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ANTICROSSING EFFECT IN THE CASE OF LEVELS WITH DIFFERENT WIDTHS

By T. Dohnalik

Institute of Physics, Jagellonian University, Cracow*

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The anticrossing of levels with different widths in resolvent formalism is described. One of the obtained results is the general expression for the anticrossing resonance width and the possibility of narrowing the resonance curve by an additional coupling between the levels is shown. This result has been tested experimentally.

1. Introduction

The anticrossing method is often used for fine structure and for the Lamb shift determination of hydrogenic atoms (see for example Beyer, Kleinpoppen 1977, Beyer 1977). If the crossing levels with opposite parities are mixed by a static electric field we obtain, due to anticrossing effect, the signal called Stark mixing signal.

A theory of the effect given by Wieder and Eck (1967) was generalized by Glass-Maujean and Descoubes (1972) for the case in which the crossing levels were coupled through the intermediate level. In the latter theory calculations were performed by solving the evolution equation for density operator describing three coupled levels. Such an approach is correct and it allows one to calculate properly all possible signals, e. g. total intensity of fluorescent light, the degree of polarization, coherent effects. However, the results obtained in such a way are very complicated and it is impossible to take into account (because of complicated calculation) the influence of several other levels which take part in the experimental results. In both theories, which we mentioned above, the coupling of studied levels with the other fine structure levels may be introduced by a global correction of the levels energy calculated in the second order of perturbation theory. Such a procedure does not include all aspects of level coupling with different energies and widths. In this situation an attempt was made to describe the anticrossing of levels with different widths in resolvent formalism allowing one to get very simple formulas,

* Address: Instytut Fizyki UJ, Reymonta 4, 30-059 Kraków, Poland.

easy to interpret. This formalism has been tested in several situations in which it was possible to find a new, important property of anticrossings. In some other cases it does not give a proper description of anticrossing effect.

2. The principles of theoretical approach (see e. g. Messiah 1964)

We consider the two levels $|a\rangle$ and $|b\rangle$ belonging to an effective Hamiltonian which takes into account the finite lifetime (e. g. radiative lifetime) of $|a\rangle$ and $|b\rangle$ states.

Let us call this Hamiltonian $\mathscr{H}_0$. The $|a\rangle$ and $|b\rangle$ levels have the energies E_a, E_b and widths Γ_a, Γ_b, and they cross each other for the E_0 value of the energy. The other levels $|c\rangle$, $|d\rangle$, ... have the energies E_c, E_d, ... and widths Γ_c, Γ_d, ... The levels may be coupled through some perturbation which we describe by shift operator $\bar{R}$. We distin-

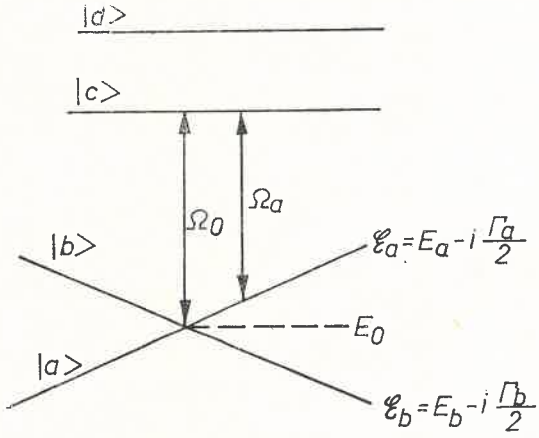

Fig. 1. The diagram of the considered levels and the used symbols

guish the $|a\rangle$, $|b\rangle$ levels from the others using projective operators $P = |a\rangle\langle a| + |b\rangle\langle b|$ and $Q = |c\rangle\langle c| + |d\rangle\langle d| + \ldots$ This separation has a physical meaning when some conditions are fulfilled: the coupling between levels $|a\rangle$ and $|b\rangle$ should be, thanks to their close energies E_a and E_b, much stronger than the coupling between $|a\rangle$ and $|c\rangle$, $|d\rangle$ ... and between $|b\rangle$ and $|c\rangle$, $|d\rangle$ ... It means that $P_{ab} \gg P_{ac}, P_{ad}$, where P_{ab} is the transition probability between the levels $|a\rangle$ and $|b\rangle$ and, respectively, P_{ac} etc.

Let G_{aa}, G_{bb}, G_{ab} be the matrix elements of resolvent G. They are obtained from the PGP ones given by formula:

$$PGP = \frac{1}{z - P\mathscr{H}_0 P - \bar{R}}. \tag{2.1}$$

The matrix element of the evolution operator $U(t)$ is related to G matrix elements by formula:

$$PU(t)P = \frac{1}{2\pi i} \int_{C_+} e^{-izt} PG(z)P dz, \tag{2.2}$$

where the path C_+ is shown by Fig. 2 and $\mathscr{E}_+$ and $\mathscr{E}_-$ are the poles of the resolvent.

