}{3}f_0+\tfrac{1}{2}C_0^1+\tfrac{1}{2}C_2^1)(1+b_0f_0-b_0C_0^0)$$

$$-k^2v^2(\tfrac{4}{15}-\tfrac{4}{15}f_0+\tfrac{1}{2}C_2^1+\tfrac{1}{2}C_4^1)(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C_0^0+\tfrac{1}{2}b_0C_0^1+\tfrac{1}{2}b_0C_2^1),$$

$$V = \omega^2(\tfrac{2}{3}-\tfrac{2}{3}f_0+\tfrac{1}{2}C_0^1+\tfrac{1}{2}C_2^1)^2,$$

$$W = 1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C_0^0+b_0C_0^1-b_0C_2^1,$$

$$X = \tfrac{2}{3}+\tfrac{1}{3}f_0-C_0^0+C_0^1-C_2^1,$$

$$Y = \omega^2(\tfrac{1}{3}-\tfrac{1}{3}f_0+\tfrac{1}{2}C_0^1-\tfrac{1}{2}C_2^1)(1+b_0f_0-b_0C_0^0)$$

$$-k^2v^2(\tfrac{1}{15}-\tfrac{1}{15}f_0+\tfrac{1}{2}C_2^1-\tfrac{1}{2}C_4^1)(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C_0^0+b_0C_0^1-b_0C_2^1),$$

$$Z = \omega^2 2(\tfrac{1}{3}-\tfrac{1}{3}f_0+\tfrac{1}{2}C_0^1-\tfrac{1}{2}C_2^1)^2.$$

Here the expressions $S\ldots Z$ are given when $b_1 = 0$. The complete forms of these expressions are presented in Appendix B.

Let us discuss now some properties of the obtained formula (9). Using (A12) we see that this formula is in agreement with those all known limits (cf. [5, 6, 9, 19]). It is obvious, since the kernels L, M, N, O have these properties, too.

Spin waves are connected with poles having spin susceptibility (9). Some of the poles, considered by Combescot [10] and Maki [11], corresponded to zeros of U and Y.

Let us go onward to investigate the spin wave velocity for the transversal mode if $T\to T_c$. Applying (A12) we see that the internal fraction $U^{-1}V$ vanishes in this limit. It is therefore understood that the superfluid phase vanishes and this type of spin waves has to vanish, too. In spite of this, let us consider the denominator U of this fraction, mainly its zeros. Applying (A10) and (A12) again we obtain that the solutions of this equation $s\left(=\dfrac{\omega}{kv}\right)$ tend to zero if $T \to T_c$. Hence for $T = T_c$ the transversal spin waves velocity equal zero. A similar result can be obtained for the longitudinal mode. The external fraction has no poles in this limit if $l_0 = 0$ (cf. [5]).

5. *Conclusions*

The method developed in this paper can be applied in all cases considered by the LMC theory. One can extend this method for the case when more than two Landau parameters do not vanish. The difficulties connected with the computation of the higher rank determinants are the sole difficulties. In the conclusion we should emphasize that the information obtained about the system ^{3}He$-$B ought to satisfy the present experiments.

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APPENDIX A

In this Appendix all introduced symbols are collected and explained. We also give some characteristic properties of the functions used.

The bracket $\langle ... \rangle$ denotes the averaging over spherical angles. The ω and kv are measured in units of 2Δ. The normal vertex function (cf. [3])

$$\hat{\mathscr{T}}_j(\hat{p}) = \mathscr{T}^i_j(\hat{p})\sigma^i \equiv \mathscr{T}^i_j\sigma^i,$$

$$\overset{s}{\mathscr{T}}{}^i_j = \tfrac{1}{2}\,[\mathscr{T}^i_j(\hat{p})+\mathscr{T}^i_j(-\hat{p})],$$

$$\overset{a}{\mathscr{T}}{}^i_j = \tfrac{1}{2}\,[\mathscr{T}^i_j(\hat{p})-\mathscr{T}^i_j(-\hat{p})]. \tag{A1}$$

