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CORRELATION FUNCTIONS OF HEISENBERG FERROMAGNET WITH A SINGLE-ION ANISOTROPY. SPIN $S = 1$. II

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The correlation functions of the Heisenberg ferromagnet with a single-ion anisotropy in the form $-\sum_f D(S_f^3)^2$ were calculated for the case of spin $S = 1$. The procedure of decoupling of higher order Green functions based on the Ishikawa and Oguchi method was applied. The temperature dependences of magnetic susceptibility, specific heat and internal energy for the case when $\frac{D}{J_0} \ll 1$ were determined. The magnetic susceptibility critical index γ rapidly decreases from $\gamma = 2$ for the isotropic Heisenberg ferromagnet to $\gamma = 1$ for higher values of the crystal field anisotropy. The Fischer critical index η for the correlation functions equals zero. The correlation functions in a long-range limit assume the Ornstein-Zernicke form. The length of transverse correlations assumes finite values at the phase transition point. The length of longitudinal correlations appears to be convergent for $T = T_c$ in the approximation of $\frac{D}{J_0} \ll 1$.

1. *Introduction*

Several authors have investigated the effect of single-ion anisotropy of the $-D\sum_f (S_f^3)^2$ type on the thermodynamic properties of exchange interacting spin systems [1]. Some of them used the method of two-time [2] retarded Green functions [1] while other employed the diagram method [3]. However, for several years the problem of calculating the longitudinal correlation functions, needed for determining several thermodynamic properties, has not been solved. To calculate the functions of longitudinal correlations we shall also apply the method of Green function equations of motion. Since the new method of the Green functions decoupling has been tested for the case of the isotropic Heisenberg ferromagnet [4], the result obtained in the present paper seems to be credible. The

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obtained critical indices of magnetization, susceptibility and specific heat support the scaling hypothesis, however lengths of the longitudinal and transverse correlations at the phase transition point are finite. The single-ion anisotropy is distinguished by its own axis in a crystal and that is why the problem of limiting values of critical indices at $D \to 0$ seems to be very interesting. The smoothness hypothesis predicts for such a case very rapid changes of critical indices. We have tested this problem for the case of the magnetic susceptibility critical index γ.

2. *Green functions*

The Hamiltonian of the problem has the form

$$H = -\mu h \sum_f S_f^3 - D \sum_f (S_f^3)^2 - \sum_{f \neq g} J_{fg}(S_f^3 S_g^3 + S_f^+ S_g^-), \tag{2.1}$$

where h is an external magnetic field, D is the single-ion anisotropy parameter, J_{fg} is exchange integral, $J_{fg} = 0$ for $f = g$. For such a system we introduce Green functions [2]

$$G_{f,g}^\alpha(t) = \langle\langle L_{\alpha+1,\alpha}^f(t) \,|\, S_g^- \rangle\rangle, \tag{2.2}$$

$$F_{f,gh}^\alpha(t) = \langle\langle L_{\alpha+1,\alpha}^f(t) \,|\, \hat{S}_h^3 S_g^- \rangle\rangle, \tag{2.3}$$

$$\alpha = -S, \ldots, S-1, \quad \hat{S}_h^3 = S_h^3 - \langle S_h^3 \rangle,$$

where S is a value of the spin for each lattice site, operators $L_{\alpha+1,\alpha}^f$ fulfill the following commutation relations

$$[-\mu h \sum_f S_f^3 - D \sum_f (S_f^3)^2, L_{\alpha+1,\alpha}^g]_- = [\mu h + D(2\alpha+1)] L_{\alpha+1,\alpha}^g, \tag{2.4}$$

and the spin operators are connected with them by

$$S_f^+ = \sum_{\alpha=-S}^{S} C_\alpha^S L_{\alpha+1,\alpha}^f,$$

$$S_f^- = \sum_{\alpha=-S}^{S} C_\alpha^S L_{\alpha,\alpha+1}^f,$$

$$(S_f^3)^N = \sum_{\alpha=-S}^{S} \alpha^N L_{\alpha,\alpha}^f, \quad C_\alpha^S = \sqrt{S(S+1) - \alpha(\alpha+1)}\,. \tag{2.5}$$