Let us assume that at the moment $t = 0$ we excite the system to the level $|a\rangle$ and at the moment t the state of system is $|\psi(t)\rangle = U(t)|a\rangle$. The probability of transition from the state $|a\rangle$ to $|b\rangle$ given by $P_{ab}(t) = \langle b|\psi(t)\rangle\langle\psi(t)|b\rangle$ equals

$$P_{ab}(t) = \langle b|U(t)\,|a\rangle\langle a|U^{\dagger}(t)\,|b\rangle = |U_{ab}|^2. \tag{2.3}$$

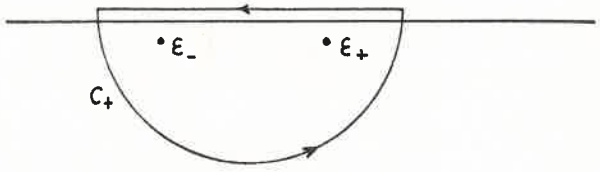

Fig. 2

We call the term

$$C_{ab}(t) = \langle a|\psi(t)\rangle\langle\psi(t)\,|b\rangle = \langle a|U(t)\,|a\rangle\langle a|U^{\dagger}(t)\,|b\rangle = U_{aa}U^{*}_{ba} \tag{2.4}$$

the coherence between the states $|a\rangle$ and $|b\rangle$. Both these terms: transition probability and coherence between states $|a\rangle$ and $|b\rangle$ influence the intensity of light emitted from these states and may be observed experimentally. The effect of the P_{ab} term is observed when the levels $|a\rangle$ and $|b\rangle$ have different populations and the resonance in probability changes them. It may also give rise to a change of the polarization of emitted light in a given direction.

In order to observe the coherence C_{ab} we should have both states coherent excitation and the so-called coherent observation: that means that the observed light should be emitted from both states and give an interference term different from zero. Now, without going into details of the $\overline{R}$ operator structure let us assume that it does not depend on z and that we know the poles of resolvent $\mathscr{E}_+$ and $\mathscr{E}_-$ which may have complex values

$$\begin{aligned}\mathscr{E}_+ &= E_+ - i\frac{\Gamma_+}{2}\\ \mathscr{E}_- &= E_- - i\frac{\Gamma_-}{2}.\end{aligned} \tag{2.5}$$

Let us evaluate the transition probability and coherence between the states $|a\rangle$ and $|b\rangle$. Using the formulae given by Cohen-Tannoudji (1968) we get

$$G_{ab} = \frac{\overline{R}_{ab}}{(z-\mathscr{E}_a-\overline{R}_{aa})\,(z-\mathscr{E}_b-\overline{R}_{bb})-\overline{R}_{ba}\overline{R}_{ab}} = \frac{\overline{R}_{ab}}{(z-\mathscr{E}_+)\,(z-\mathscr{E}_-)}. \tag{2.6}$$

Evaluating $U_{ab}(t)$ and $U^{*}_{ab}(t)$ we get $P_{ab}(t)$. Assuming the excitation at the moment $t = 0$ and observation after very long time at $t = \infty$ we have to calculate in fact

$$\overline{P}_{ab} = N\int_0^{\infty} P_{ab}(t)dt, \tag{2.7}$$

which is the time average of the so obtained value, where N is a normalization constant. After evaluating it gives

$$\overline{P}_{ab} = \frac{N|\overline{R}_{ab}|^2(\Gamma_+ + \Gamma_-)}{\Gamma_+\Gamma_-\left[(E_+ - E_-)^2 + \left(\frac{\Gamma_+ + \Gamma_-}{2}\right)^2\right]}. \tag{2.8}$$

We can calculate the coherence $\overline{C}_{ab}$ created by interaction $\overline{R}$ between the states $|a\rangle$ and $|b\rangle$ in a similar way under the same assumption that at $t = 0$ the system is excited to the state $|a\rangle$:

$$\overline{C}_{ab} = \frac{N\overline{R}^*_{ba}}{(E_+ - E_-)^2 + \left(\frac{\Gamma_- - \Gamma_+}{2}\right)^2}\left[\frac{1}{\Gamma_+}\left(E_+ - E_b + i\,\frac{\Gamma_b - \Gamma_+}{2}\right)\right.$$

$$\left. + \frac{1}{\Gamma_-}\left(E_- - E_b + i\,\frac{\Gamma_b - \Gamma_-}{2}\right) - \frac{E_+ - E_b + i\,\frac{\Gamma_b - \Gamma_+}{2}}{i(E_+ - E_-) + \frac{\Gamma_- + \Gamma_+}{2}} - \frac{E_- - E_b + i\,\frac{\Gamma_b - \Gamma_-}{2}}{i(E_- - E_+) + \frac{\Gamma_- + \Gamma_+}{2}}\right]. \tag{2.9}$$

We see that both quantities have a resonant character with respect to the difference $E_+ - E_-$.

3. *The form of* $\overline{R}$

The expansion of $\overline{R}$ has the form:

$$\overline{R} = PVP + PV\frac{Q}{z - \mathscr{H}_0}VP + PV\frac{Q}{z - \mathscr{H}_0}PV\frac{Q}{z - \mathscr{H}_0}PV + \ldots \tag{3.1}$$

where V is the perturbing term which mixes states of $\mathscr{H}_0$. We will consider this expansion for some specific cases.

3.1. The levels $|a\rangle$ and $|b\rangle$ are coupled only to each other. Then $\overline{R}_{ab} = V_{ab}$ and $\overline{R}_{aa} = \overline{R}_{bb} = 0$.

3.2. The levels $|a\rangle$ and $|b\rangle$ are directly coupled to each other and also to the levels $|c\rangle$, $|d\rangle$ ...