The anomalous vertex function (cf. [3, 9])

$$\hat{\tau}_j(\hat{p}) = \tau^i_j(\hat{p})\sigma^i, \quad \tau^i_j(\hat{p}) = \tau^{in}_j\hat{p}_n. \tag{A2}$$

The exchange part of the dimensionless effective interaction in the particle-hole channel, i.e. the antisymmetric Fermi-liquid interaction is

$$B(\hat{p}\hat{p}') = \sum_{l=0}^{\infty}(2l+1)b_lP_l(\hat{p}\hat{p}'),$$

$$\overset{s}{B} = \tfrac{1}{2}\,[B(\hat{p}\hat{p}')+B(-\hat{p}\hat{p}')],$$

$$\overset{a}{B} = \tfrac{1}{2}\,[B(\hat{p}\hat{p}')-B(-\hat{p}\hat{p}')], \tag{A3}$$

and

$$b_l = \frac{F_l^a}{2l+1} = \frac{\frac{1}{4} Z_l}{2l+1}.$$

For a different definition of Landau's parameters see, e.g. [5]. The kernels L, M, N, O in a matrix notation (for details see [3, 4, 6] and [18])

$$L \equiv G_s(p_+)G_s(p_-)-(G^2(p))^{\omega},$$

$$M \equiv G_s(p_+)F(p_-),$$

$$N \equiv G_s(p_+)G_s^-(p_-)-G_s(p)G^-(p),$$

$$O \equiv F(p_+)F(p_-), \tag{A4}$$

where $p_\pm = p \pm q$ and $q = (k, \omega)$. The introduced function is

$$f_0 = \int_0^\infty d\varepsilon \left(\frac{dn}{dE}\right) \quad - \text{ Yosida function,}$$

$$f_1 = \int_0^\infty d\varepsilon \left(\frac{dn}{dE}\right)\frac{\Delta^2}{E^2}. \tag{A5}$$

The introduced functionals are

$$F_0 \cdot = \int_0^\infty d\varepsilon \left(\frac{dn}{dE}\right) \cdot, \qquad F_1 \cdot = \int_0^\infty d\varepsilon \left(\frac{dn}{dE}\right)\frac{\Delta^2}{E^2} \cdot, \tag{A6}$$

where

$$\left(\frac{dn}{dE}\right) = \frac{1}{2T} ch^{-2} \frac{E}{2T}, \qquad E^2 = \varepsilon^2 + \Delta^2.$$

The formulae used in this paper are

$$\varepsilon_{ijn} = \varepsilon_{ijp}\hat{k}_p\hat{k}_n + \varepsilon_{ipn}\hat{k}_p\hat{k}_j + \varepsilon_{pjn}\hat{k}_p\hat{k}_i, \tag{A7}$$

cf. [9]. The averaging formulae are

$$\langle \hat{p}_{a_1} \dots \hat{p}_{a_n} \rangle = \frac{1}{(n+1)!!}\left(\sum \delta_{a_1 a_i} \dots \delta_{a_j a_k}\right), \tag{A8}$$

where the product of deltas contains $n/2$ factors and the sum $(n-1)!!$ terms.

$$\langle (\hat{k}\hat{p})^n \hat{p}_i \hat{p}_j \hat{p}_l \rangle = \frac{1}{(n+2)(n+4)}(\hat{k}_i\delta_{jl} + \hat{k}_j\delta_{il} + \hat{k}_l\delta_{ij}) + \frac{(n-1)}{(n+2)(n+4)}\hat{k}_i\hat{k}_j\hat{k}_l, \tag{A9}$$

$$\langle (\hat{k}\hat{p})^n \hat{p}_i \hat{p}_j \rangle = \frac{1}{(n+1)(n+3)}\delta_{ij} + \frac{n}{(n+1)(n+3)}\hat{k}_i\hat{k}_j,$$