Equations of motion of the Green functions $G_{f,g}^\alpha$, $F_{f,gh}^\alpha$ assume the form [2]

$$E\langle\langle L_{\alpha+1,\alpha}^f | A_h S_g^- \rangle\rangle = \langle [L_{\alpha+1,\alpha}^f, A_h S_g^-]_- \rangle + [\mu h + D(2\alpha+1)] \langle\langle L_{\alpha+1,\alpha}^f | A_h S_g^- \rangle\rangle$$

$$+ 2 \sum_p J_{pf} \sum_{\beta=-S}^{S} \beta \langle\langle L_{\beta,\beta}^p L_{\alpha+1,\alpha}^f | A_h S_g^- \rangle\rangle$$

$$- \sum_p J_{pf} \sum_{\gamma=-S}^{S} C_\gamma^S C_\alpha^S \langle\langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) L_{\gamma+1,\gamma}^p | A_h S_g^- \rangle\rangle$$

$$- \sum_p J_{pf} \sum^{S} C_\beta^S \langle\langle L_{\beta,\beta+1}^p (C_{\alpha-1}^S L_{\alpha+1,\alpha-1}^f - C_{\alpha+1}^S L_{\alpha+2,\alpha}^f) | A_h S_g^- \rangle\rangle. \tag{2.6}$$

Now we decouple the Green functions appearing in these equations ($A_h = \hat{S}_h^3$ or $A_h = 1$) [4]

$$\langle\langle L^p_{\beta,\beta} L^f_{\alpha+1,\alpha} | S^3_h S^-_g \rangle\rangle \approx \langle L^p_{\beta,\beta} S^3_h \rangle G^\alpha_{fg} + \langle L^p_{\beta,\beta} \rangle F^\alpha_{fgh},$$

$$\langle\langle L^p_{\beta,\beta} L^f_{\alpha+1,\alpha} | S^-_g \rangle\rangle \approx \langle L^p_{\beta,\beta} \rangle G^\alpha_{fg},$$

$$\langle\langle L^p_{\beta,\beta} (C^S_{\alpha-1} L^f_{\alpha+1,\alpha-1} - C^S_{\alpha+1} L^f_{\alpha+2,\alpha}) \, | A_h S^-_g \rangle\rangle \approx 0. \tag{2.7}$$

Here the decoupling procedure is of the nature of the generalized RPA decoupling scheme [5], [6] and lies in neglecting of the highest order cumulant corresponding to the given Green function.

After applying the approximations mentioned above we make use of the Fourier transformation [7] and the so-called nearest neighbours approximation

$$\sum_{\gamma=-S}^{S-1} \{[E - \omega_0 - D(2\gamma+1)]\delta_{\alpha,\gamma} + P^S_\alpha C^S_\gamma J_k\} G^\gamma_k(E) = P^S_\alpha, \tag{2.8}$$

$$\sum_{\gamma=-S}^{S-1} \{[E - \omega_0 - D(2\gamma+1)]\delta_{\alpha,\gamma} + P^S_\alpha C^S_\gamma J_{k+q}\} F^\gamma_{k,q}(E) = A^S_\alpha, \tag{2.9}$$

where

$$P^S_\alpha = C^S_\alpha \langle L^f_{\alpha+1,\alpha+1} - L^f_{\alpha,\alpha} \rangle,$$
$$\omega_0 = \mu h + 2\sigma J_0, \qquad \sigma = \langle S^3_f \rangle, \tag{2.10}$$

and

$$A^S_\alpha = C^S_\alpha \langle (L^k_{\alpha+1,\alpha+1} - L^k_{\alpha,\alpha}) \hat{S}^3_{-k} \rangle - \langle L^q_{\alpha+1,\alpha} S^-_{-q} \rangle$$
$$- C^S_\alpha \langle (L^k_{\alpha+1,\alpha+1} - L^k_{\alpha,\alpha}) \hat{S}^3_{-k} \rangle J_q \sum_{\beta=-S}^{S-1} C^S_\beta G^\beta_q(E) + 2 \sum_{\beta=-S}^{S} \beta \langle L^k_{\beta,\beta} \hat{S}^3_{-k} \rangle J_k G^\alpha_q(E). \tag{2.11}$$