Now $\overline{R}_{ab} = V_{ab}$ and it is independent of z, whilst

$$\overline{R}_{aa} = \sum_{\gamma}\frac{|\langle a|V|\gamma\rangle|^2}{z - \mathscr{E}_\gamma} + \text{terms of higher order},$$

where γ denotes one of the levels $|c\rangle$, $|d\rangle$ ... $\overline{R}$ is a very weak perturbation, therefore only the z range values very close to $\mathscr{E}_a$ and $\mathscr{E}_b$ influence the value of the integral over the C_+ path which we calculate for obtaining P_{ab} and C_{ab}. We do not make a big mistake in

putting $\mathscr{E}_a$ instead of z in the element $\overline{R}$. It is possible to prove that this intuitive choice is really correct. The results obtained by this method are the same as those of Dupont-Roc (1973). In his calculation $\overline{R}$ depends on z by expansion in power series in which all terms starting from the third are cut. Finally we assume

$$\overline{R}_{aa} \simeq \sum_{\gamma} \frac{|\langle a|V|\gamma\rangle|^2}{\mathscr{E}_a - \mathscr{E}_\gamma} \simeq \sum_{\gamma} \frac{|\langle a|V|\gamma\rangle|^2}{E_a - E_\gamma} - \frac{i}{2} \sum_{\gamma} \frac{(\Gamma_\gamma - \Gamma_a)\,|\langle a|V|\gamma\rangle|^2}{(E_a - E_\gamma)^2}$$

$$= r_{aa} - \frac{i}{2} u_{aa} \tag{3.2}$$

and similarly

$$\overline{R}_{bb} \simeq \sum_{\gamma} \frac{|\langle b|V|\gamma\rangle|^2}{\mathscr{E}_b - \mathscr{E}_\gamma} \simeq r_{bb} - \frac{i}{2} u_{bb}. \tag{3.3}$$

3.3. The levels $|a\rangle$ and $|b\rangle$ are not directly coupled, but are coupled through the level $|c\rangle$ and additionally they are coupled with the levels $|d\rangle$, $|e\rangle$ ... The matrix elements $\overline{R}_{aa}$ and $\overline{R}_{bb}$ are the same as in 3.2 while $\overline{R}_{ab}$ in the lowest order is

$$\overline{R}_{ab} = \frac{\langle a|V|c\rangle\,\langle c|V|b\rangle}{z - \mathscr{E}_c}. \tag{3.4}$$

We may, as in the previous case, put for z values close to $\mathscr{E}_a$ and $\mathscr{E}_b$. The best substitution for the real part of energy is, of course, the crossing energy E_0 of both levels. The imaginary parts of energies are always different and it is impossible to find their common value. Let us put $\mathscr{E}_0 = E_0 - i\frac{\Gamma_0}{2}$ for z, remembering that Γ_0 should have a value between Γ_a and Γ_b. Further, we will show that in the cases interesting for us we may omit the effects for which the choice of Γ_0 is important. Such effects cannot be described by the formalism for $\Gamma_a \neq \Gamma_b$ discussed in this paper.

We can imagine many other ways of coupling the $|a\rangle$ and $|b\rangle$ levels e.g. they are not coupled directly but through two levels $|c\rangle$ and $|d\rangle$. $\overline{R}_{aa}$ and $\overline{R}_{bb}$ do not change in that case and $\overline{R}_{ab}$ may be easily evaluated from the general formula (3.1).

4. Evaluating $\mathscr{E}_+$ and $\mathscr{E}_-$

In order to find the values of $\mathscr{E}_+$ and $\mathscr{E}_-$ we have to find the poles of the denominator in equation (2.6) by solving the equation

$$(z - \mathscr{E}_a - \overline{R}_{aa})\,(z - \mathscr{E}_b - \overline{R}_{bb}) - \overline{R}_{ba}\overline{R}_{ab} = 0. \tag{4.1}$$

We put:

$$z = E - i\frac{\Gamma}{2}, \qquad \tilde{E}_a = E_a + r_{aa}, \qquad \tilde{E}_b = E_b + r_{bb}, \qquad \tilde{\Gamma}_a = \Gamma_a + u_{aa},$$

$$\tilde{\Gamma}_b = \Gamma_b + u_{bb} \tag{4.2}$$

and separate real and imaginary parts in the obtained solution. Putting

$$x = E - \frac{\tilde{E}_a + \tilde{E}_b}{2}, \quad y = \Gamma - \frac{\tilde{\Gamma}_a + \tilde{\Gamma}_b}{2} \tag{4.3}$$

we obtain the system of equations:

$$4x^2 - y^2 = 4\,\mathrm{Re}\,(\bar{R}_{ba}\bar{R}_{ab}) + (\tilde{E}_a - \tilde{E}_b)^2 - \left(\frac{\tilde{\Gamma}_a - \tilde{\Gamma}_b}{2}\right)^2,$$

$$xy = \tfrac{1}{2}(\tilde{E}_a - \tilde{E}_b)\frac{\tilde{\Gamma}_a - \tilde{\Gamma}_b}{2} - \mathrm{Im}\,(\bar{R}_{ba}\bar{R}_{ab}) \tag{4.4}$$

which have two solutions

$$E_\pm = \frac{\tilde{E}_a + \tilde{E}_b}{2} \pm x, \quad \Gamma_\pm = \frac{\tilde{\Gamma}_a + \tilde{\Gamma}_b}{2} \pm y. \tag{4.5}$$

We shall not write in details the very complicated form of $E_\pm$ and $\Gamma_\pm$ since the relations obtained and given above are satisfactory for eliminating them from the formulas for $\bar{P}_{ab}$ and $\bar{C}_{ab}$.