$$\langle (\hat{k}\hat{p})^n \hat{p}_i \rangle = \frac{1}{n+2}\hat{k}_i,$$

$$\langle(\hat{k}\hat{p})^n\rangle = \frac{1}{n+1},$$

$$\langle[1-\beta^2(\hat{k}\hat{p})^2]^{-1}(\hat{k}\hat{p})^{n-3}\hat{p}_i\hat{p}_j\hat{p}_l\rangle$$
$$= A_n(\hat{k}_i\delta_{jl}+\hat{k}_j\delta_{il}+\hat{k}_l\delta_{ij})+B_n\hat{k}_i\hat{k}_j\hat{k}_l,$$
$$\langle[1-\beta^2(\hat{k}\hat{p})^2]^{-1}(\hat{k}\hat{p})^{n-2}\hat{p}_i\hat{p}_j\rangle$$
$$= A_n\delta_{ij}+(2A_n+B_n)\hat{k}_i\hat{k}_j,$$
$$\langle[1-\beta^2(\hat{k}\hat{p})^2]^{-1}(\hat{k}\hat{p})^{n-1}\hat{p}_i\rangle$$
$$= (3A_n+B_n)\hat{k}_i,$$
$$\langle[1-\beta^2(\hat{k}\hat{p})^2]^{-1}(\hat{k}\hat{p})^{n}\rangle = 3A_n+B_n \equiv C_n, \tag{A10}$$

where $\beta = \frac{kv}{\omega}\frac{\varepsilon}{E}$. It is easy to remark that

$$A_n = \tfrac{1}{2}C_{n-2}-\tfrac{1}{2}C_n,$$
$$B_n = \tfrac{5}{2}C_n-\tfrac{3}{2}C_{n-2},$$

and

$$C_n = \beta^{-n}\left(C_0-1-\tfrac{1}{3}\beta^2-\ \dots\ -\frac{1}{n-1}\beta^{n-2}\right),$$

where the function $1-C_0$ is the same as the Lindhard function (cf. [5]). The form of the functions C_n^0 and C_n^1 are

$$C_n^0 = F_0C_n = \int_0^\infty d\varepsilon\left(\frac{dn}{dE}\right)C_n,$$

$$C_n^1 = F_1C_n = \int_0^\infty d\varepsilon\left(\frac{dn}{dE}\right)\frac{\Delta^2}{E^2}C_n \tag{A11}$$

and A_n^0, A_n^1 and B_n^0, B_n^1 are analogous to (A11). The properties of functions f_0, f_1, C_n^0, C_1^n are

$$\lim_{\omega=0}\frac{kv}{\omega}C_n^0 = \lim_{\omega=0}\frac{kv}{\omega}C_n^1 = 0,$$

$$\lim_{k=0}C_n^0 = \frac{1}{n+1}f_0,\quad \lim_{k=0}C_n^1 = \frac{1}{n+1}f_1,$$
$$T=0;\quad f_0=f_1=C_n^0=C_n^1=0,$$
$$T=T_c;\quad f_0=1,\quad C_n^0=C_n,\quad f_1=C_n^1=0,$$
$$T\to T_c\quad (1-f_0)\sim\Delta^2,\quad C_n^1\sim\Delta C_n. \tag{A12}$$

APPENDIX B

In this Appendix the forms of expressions $S \ldots Z$ are presented for the case where two Landau parameters b_0 and b_1 do not vanish. Since we do not want to introduce extra symbols, they are quite lenghty.