The following are the solutions of matrix equations (2.8) and (2.9)

$$G^\gamma_k(E) = \frac{\Delta^S_\gamma(E)}{1 + J_k \sum_{\beta=-S}^{S-1} C^S_\beta \Delta^S_\beta(E)}, \tag{2.12}$$

$$F^\gamma_{k,q}(E) = \frac{A^S_\gamma + J_{k+q} \sum_{\beta=-S}^{S-1} \dfrac{C^S_\beta (A^S_\gamma P^S_\beta - A^S_\beta P^S_\gamma)}{E - \omega_0 - D(2\beta+1)}}{[E - \omega_0 - D(2\gamma+1)]\,[1 + J_{k+q} \sum_{\beta=-S}^{S-1} C^S_\beta \Delta^S_\beta(E)]}, \tag{2.13}$$

where

$$\Delta^S_\beta = \frac{P^S_\beta}{E - \omega_0 - D(2\beta+1)}. \tag{2.14}$$

Further calculations for spins $S > 1$ can be performed only by laborious numerical calculations. Let us restrict ourselves to the case when $S = 1$. In this case

$$G_k^0(E) = \frac{\sqrt{2}}{2}(\sigma+\lambda)\frac{E-\omega_0+D}{(E-\Omega_k^1)(E-\Omega_k^2)}, \tag{2.15}$$

$$G_k^{-1}(E) = \frac{\sqrt{2}}{2}(\sigma-\lambda)\frac{E-\omega_0-D}{(E-\Omega_k^1)(E-\Omega_k^2)}, \tag{2.16}$$

and

$$F_{k,q}^0(E) = \frac{A_0^1[E-\omega_0+D+(\sigma-\lambda)J_{k+q}]-A_{-1}^1(\sigma+\lambda)J_{k+q}}{(E-\Omega_{k+q}^1)(E-\Omega_{k+q}^2)}, \tag{2.17}$$

$$F_{kq}^{-1}(E) = \frac{A_{-1}^1[E-\omega_0-D+(\sigma+\lambda)J_{k+q}]-A_0^1(\sigma-\lambda)J_{k+q}}{(E-\Omega_{k+q}^1)(E-\Omega_{k+q}^2)}, \tag{2.18}$$

where with the use of (2.10) and (2.5) we have substituted

$$C_0^1 = C_{-1}^1 = \sqrt{2},$$

$$P_0^1 = \frac{\sqrt{2}}{2}(\sigma+\lambda), \quad P_{-1}^1 = \frac{\sqrt{2}}{2}(\sigma-\lambda),$$

$$\lambda = 3\langle (S_f^3)^2\rangle - 2. \tag{2.19}$$

The functions A_0^1 and A_{-1}^1 are determined by the equation (2.11) and $\Omega_k^{1,2}$ are the solutions of

$$E^2-2E(\omega_0-\sigma J_k)+\omega_0^2-D^2-2J_k(\omega_0\sigma-\lambda D) = 0. \tag{2.20}$$

Using the spectral theorem [2, 8] from (2.17), (2.18) for the correlation functions we get the expressions

$$\langle S_k^- L_{1,0}^{-k}\rangle = \frac{\sqrt{2}}{2}\,\frac{\sigma+\lambda}{\Omega_k^1-\Omega_k^1}\,[(\Omega_k^1-\omega_0+D)N(\Omega_k^1)-(\Omega_k^2-\omega_0+D)N(\Omega_k^2)], \tag{2.21}$$

$$\langle S_k^- L_{0,-1}^{-}\rangle = \frac{\sqrt{2}}{2}\,\frac{\sigma-\lambda}{\Omega_k^1-\Omega_k^2}\,[(\Omega_k^1-\omega_0-D)N(\Omega_k^1)-(\Omega_k^2-\omega_0-D)N(\Omega_k^2)]. \tag{2.22}$$