5. *The discussion of properties of* $\bar{P}_{ab}$

Putting into formula for $\bar{P}_{ab}$ the relations obtained above it is easy to see that

$$\bar{P}_{ab} = \frac{N|\bar{R}_{ab}|^2(\tilde{\Gamma}_a + \tilde{\Gamma}_b)}{\tilde{\Gamma}_a\tilde{\Gamma}_b\left[(\tilde{E}_a - \tilde{E}_b)^2 + \left(\frac{\tilde{\Gamma}_a + \tilde{\Gamma}_b}{2}\right)^2\left(1 + \frac{4\,\mathrm{Re}\,(\bar{R}_{ab}\bar{R}_{ba})}{\tilde{\Gamma}_a\tilde{\Gamma}_b}\right)\right] + 2(\tilde{E}_a - \tilde{E}_b)(\tilde{\Gamma}_a - \tilde{\Gamma}_b)\,\mathrm{Im}\,(\bar{R}_{ab}\bar{R}_{ba}) - 4[\mathrm{Im}\,(\bar{R}_{ab}\bar{R}_{ba})]^2}. \tag{5.1}$$

5.1. The case of the two levels $|a\rangle$ and $|b\rangle$ coupled only to each other.

Taking the advantage of the discussed in 3.1 form of $\bar{R}$ we see that $V_{ab}V_{ba} = |V_{ab}|^2$ hence $\mathrm{Re}\,(V_{ab}V_{ba}) = |V_{ab}|^2$ and $\mathrm{Im}(V_{ab}V_{ba}) = 0$. It gives

$$\bar{P}_{ab} = \frac{N|V_{ab}|^2(\Gamma_a + \Gamma_b)}{\Gamma_a\Gamma_b\left[(E_a - E_b)^2 + \left(\frac{\Gamma_a + \Gamma_b}{2}\right)^2\left(1 + \frac{4|V_{ab}|^2}{\Gamma_a\Gamma_b}\right)\right]}. \tag{5.2}$$

The populations of the levels become equal in the centre of the resonance when $E_a = E_b$ for $V_{ab} \gg \Gamma_a, \Gamma_b$, it means $\frac{1}{\Gamma_a}(1 - \bar{P}_{ab}) = \frac{1}{\Gamma_b}\bar{P}_{ab}$. Finally, we find $N = \Gamma_b$ and

$$\bar{P}_{ab} = \frac{|V_{ab}|^2(\Gamma_a + \Gamma_b)}{\Gamma_a\left[(E_a - E_b)^2 + \left(\frac{\Gamma_a + \Gamma_b}{2}\right)^2\left(1 + \frac{4|V_{ab}|^2}{\Gamma_a\Gamma_b}\right)\right]}. \tag{5.3}$$

It is easy to find the relation between the change of intensity of polarized light emitted from the levels $|a\rangle$ and $|b\rangle$ and the calculated probability. We get the result obtained previously by other methods (Wieder, Eck 1967, Glass-Maujean, Descoubes 1971). The resonance broadens proportionally to $|V_{ab}|^2$, but its position is not shifted.

5.2. The levels $|a\rangle$ and $|b\rangle$ are directly coupled to each other and also to the levels $|c\rangle$, $|d\rangle$ ...

Now $\bar{R}_{aa} \neq 0$, $\bar{R}_{bb} \neq 0$, and $\bar{R}_{ab}$ is equal to V_{ab} as in the previous case and has the same properties. The final form of the formula is obtained including a normalization constant calculated in 5.1.

$$\bar{P}_{ab} = \frac{|V_{ab}|^2(\Gamma_a+u_{aa}+\Gamma_b+u_{bb})}{(\Gamma_a+u_{aa})\left[(E_a-E_b+r_{aa}-r_{bb})^2+\frac{1}{4}(\Gamma_a+u_{aa}+\Gamma_b+u_{bb})^2\left(1+\frac{4|V_{ab}|^2}{(\Gamma_a+u_{aa})(\Gamma_b+u_{bb})}\right)\right]}. \tag{5.4}$$

All properties of the resonance are seen after putting an explicit form of u_{aa}, u_{bb}, r_{aa}, r_{bb}. The broadening of the resonance is still practically proportional to $|V_{ab}|^2$ but in the same time the position of the center of the resonance changes by $(r_{aa}-r_{bb})$ which is also proportional to the second power of V. An asymmetry appears in the shape of the resonance which is due to the lack of symmetry of E_a-E_c and E_b-E_c in the denominators of u_{aa}, u_{bb}, r_{aa}, r_{bb}. A more detailed discussion of the broadening of resonance will be given in Chapter 7.

5.3. The levels $|a\rangle$ and $|b\rangle$ are not coupled directly but through the level $|c\rangle$ and additionally they are coupled with the levels $|d\rangle$, $|e\rangle$ ...