$$S = (1+\tfrac{2}{3}\,b_0+\tfrac{1}{3}\,b_0 f_0+b_0 C_0^0+\tfrac{1}{2}\,b_0 C_0^1+\tfrac{1}{2}\,b_0 C_2^1)$$
$$\times[(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-3b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1)$$
$$\times(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-\tfrac{3}{2}\,b_1 C_0^0+\tfrac{3}{2}\,b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1)$$
$$-b_1^2(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}\,C_2^1-\tfrac{3}{2}\,C_4^1)^2]$$
$$-\frac{k^2v^2}{\omega^2}\,3b_0b_1(C_2^0-C_2^1)^2$$
$$\times(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-\tfrac{3}{2}\,b_1 C_0^0+\tfrac{3}{2}\,b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1),$$

$$T = (\tfrac{2}{3}+\tfrac{1}{3}f_0-C_0^0+\tfrac{1}{2}\,C_0^1+\tfrac{1}{2}\,C_2^1)$$
$$\times[(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-3b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1)$$
$$\times(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-\tfrac{3}{2}\,b_1 C_0^0+\tfrac{3}{2}\,b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1)$$
$$-b_1^2(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}\,C_2^1-\tfrac{3}{2}\,C_4^1)^2]$$
$$-\frac{k^2v^2}{\omega^2}\,3b_1(C_2^0-C_2^1)^2$$
$$\times(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-\tfrac{3}{2}\,b_1 C_0^0+\tfrac{3}{2}\,b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1),$$

$$U = \omega^2(\tfrac{2}{3}-\tfrac{2}{3}f_0+\tfrac{1}{2}\,C_0^1+\tfrac{1}{2}\,C_2^1)\,\{(1+b_0 f_0-b_0 C_0^0)$$
$$\times[(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-3b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1)$$
$$\times(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-\tfrac{3}{2}\,b_1 C_0^0+\tfrac{3}{2}\,b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1)$$
$$-b_1^2(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}\,C_2^1-\tfrac{3}{2}\,C_4^1)^2]$$
$$-\frac{k^2v^2}{\omega^2}\,b_0b_1(C_2^0-C_2^1)\,[(\tfrac{4}{5}-\tfrac{4}{5}f_0+3C_2^0-\tfrac{3}{2}\,C_2^1+\tfrac{3}{2}\,C_4^1)$$
$$\times(1+\tfrac{1}{5}\,b_1+\tfrac{4}{5}\,b_1 f_0-\tfrac{3}{2}\,b_1 C_0^0+\tfrac{3}{2}\,b_1 C_2^0+\tfrac{3}{2}\,b_1 C_2^1-\tfrac{3}{2}\,b_1 C_4^1)$$
$$+2b_1(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}\,C_2^1-\tfrac{3}{2}\,C_4^1)^2]\}$$
$$-k^2v^2(\tfrac{4}{15}-\tfrac{4}{15}f_0+\tfrac{1}{2}\,C_2^1+\tfrac{1}{2}\,C_4^1)$$
$$\times\Big\{(1+\tfrac{2}{3}\,b_0+\tfrac{1}{3}\,b_0 f_0-b_0 C_0^0+\tfrac{1}{2}\,b_0 C_0^1+\tfrac{1}{2}\,b_0 C_2^1)$$
$$\times(1+b_1-3b_1 C_2^0+3b_1 C_2^1)+3b_0b_1(C_2^0-C_2^1)$$

$$\times\left[\left(\tfrac{2}{3}-\tfrac{2}{3}f_0+\tfrac{1}{2}C_0^1+\tfrac{1}{2}C_2^1\right)-\frac{k^2v^2}{\omega^2}\left(C_2^0-C_2^1\right)\right]\Bigg\}$$

$$\times\left(1+\tfrac{1}{5}b_1+\tfrac{4}{5}b_1f_0-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_2^1-\tfrac{3}{2}b_1C_4^1\right)$$

$$-k^2v^2 3b_1\left(\tfrac{1}{15}-\tfrac{1}{15}f_0+\tfrac{1}{2}C_2^1-\tfrac{1}{2}C_4^1\right)^2$$

$$\times\Bigg[\left(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C_0^0+\tfrac{1}{2}b_0C_0^1+\tfrac{1}{2}b_0C_2^1\right)$$