Then we can calculate by the same means from Green's function $F_{k,q}^{\alpha}(E)$

$$\langle \hat{S}_{-k}^3 S_{-q}^- L_{1,0}^{k+q}\rangle = \sum_{r=1}^{2}(-1)^{r+1}\frac{1}{(\Omega_{k+q}^1-\Omega_{k+q}^2)(\Omega_q^1-\Omega_q^2)}$$
$$\times\{[\Omega_{k+q}^r-\omega_0+D+(\sigma-\lambda)J_{k+q}]B_r^1(k,q)-(\sigma+\lambda)J_{k+q}B_r^2(k,q)\}, \tag{2.23}$$

$$\langle \hat{S}_{-k}^3 S_{-q}^- L_{0,-1}^{k+q}\rangle = \sum_{r=1}^{2}(-1)^{r+1}\frac{1}{(\Omega_{k+q}^1-\Omega_{k+q}^2)(\Omega_q^1-\Omega_q^2)}$$
$$\times\{[\Omega_{k+q}^r-\omega_0-D+(\sigma+\lambda)J_{k+q}]B_r^2(k,q)-(\sigma-\lambda)J_{k+q}B_r^1(k,q)\}, \tag{2.24}$$

where

$$B_r^1(k,q) = \sqrt{2}\,(X_k^1 - X_k^0)J_q \sum_{j=-1,1} \Gamma_{j,r}(k,q) - \sqrt{2}\,(X_k^1 - X_k^{-1})J_k\Gamma_{1,r}(k,q)$$

$$+[\sqrt{2}\,(X_k^1 - X_k^0) - \langle L_{1,0}^q S_{-q}^- \rangle] N(\Omega_{k+q}^r)\,(\Omega_q^1 - \Omega_q^2),$$

$$B_r^2(k,q) = \sqrt{2}\,(X_k^0 - X_k^{-1})J_q \sum_{j=-1,1} \Gamma_{j,r}(k,q) - \sqrt{2}\,(X_k^1 - X_k^{-1})J_k\Gamma_{-1,r}(k,q)$$

$$+[\sqrt{2}\,(X_k^0 - X_k^{-1}) - \langle L_{0,-1}^q S_{-q}^- \rangle] N(\Omega_{k+q}^r)\,(\Omega_q^1 - \Omega_q^2),$$

where

$$X_k^\alpha = \langle \hat{S}_k^3 L_{\alpha,\alpha}^{-k} \rangle, \qquad N(\Omega) = (e^{\beta\Omega} - 1)^{-1}, \tag{2.25}$$

and function $\Gamma_{i,r}$ assumes the form

$$\Gamma_{i,r} = \sum_{j=1}^{2} (-1)^j \frac{(\sigma + i\lambda)(\Omega_q^j - \omega_0 + iD)}{\Omega_q^j - \Omega_{k+q}^r} [N(\Omega_q^j) - N(\Omega_{k+q}^r)]. \tag{2.26}$$

From the identity

$$\frac{1}{N} \sum_q \langle S_{-k}^3 S_{-q}^{-k} L_{\alpha+1,\alpha}^{k+q} \rangle = C_\alpha^S X_k^\alpha, \tag{2.27}$$

after substituting the explicit form of the correlation functions (2.23) and (2.24) we obtain the set of equations for the correlation functions X_k^α, $\alpha = -1, 0, 1$. Then we get

$$\langle \hat{S}_k^3 S_{-k}^3 \rangle = X_k^1 - X_k^{-1}, \tag{2.28}$$

and using (2.21) and (2.22)

$$\langle S_k^- S_{-k}^+ \rangle = \sqrt{2}\,[\langle S_k^- L_{1,0}^{-k} \rangle + \langle S_k^- L_{0,-1}^{-k} \rangle]. \tag{2.29}$$