To obtain the properties of resonance we should calculate $\mathrm{Re}(\bar{R}_{ab}\bar{R}_{ba})$ and $\mathrm{Im}\,(\bar{R}_{ab}\bar{R}_{ba})$. An accurate form of $\bar{R}$ is given in 3.3. It is easy to evaluate that

$$\mathrm{Re}\,(\bar{R}_{ab}\bar{R}_{ba}) \simeq \frac{|V_{ac}|^2|V_{bc}|^2}{(E_0-E_c)^2} = |\bar{R}_{ab}|^2 \tag{5.5}$$

The term $(\Gamma_0-\Gamma_c)^2 \sim 10^2\ (\mathrm{MHz})^2$ is neglected in comparison with $(E_0-E_c)^2 \sim 10^6(\mathrm{MHz})^2$ in the denominator. After such an approximation we get also

$$\mathrm{Im}\,(\bar{R}_{ab}\bar{R}_{ba}) \simeq \frac{\Gamma_0-\Gamma_c}{E_0-E_c}|\bar{R}_{ab}|^2. \tag{5.6}$$

Now, we can write an explicit formula for $\bar{P}_{ab}$. From the relation describing $\mathrm{Im}\,(\bar{R}_{ab}\bar{R}_{ba})$ we see that it is very small and the term $[\mathrm{Im}\,(\bar{R}_{ab}\bar{R}_{ba})]^2$ may be omitted. The term $(\tilde{E}_a-\tilde{E}_b)$ $(\tilde{\Gamma}_a-\tilde{\Gamma}_b)$ $\mathrm{Im}\,(\bar{R}_{ab}\bar{R}_{ba})$ gives the shift and broadening of the resonance. The value of the latter is proportional to $(\Gamma_a-\Gamma_b)$ $\mathrm{Im}\,(\bar{R}_{ab}\bar{R}_{ba}) = \dfrac{(\Gamma_a-\Gamma_b)(\Gamma_0-\Gamma_c)}{(E_0-E_c)}|\bar{R}_{ab}|^2$. In comparison with the width of resonance $\sim(\Gamma_a+\Gamma_b)$ and with the shift due to $\bar{R}_{aa}$ and $\bar{R}_{bb}$ this term

is usually very small and we also neglect it[1]. As a conclusion of the above given discussion we may rewrite the formula describing $\overline{P}_{ab}$ in analogical form as in 5.2, but we put $|\overline{R}_{ab}|^2$ in place of $|V_{ab}|^2$. This changes radically the properties of the resonance since its amplitude and broadening depend now on $|V_{ab}|^4$, while the shift of the resonance still depends on V_{ab} to the second power. The formula (5.1) is easy to generalize for other, more complicated cases of levels coupling.

6. *The properties of* $\overline{C}_{ab}$

As in the previous case of probability we will discuss the properties of the term describing the coherence introduced into the levels system by their coupling. In order to observe an effect belonging to the coherence between the levels we should be able to detect simultaneously the light emitted from both levels in a way which allows interference of its components. If we observe, as we do in all interesting experiments, the anti-crossings of fine structure levels coupled by an electric field, the above condition introduces the selection rule $\Delta l = 2$ of studied levels. Such levels can be coupled only through one intermediate level because an electric field couples the levels with $\Delta l = 1$. Hence, the only interesting case is 3.3. Neglecting, as in the case of probability, an effect belonging to the imaginary part of $\overline{R}_{ab}\overline{R}_{ba}$, we obtain

$$\overline{C}_{ab} = \frac{\overline{R}^*_{ab}\left[\tilde{\Gamma}_a(\tilde{E}_a-\tilde{E}_b)+i\frac{\Gamma_b}{2}(\tilde{\Gamma}_a+\tilde{\Gamma}_b)\right]}{\tilde{\Gamma}_a\tilde{\Gamma}_b\left[(\tilde{E}_a-\tilde{E}_b)^2+\left(\frac{\tilde{\Gamma}_a+\tilde{\Gamma}_b}{2}\right)^2\left(1+\frac{4|\overline{R}_{ab}|^2}{\tilde{\Gamma}_a\tilde{\Gamma}_b}\right)\right]}. \tag{6.1}$$

The signal depends on the second power of V and has both the dispersion and absorption parts whose widths and shifts of the center depend on V^2, similarly to the case of a signal connected with the population. Choosing such a geometry, that the product $V_{bc}V_{ca}$ in $\overline{R}_{ab}$ is either real or imaginary, we can obtain the resonance of an almost dispersion (neglecting the small contribution of the absorption curve) or absorption (neglecting the contribution of the dispersion curve) shape. We may also change the sign of a signal by changing the sign of the product $V_{bc}V_{ca}$ (it was impossible for the population signal since it depends on the square of the module of the V perturbation). It plays a very important role in our experiment because due to a cylindrical symmetry of the coupling electric field, the above product gives, in average, the value of 0, and the terms connected with the coherence were not observed.

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[1] However, if we described the system with E_0-E_c comparatively small and with a strong coupling potential and big difference Γ_c and Γ_a, Γ_b the correction corresponding to this term could be noticeable. Unfortunately, our calculation cannot define its absolute value since we do not know how to choose Γ_0 and its arbitrariness.

7. The properties of the resonance width

The possibility of narrowing the resonance curve by an additional coupling between the levels

7.1. The formula describing the width

In the general case, when we neglect an imaginary part of the product $\overline{R}_{ab}\overline{R}_{ba}$ the halfwidth $\Delta\omega_{1/2}$ of resonance connected with levels anticrossing is

$$\Delta\omega_{1/2} = \frac{\tilde{\Gamma}_a+\tilde{\Gamma}_b}{2}\left(1+\frac{4|\overline{R}_{ab}|^2}{\tilde{\Gamma}_a\tilde{\Gamma}_b}\right)^{1/2}. \tag{7.1}$$

Putting $\frac{\tilde{\Gamma}_a}{\tilde{\Gamma}_b} = \kappa$ and $\frac{4|\overline{R}_{ab}|^2}{\tilde{\Gamma}_b^2} = |Q_{ab}|^2$ we get

$$(\Delta\omega_{1/2})^2 = \frac{\tilde{\Gamma}_b^2}{4}(\kappa+1)^2\left(1+\frac{|Q_{ab}|^2}{\kappa}\right). \tag{7.2}$$

Let us assume that we change κ without changing Q_{ab}. For small κ values ($\tilde{\Gamma}_a \ll \tilde{\Gamma}_b$) the term $\frac{|Q_{ab}|^2}{\kappa}$ decides of the width. For large κ values the width is decided by κ^2. Hence, there is one particular value of κ for which at a defined Q_{ab} the resonance width is the smallest, despite the increasing width $\tilde{\Gamma}_a$ of levels $|a\rangle$.