$$\times\left(1+b_1-3b_1C_2^0+3b_1C_2^1\right)-\frac{k^2v^2}{\omega^2}3b_0b_1\left(C_2^0-C_2^1\right)^2\Bigg],$$

$$V=\omega^2\Bigg\{\left(\tfrac{2}{3}-\tfrac{2}{3}f_0+\tfrac{1}{2}C_0^1+\tfrac{1}{2}C_2^1\right)$$

$$\times\Big[\left(1+\tfrac{1}{5}b_1+\tfrac{4}{5}b_1f_0-3b_1C_2^0+\tfrac{3}{2}b_1C_2^1-\tfrac{3}{2}b_1C_4^1\right)$$

$$\times\left(1+\tfrac{1}{5}b_1+\tfrac{4}{5}b_1f_0-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_2^1-\tfrac{3}{2}b_1C_4^1\right)$$

$$-b_1^2\left(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1\right)^2\Big]+\frac{k^2v^2}{\omega^2}b_1\left(C_2^0-C_2^1\right)$$

$$\times\Big[\left(1+\tfrac{1}{5}b_1+\tfrac{4}{5}b_1f_0-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_2^1-\tfrac{3}{2}b_1C_4^1\right)$$

$$\times\left(\tfrac{4}{5}-\tfrac{4}{5}f_0+\tfrac{3}{2}C_2^1+\tfrac{3}{2}C_4^1\right)+b_1\left(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1\right)^2\Big]\Bigg\}^2,$$

$$W=\left(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C_0^0+b_0C_0^1-b_0C_2^1\right)$$

$$\times\Big[\left(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_1f_0-3b_1C_2^0-\tfrac{3}{2}b_1C_2^1+\tfrac{9}{2}b_1C_4^1\right)$$

$$\times\left(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1\right)$$

$$-\tfrac{3}{2}b_1^2\left(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1\right)\left(C_0^0-3C_2^0-C_0^1+3C_2^1\right)\Big]$$

$$-\frac{k^2v^2}{\omega^2}3b_0b_1\left(C_2^0-C_2^1\right)^2$$

$$\times\left(1+\tfrac{4}{5}b_1+\tfrac{1}{5}b_1f_0-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-3b_1C_2^1+\tfrac{3}{2}b_1C_4^1\right),$$

$$X=\left(\tfrac{2}{3}+\tfrac{1}{3}f_0-C_0^0+C_0^1-C_2^1\right)$$

$$\times\Big[\left(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_1f_0-3b_1C_2^0-\tfrac{3}{2}b_1C_2^1+\tfrac{9}{2}b_1C_4^1\right)$$

$$\times\left(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1\right)$$

$$-\tfrac{3}{2}b_1^2\left(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1\right)\left(C_0^0-3C_2^0-C_0^1+3C_2^1\right)\Big]$$

$$-\frac{k^2v^2}{\omega^2}3b_1\left(C_2^0-C_2^1\right)^2$$

$$\times\left(1+\tfrac{4}{5}b_1+\tfrac{1}{5}b_1f_0-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-3b_1C_2^1+\tfrac{3}{2}b_1C_4^1\right),$$

$$Y = \omega^2(\tfrac{1}{3}-\tfrac{1}{3}f_0+\tfrac{1}{2}C_0^1-\tfrac{1}{2}C_2^1)$$

$$\times\Big\{(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)$$

$$\times\Big\{(1+b_0f_0-b_0C_0^0)$$

$$\times[(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_1f_0-3b_1C_2^0-\tfrac{3}{2}b_1C_2^1+\tfrac{9}{2}b_1C_4^1)$$

$$\times(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)$$

$$-\tfrac{3}{2}b_1^2(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1)(C_0^0-3C_2^0-C_0^1+3C_2^1)]$$

$$-\frac{k^2v^2}{\omega^2}3b_0b_1(C_2^0-C_2^1)^2$$

$$\times(1+\tfrac{4}{5}b_1+\tfrac{1}{5}b_1f_0-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-3b_1C_2^1+\tfrac{3}{2}b_1C_4^1)\Big\}$$