3. *Thermodynamic properties*

Now we can calculate the magnetization and the second moment of the z-component of the spin vector. Using the formulas [4]

$$\sigma = \frac{1 + \varphi_{-1} + \varphi_0}{1 + \varphi_{-1} + 2\varphi_0 + 3\varphi_0\varphi_{-1}}, \tag{3.1}$$

and

$$\lambda = \frac{1 + \varphi_{-1} - \varphi_0}{1 + \varphi_{-1} + 2\varphi_0 + 3\varphi_0\varphi_{-1}}, \tag{3.2}$$

where functions φ_α are defined by (2.21) and (2.22). We get

$$\varphi_0 = \frac{1}{N}\sum_k \frac{1}{\Omega_k^1-\Omega_k^2}\left[(\Omega_k^1-\omega_0+D)N(\Omega_k^1)-(\Omega_k^2-\omega_0+D)N(\Omega_k^2)\right],$$

$$\varphi_{-1} = \frac{1}{N}\sum \frac{1}{\Omega_k^1-\Omega_k^2}\left[(\Omega_k^1-\omega_0-D)N(\Omega_k^1)-(\Omega_k^2-\omega_0-D)N(\Omega_k^2)\right]. \qquad (3.3)$$

To calculate T_c we use the method given in [9] expanding the expressions for σ and λ with respect to σ,

$$\sigma = \left[U(\beta)-\frac{4V(\beta)}{1+3U(\beta)}\right]\sigma+O(\sigma^2), \qquad (3.4)$$

where

$$U(\beta) = \frac{1}{N}\sum_k \frac{D}{\sqrt{D^2-2\lambda DJ_k}}\,\mathrm{cth}\,\frac{\beta\sqrt{D^2-2\lambda DJ_k}}{2}$$

$$V(\beta) = U(\beta)+\frac{\beta}{2N}\sum_k\left(\frac{1}{\chi}+2J_0-J_k\right)\mathrm{cosech}^2\,\frac{\beta\sqrt{D^2-2\lambda DJ_k}}{2}$$

$$+\frac{1}{N}\sum_k \frac{J_k-D}{\sqrt{D^2-2\lambda DJ_k}}\,\mathrm{cth}\,\frac{\beta\sqrt{D^2-2\lambda DJ_k}}{2}. \qquad (3.5)$$

Now we can use the expression

$$\lambda = \frac{4}{1+3U(\beta)}+O(\sigma^2), \qquad (3.6)$$

which enables us to write down for the case $\dfrac{D}{J_0} \ll 1$

$$\frac{D}{\lambda} = \frac{1}{\chi}+2J_0+\tfrac{3}{4}D, \qquad (3.7)$$

and

$$\frac{kT}{J_0}I(Y) = \tfrac{4}{3}-\frac{D}{6J_0}Y, \qquad (3.8)$$

where

$$\frac{1}{Y} = \frac{D}{2J_0\lambda}, \qquad I(Y) = \frac{1}{N}\sum \frac{1}{\frac{1}{Y}-\frac{J_k}{J_0}}. \qquad (3.9)$$

Equation (3.8) enables us to calculate the magnetic susceptibility and the temperature of the phase transition

$$kT_c = \frac{4J_0}{3I(1)}\left[1+\frac{\sqrt{3}\,A}{2I(1)}\left(\frac{D}{2J_0}\right)^{1/2}+\dots\right], \quad A = \frac{3\sqrt{3}}{\pi\sqrt{2}}, \text{ s.c.} \tag{3.10}$$

Above the transition point, when $\sigma = 0$ and $h = 0$, $\frac{\sigma}{h} \to \chi$ the transverse correlation function calculated in (2.29) takes on the form

$$\langle S_k^- S_{-k}^+\rangle = \frac{\dfrac{kT}{J_0}}{\dfrac{D}{2\lambda J_0} - \dfrac{J_k}{J_0}}, \tag{3.11}$$

and within the long-range limit $a|\boldsymbol{k}| \ll 1$ (simple cubic lattice)

$$\langle S_k^- S_{-k}^+\rangle = \frac{6kT}{J_0 a^2}\,\frac{\kappa_\perp^2}{1+\kappa_\perp^2|\boldsymbol{k}|^2}, \tag{3.12}$$

where

$$\kappa_\perp^2 = \frac{a^2}{6}\left[\frac{1}{2\chi J_0} + \tfrac{3}{8}\frac{D}{J_0}\right]^{-1}. \tag{3.13}$$