7.2. Experimental realization of the discussed situation

The situation in which we change κ without changing Q_{ab} may be easily fulfilled in some specific physical cases. We can see that for two crossing levels the width of only one should be changed and the coupling potential should remain unchanged. In the case of the crossing of the Zeeman sublevels with opposite parities coupled by an electric field, the $E_\parallel$ component parallel to the magnetic field couples the sublevels with $\Delta M = 0$ and the component $E_\perp$, perpendicular to the magnetic field, couples sublevels such that $\Delta M = 1$.

Let us consider the level diagram shown in Fig. 3, which shows the Zeeman sublevels for $n = 3$ in hydrogen. In weak magnetic fields we see two level crossings of S and P levels:

$$^2S_{1/2},\ m_J = -\tfrac{1}{2} \text{ with } {}^2P_{1/2},\ m_J = +\tfrac{1}{2} \text{ coupled by } E_\perp$$

and

$$^2S_{1/2},\ m_J = -\tfrac{1}{2} \text{ with } {}^2P_{1/2},\ m_J = -\tfrac{1}{2} \text{ coupled by field } E_\parallel.$$

The widths of levels are $\Gamma_S = 1$ MHz and $\Gamma_P = 30$ MHz. In the first crossing the field $E_\parallel$ couples very strongly the level $^2S_{1/2}$, $m_J = -\frac{1}{2}$ with the level $^2P_{1/2}$, $m_J = -\frac{1}{2}$, changing considerably its width, while the level $^2P_{1/2}$, $m_J = \frac{1}{2}$ with $^2S_{1/2}$, $m_J = \frac{1}{2}$ presents a much

weaker coupling (large distance, small relative change of the width due to the large width Γ_P). This field does not change the level direct coupling which depends on $E_\perp$ only. Substituting $\Gamma_a = \Gamma_S$ and $\Gamma_b = \Gamma_P$ we can see that κ is very small. By introducing the field $E_\parallel$ we may change κ through the change of Γ_S, keeping Γ_P and $|Q_{ab}|^2$ practically unchanged. Hence for a given $E_\perp$ it should exist such $E_\parallel$ values for which the width of resonance is minimal.

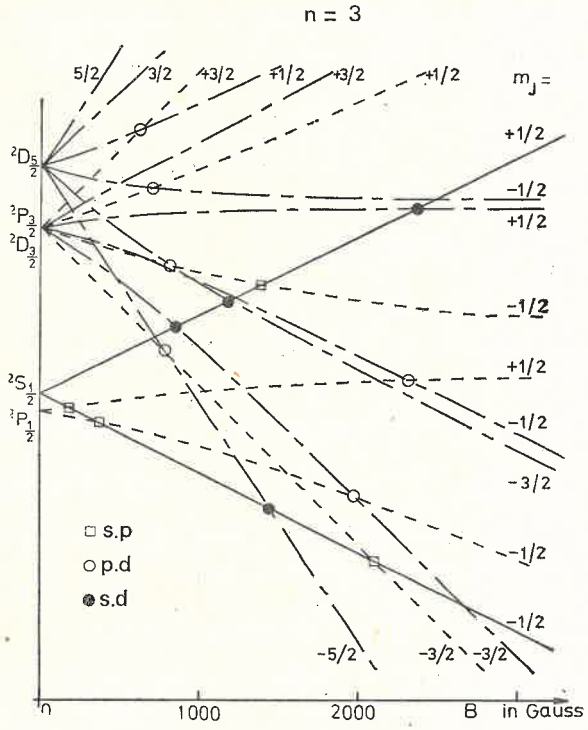

Fig. 3. The diagram of levels with $n = 3$ for hydrogen

In the case of the second mentioned crossing the situation is similar, but the fields $E_\parallel$ and $E_\perp$ play opposite roles. It is very important to know exactly the above mentioned resonances and the fields $E_\perp$ and $E_\parallel$ occurring in the experiment because from their position we calculate the Lamb shift. Hence, the results obtained in the experiment may be optimized when studying simultaneously resonance width and position and influencing them by $E_\perp$ and $E_\parallel$.

The level crossings ^2S, $m_J = -\frac{1}{2}$ and ^2D, $m_J = -\frac{5}{2}$ (J is not a good quantum number in the magnetic field range used in the experiment) at the field 1433 G may show another example of the described situation, still for $n = 3$. These levels with the widths $\Gamma_S = 1$ MHz and $\Gamma_D = 10$ MHz are coupled through the ^2P, $m_J = -\frac{1}{2}$ level by field $E_\perp$. The field $E_\parallel$ couples with other levels (mainly with ^2P, $m_J = -\frac{1}{2}$) the S level only. The ^2D, $m_J = -\frac{5}{2}$ level is not coupled by the field $E_\parallel$ with any level due to the value of m_J, so it does not change its width. The width of this anticrossing was investigated experimentally as a function of $E_\parallel$ when the positions of level crossings were measured

in order to find the Lamb shifts in the $n = 3$ state. It should be mentioned that in fact the crossings described before are much more complicated because of hf structure of the levels. However, to avoid the complications of the whole picture we shall not consider it here.