$$+\frac{k^2v^2}{\omega^2}b_0b_1(C_2^0-C_2^1)(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1)$$

$$\times[(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_1f_0-3b_1C_2^0-\tfrac{3}{2}b_1C_2^1+\tfrac{9}{2}b_1C_4^1)$$

$$\times(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)$$

$$-\tfrac{3}{2}b_1^2(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1)(C_0^0-3C_2^0-C_0^1+3C_2^1)$$

$$-3(1+\tfrac{4}{5}b_1+\tfrac{1}{5}b_1f_0-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-3b_1C_2^1+\tfrac{3}{2}b_1C_4^1)$$

$$\times(1+b_1-b_1C_0^0+b_1C_0^1)]\Big\}$$

$$-k^2v^2(\tfrac{1}{15}-\tfrac{1}{15}f_0+\tfrac{1}{2}C_2^1-\tfrac{1}{2}C_4^1)$$

$$\times(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)^2$$

$$\times\Big\{(1+\tfrac{2}{3}b_0+\tfrac{1}{3}b_0f_0-b_0C_0^0+b_0C_0^1-b_0C_2^1)$$

$$\times(1+b_1-3b_1C_2^0+3b_1C_2^1)+3b_0b_1(C_2^0-C_2^1)$$

$$\times\left[(\tfrac{2}{3}-\tfrac{2}{3}f_0+C_0^1-C_2^1)-\frac{k^2v^2}{\omega^2}(C_2^0-C_2^1)\right]\Big\},$$

$$Z = \{\omega^2(\tfrac{2}{3}-\tfrac{2}{3}f_0+C_0^1-C_2^1)$$

$$\times[(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_1f_0-3b_1C_2^0-\tfrac{3}{2}b_1C_2^1+\tfrac{9}{2}b_1C_4^1)$$

$$\times(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)$$

$$-\tfrac{3}{2}b_1^2(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1)(C_0^0-3C_2^0-C_0^1+3C_2^1)]$$

$$\times(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)$$
$$-k^2v^2b_1(C_2^0-C_2^1)(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1)$$
$$\times[(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_1f_0-3b_1C_2^0-\tfrac{3}{2}b_1C_2^1+\tfrac{9}{2}b_1C_4^1)$$
$$\times(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)$$
$$-\tfrac{3}{2}b_1^2(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1)(C_0^0-3C_2^0-C_0^1+3C_2^1)$$
$$-3(1+\tfrac{4}{5}b_1+\tfrac{1}{5}b_1f_0-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-3b_1C_2^1+\tfrac{3}{2}b_1C_4^1)$$
$$\times(1+b_1-b_1C_0^0+b_1C_0^1)]\}\Big\{(\tfrac{1}{3}-\tfrac{1}{3}f_0+\tfrac{1}{2}C_0^1-\tfrac{1}{2}C_2^1)$$
$$\times[(1+\tfrac{2}{5}b_1+\tfrac{3}{5}b_1f_0-3b_1C_2^0-\tfrac{3}{2}b_1C_2^1+\tfrac{9}{2}b_1C_4^1)$$
$$\times(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)$$
$$-\tfrac{3}{2}b_1^2(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{3}{2}C_4^1)(C_0^0-3C_2^0-C_0^1+3C_2^1)]$$
$$+\frac{k^2v^2}{\omega^2}b_1(C_2^0-C_2^1)(\tfrac{1}{5}-\tfrac{1}{5}f_0+\tfrac{3}{2}C_2^1-\tfrac{5}{2}C_4^1)$$
$$\times(1+b_1-\tfrac{3}{2}b_1C_0^0+\tfrac{3}{2}b_1C_2^0+\tfrac{3}{2}b_1C_0^1-\tfrac{3}{2}b_1C_2^1)\Big\}.$$

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