In a paramagnetic region the function of longitudinal correlations (2.28) equals

$$\langle \hat{S}_k^3 S_{-k}^3\rangle = \frac{\dfrac{I(Y)}{2\beta J_0}\left[\tfrac{3}{2} - \dfrac{1}{2D\beta} - \dfrac{3}{D^2Y^2\beta^2}F(Y,\boldsymbol{k})\right]}{1+\dfrac{3F(Y,\boldsymbol{k})}{2D\beta Y^2} - I(Y)\left[\dfrac{1}{Y}-\dfrac{J_k}{J_0}\right]\left[\dfrac{1}{2D\beta}+\dfrac{3F(Y,\boldsymbol{k})}{D^2Y^2\beta^2}\right]}, \tag{3.14}$$

$$F(Y,\boldsymbol{k}) = \frac{1}{N}\sum_q \frac{1}{\left[\dfrac{1}{Y}-\dfrac{J_{k+q}}{J_0}\right]\left[\dfrac{1}{Y}-\dfrac{J_k}{J_0}\right]}. \tag{3.15}$$

For the case of small $\frac{D}{J_0}$ and in the long-range limit the function (3.14) reduces to the Ornstein-Zernicke form

$$\langle \hat{S}_k^3 S_{-k}^3\rangle = \frac{6kT}{2J_0a^2}\,\frac{\kappa_\parallel^2}{1+\kappa_\parallel^2|\boldsymbol{k}|^2}, \tag{3.16}$$

with correlation length

$$\kappa_{\parallel}^2 = \frac{a^2}{6}\left[\frac{1}{2\chi J_0}\right]^{-1}. \tag{3.17}$$

At the phase transition point the longitudinal correlation function (3.16) approaches finite value and the correlation length is divergent.

The magnetic susceptibility near the critical point (3.8) assumes various values depending on the value of the parameter $\frac{D}{J_0}$. The critical index γ depends on this parameter

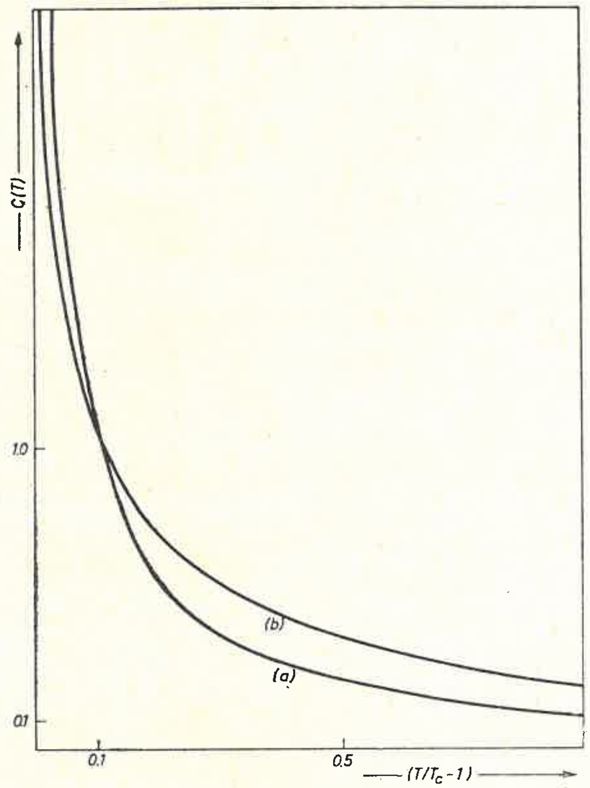

Fig. 1. Specific heat as a function of temperature for $\frac{D}{J} = 0.1$, (*a*) is our result and (*b*) is the result of Tanaka and Tani

too (Fig. 3) and rapidly approaches its limiting value of 2 at $D \to 0$. It is worthwhile to mention that such kind of behaviour of critical indices with change of the Hamiltonian symmetry class (from spherical to axial, $h = 0$) is referred to as the smoothness hypothesis or smoothness postulate [10].