7.3. Physical interpretation of the resonance narrowing

The resonance narrowing, which occurs when the width of one of the levels increases, contradicts the intuition and that is the reason why we shall spend a little more time on its thorough interpretation. In Chapter 4 we gave equation (4.4) for $E_{\pm}$ and $\Gamma_{\pm}$ describing the energies and widths of crossing levels where all couplings were taken into consideration. These equations were solved by Dupont-Roc (1968) for the case of two levels coupled directly and by Series (1964) with an additional condition that one of the levels has $\Gamma = 0$. The following conclusions may be drawn from the discussion of the results of these works:

a. a level crossing remains level crossing all the time for $V_{ab} < \frac{|\Gamma_a - \Gamma_b|}{4}$ and at the same time the angle of intersection of the levels curves increases with increasing V_{ab} and has maximum at $V_{ab} = \frac{|\Gamma_a - \Gamma_b|}{4}$. In our opinion this effect is responsible for resonance narrowing. The decrease of the difference $\Gamma_a - \Gamma_b$ e.g. by increasing Γ_a due to an additional coupling (we assume $\Gamma_a < \Gamma_b$) causes, at V_{ab} constant, approaching to the condition $V_{ab} = \frac{|\Gamma_a - \Gamma_b|}{4}$ for which the angle of levels at their intersection is maximum so the resonance width is minimum.

b. for $V_{ab} > \frac{|\Gamma_a - \Gamma_b|}{4}$ the levels repulsion occurs, typical for anticrossing of levels with equal widths. The smaller the difference $\Gamma_a - \Gamma_b$ the further we are from the minimum of resonance width, which causes the widening of the resonance.

7.4. Experimental verification of the possibilities of narrowing the anticrossings

As it was mentioned in 7.2, in order to verify experimentally the suggested conclusions we studied the resonance corresponding to anticrossing of 3^2S, $m_J = -\frac{1}{2}$ and 3^2D, $m_J = -\frac{5}{2}$ levels. The experimental set-up described in details by Glass-Maujean, Julien and Dohnalik (1978) was used. The electron excitation transferred hydrogen atoms to the wanted state. The field $E_{\perp}$ was a motional field seen by the atoms flying through the strong magnetic field. The spatial distribution of the density of exciting electrons and hydrogen ions creates a small parallel field $E_{\parallel \mathrm{sp}}$ and in addition external field $E_{\parallel \mathrm{ex}}$ was applied. In order to keep the energy of exciting electrons constant, in the observation region, the voltage accelerating the electrons was appropriately changed with $E_{\parallel \mathrm{ex}}$. Such a procedure could, however, influence atoms velocities after an excitation and spatial distribution of electrons and ions and what follows, change simultaneously the $E_{\perp}$ and

$E_{\parallel sp}$. Since the shift of resonance given by the terms r_{aa} and r_{bb} depends on $E_\perp^2$ and $E_\parallel^2 = (E_{\parallel sp}+E_{\parallel ex})^2$ it was possible to check how big is a range of the applied fields in which the shift depends on $E_\parallel^2$ only, then $E_\perp$ and $E_{\parallel sp}$ are constant. We fulfilled this condition for $E_\parallel = 120 \div 180$ V/cm, with $E_\perp = 42.5$ V/cm and $E_{\parallel sp} = 20$ V/cm. In this range of $E_\parallel$ we have Q_{ab} constant. Typical curves obtained for $E_\parallel = 120$ V/cm and 145 V/cm are shown in Fig. 4. Large shift of the resonance center is observable, while

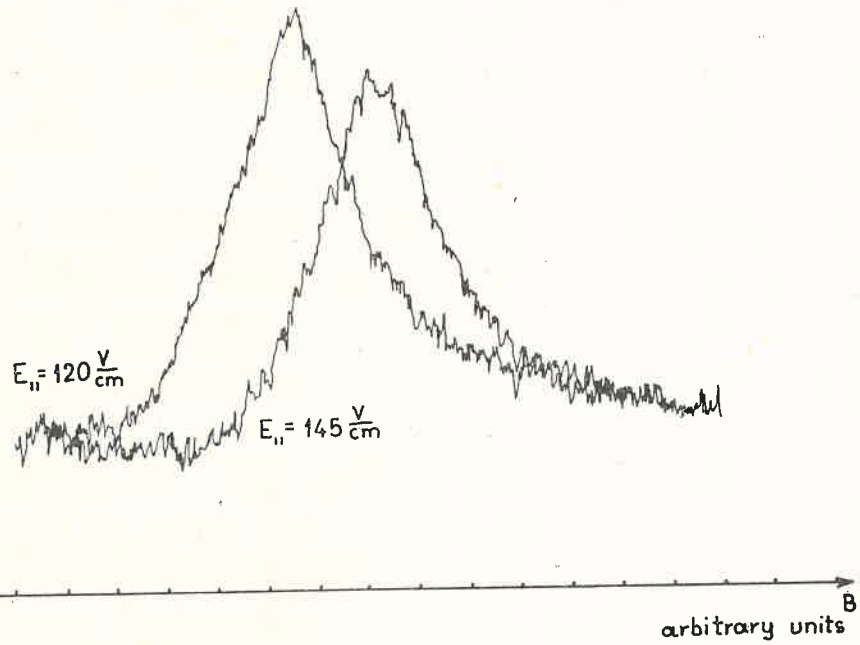

Fig. 4. Resonance curves of anticrossings between the levels 3^2S, $m_J = -\frac{1}{2}$ and 3^2D, $m_J = -\frac{5}{2}$ at the fields $E_\parallel$ equal 120 V/cm and 145 V/cm

the narrowing is very small (smaller than the experimental error). Fig. 5 shows a theoretical curve — the resonance width as a function of $\kappa = \frac{\tilde{\Gamma}_a}{\tilde{\Gamma}_b}$ for $|Q_{ab}|^2 = 181.15$ — taken from our experiment. The theoretical width is marked for $\kappa = \frac{\Gamma_a}{\Gamma_b} = 0.1$ ($E_\perp = E_\parallel = 0$) which unfortunately, due to the coupling through the intermediate state was not obtained in our case. The width for $\kappa = 0.36$ is also marked and it corresponds to the one obtained at $E_\perp = 42.5$ V/cm and $E_\parallel = 0$. Five experimental points for κ from 0.87 to 1.45 are in good agreement with the theoretical curve and obtained widths are considerably smaller than the width for $\kappa = 0.36$. The theoretical curve has a minimum for $\kappa = 1$ and it confirms our interpretation. Hence, we see that it is really possible to get the narrowing of the resonance curve. If we start from the point for $\kappa = 0.1$ (such a situation takes place for the direct coupling of crossing levels) the obtained narrowing will be considerable.

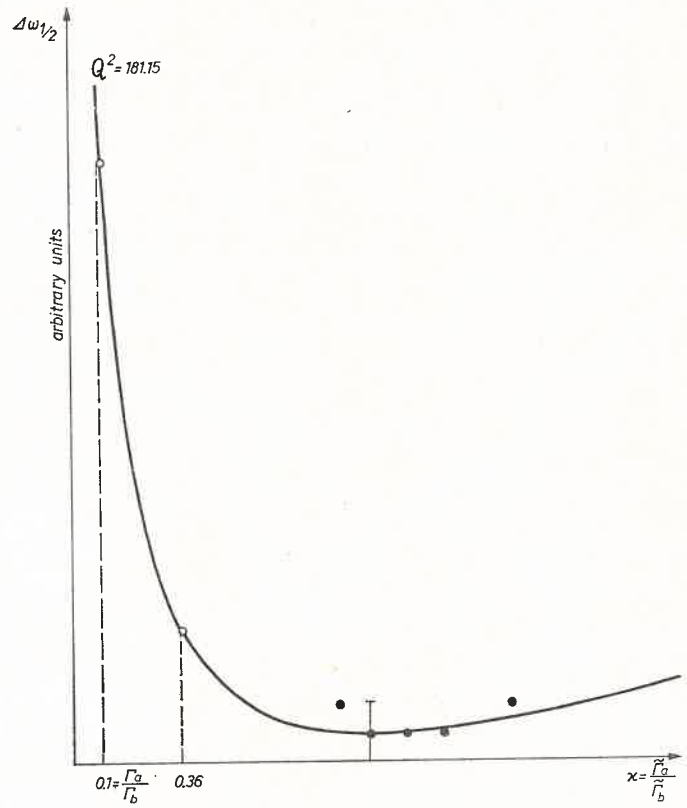

Fig. 5. Dependence of the resonance width on $\varkappa$; ○ – theoretical values for $\varkappa = 0.1$ ($E_\perp = E_{||} = 0$) and and $\varkappa = 0.36$ ($E_\perp = 42.5$ V/cm, $E_{||} = 0$), ● – experimental points

8. *Applicability conditions*

Let us racapitulate the conditions of calculation applicability

a. $(E_a - E_c)^2,\ (E_b - E_c)^2 \ldots \gg \Gamma_a^2, \Gamma_b^2, \Gamma_c^2 \ldots \approx \Gamma^2$

b. $\overline{P}_{ac} \ll \overline{P}_{ab}$

c. $\overline{R}(E_0) \ll (E_a - E_c)^2,\ (E_b - E_c)^2 \ldots \approx \Omega^2$

These conditions are usually fulfilled very simply. Let us notice the conclusion resulting from conditions b. and c. for the level coupled through an intermediate level. From c. we have $\overline{R}_{ab}(E_0) \approx \dfrac{V^2}{\Omega} \ll \Omega$ which gives $V^2 \ll \Omega^2$ and at the same time $\overline{P}_{ab} \approx \dfrac{|\overline{R}_{ab}|^2}{(E_a - E_0)^2} \sim \dfrac{V^2}{\Gamma^2 \Omega^2}$ and $\overline{P}_{ac} \sim \left(\dfrac{V}{\Omega}\right)^2$, and from b. we have $V^2 \gg \Gamma^2$, hence $\Omega^2 \gg V^2 \gg \Gamma^2$.

It is easy to fulfil the mentioned condition in the interesting us cases $n = 3$, $n = 4$ levels in hydrogen since $\Gamma \sim 10$ MHz, $\Omega \sim 10^3$ MHz. This condition is also fulfilled in our experimentally studied and described situation.

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