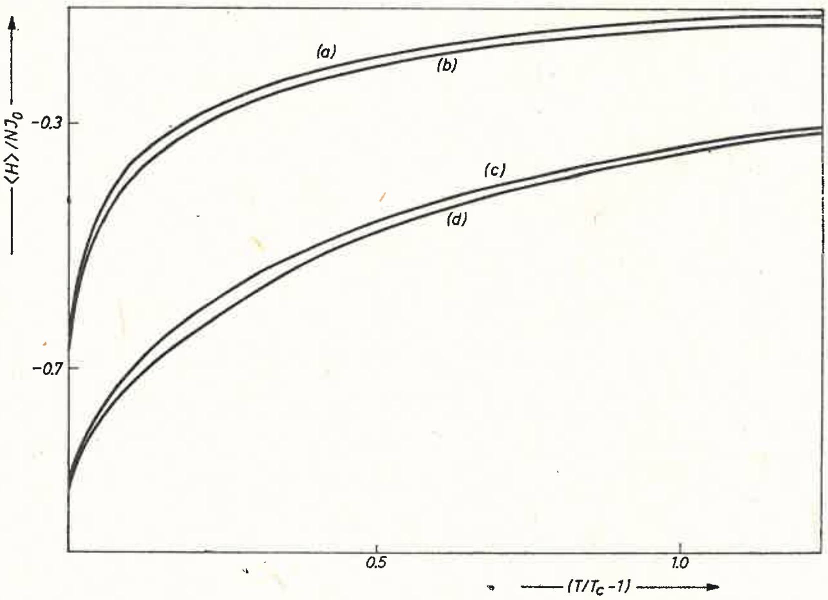

Fig. 2. Energy as a function of temperature. Curves (a) and (b) are for $\frac{D}{J} = 0.1$, (c) and (d) for $\frac{D}{J} = 0.01$. Curves (a) and (c) are our results and (b) and (d) are results of Tanaka and Tani

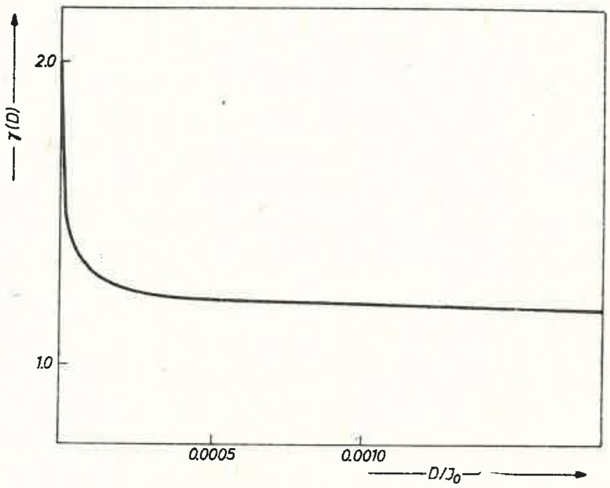

Fig. 3. The susceptibility critical index versus $\frac{D}{J_0}$

4. *Discussion*

Tanaka and Tani [1] have already calculated the longitudinal correlation function $\langle \hat{S}_k^3 S_{-k}^3 \rangle$ for the system described by the Hamiltonian (2.1). They employed then the method of Green's causal functions in combination with Kubo's method of linear reaction [11]. For anisotropic Green's functions, i. e. those occurring at the parameter D, the Narath [12] decoupling procedure was employed

$$\langle\langle S_f^+ S_f^3 + S_f^3 S_f^+ | A_g \rangle\rangle \approx 2\langle S_f^3 \rangle \langle\langle S_f^+ | A_g \rangle\rangle.$$

This procedure, however, neglects very substantial and strong single-site correlations. For such an approximation magnetization as well as Curie temperature exhibit several anomalous features. Magnetization $\sigma(J = 0) \neq 0$ and $\lim_{D\to\infty} T(D) = \infty$. Moreover the basic sum rule [13] does not hold neither in the paramagnetic nor in the ferromagnetic region. We have corrected Tanaka and Tani's results without decoupling of the single-site Green functions and taking into consideration the anisotropy of crystalline field in equations of motion of Green functions (2.8) and (2.9). We calculated the longitudinal and transverse correlation functions applying the modified Ishikawa and Oguchi method [4]. However, this method permitted one only to calculate the static longitudinal and dynamic transverse correlation functions (2.28), (2.29). Both of these functions in a long-range limit assume the Ornstein-Zernicke form (3.12) and (3.16). Moreover in both cases equal correlation lengths behave in a different manner near the phase transition point and the principal sum rule in the ferromagnetic region

$$\frac{1}{N}\sum_k (\langle S_k^3 S_{-k}^3 \rangle + \langle S_k^+ S_{-k}^- \rangle) = S(S+1),$$

is also not fulfilled. We could not, however, verify this rule in the ferromagnetic region due to a very complicated form of the longitudinal correlation functions (2.28) or (3.14). It can be easily proved that the phase transition temperature defined for arbitrary D by the formula (3.4) is finite as $D \to \infty$ and the magnetization also tends to zero at $J \to 0$. The magnetic susceptibility at $T = T_c$ is divergent for every $\frac{D}{J_0}$, but its behaviour for various $\frac{D}{J_0} \ll 1$ is different, which reflect in a rapid change of the critical index γ (Fig. 3), supporting the smoothness hypothesis (a symmetry of the Hamiltonian (2.1) turns at $D \to 0$ from the axial to spherical one). An explicit form of the correlation functions permitted one to calculate the temperature dependences of

$$E = \langle H \rangle, \quad T > T_c$$

and specific heat $C(T) = \left.\frac{\partial E}{\partial T}\right|_{h=0}$ for various $\frac{D}{J_0}$ (Fig. 1 and Fig. 2). The specific heat value near T_c derived from (3.11) and (3.14) is higher than that obtained by Tanaka and Tani [1].

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REFERENCES

[1] H. Tanaka, K. Tani, *Progr. Theor. Phys.* **41**, 590 (1969); K. Tani, H. Tanaka, *Phys. Lett.* **27A**, 25 (1968); J. F. Devlin, *Phys. Rev.* **B4**, 136 (1971); S. B. Haley, P. Erdos, *Phys. Rev.* **B5**, 1106 (1972).
[2] D. N. Zubarev, *Usp. Fiz. Nauk* **71**, 71 (1960).
[3] B. Westwański, Reports JINR E4-7624, E4-7625, Dubna 1973.
[4] J. Czakon, *Acta Phys. Pol.* **A53**, 57 (1978).
[5] T. Ishikawa, T. Oguchi, *Progr. Theor. Phys.* **50**, 807 (1973).
[6] A. Royer, *Phys. Rev.* **A6**, 1741 (1972).
[7] For example see V. N. Kashteyev, *Ferromagnetizm pri vysokikh temperaturakh*, Zinatne, Riga 1973 (in Russian).
[8] W. Borgieł, J. Czakon, *Acta Phys. Pol.* **A44**, 373 (1973).
[9] M. Tanaka, Y. Kondo, *Progr. Theor. Phys.* **48**, 1815 (1972); **50**, 708 (1973).
[10] R. B. Griffiths, *Phys. Rev. Lett.* **24**, 1479 (1970).
[11] R. Kubo, *J. Phys. Soc. Jap.* **12**, 570 (1957).
[12] A. Narath, *Phys. Rev.* **140A**, 854 (1965).
[13] For example see S. H. Liu, D. B. Siano, *Phys. Rev.* **164**, 697 (1967).