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THEORY OF MAGNON-PLASMON INTERACTION IN ANTIFERROMAGNETIC SEMICONDUCTORS*

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A model of magnon-plasmon coupling in antiferromagnetic semiconductors is proposed on the assumption that the ground state of a magnetic ion in the crystal field is orbitally non-degenerate. The model is based on indirect, via spin-orbit coupling, interaction of the spin system and electric field produced by a plasmon. The estimated value of this coupling amounts to about $10-10^{-2}$ cm^{-1}. The explicit form of the Hamiltonian, describing this interaction, is given for T_d symmetry of the environment of the magnetic ion. The differential cross section for the Raman light scattering from hybrids and its dependence on the external magnetic field are given.

1. *Introduction*

Infrared [1, 2] and neutron [3, 4] spectroscopy in connection with Raman [5–7] and resonance Raman [8, 9] spectroscopy are highly potent methods for the investigation of elementary excitations and interactions between the latter. The most striking effects of these interactions are observed, in the appropriate spectra, in those ranges of frequency in which the interactions between elementary excitations acquire the resonance nature i.e. when frequencies of the interacting excitations lie close or are equal to one another [3,6, 10–12]. Light or neutrons are then scattered not from "pure" modes but from hybridized ones. An effect of resonance coupling between elementary excitations in the solid state has first been observed in the plasmon-longitudinal optical phonon system [5, 6].

Recently, intensive studies of resonance interactions in the systems: magnon-optical phonon and magnon-acoustical phonon have been reported [3, 4, 12, 13]. The coupling mechanism of the last two kinds of elementary excitations can consist either in a phonon modulation of the crystal field transferred to the spin system by way of spin-orbit coupling [4, 14], or in a modulation of the exchange integral [15].

The aim of this paper is to draw attention to the possibility of coupling in the plasmon-magnon system due to interaction of a spin, via spin-orbit coupling, with the electric field

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of a plasmon. The experimental verification of this prediction should be sought in antiferromagnetic semiconductors with a carrier concentration of 10^{14}–10^{16} cm^{-3} and large mobility of order 10 cmV/s, of the type EuTe and that of certain spinels [16, 17]. Under these conditions, the plasmon frequency approaches a value of several cm^{-1} which should make it possible to observe resonance interaction between the magnons and plasmons. It should be emphasized that the frequency of antiferromagnons can be varied by applying an external magnetic field in the range up to 20 cm^{-1} and thus brought into line with the plasmon frequency.

In Section 2 we propose a model of the preceding magnon-plasmon coupling, which becomes essential when the ground state of a free magnetic ion is orbitally degenerate. The final form of the Hamiltonian is obtained for the T_d symmetry of the environment of the magnetic ion, but the final results can be easily generalized.

In Section 3 we discuss the problem of hybridization and Raman light scattering from hybridized modes, as well as the feasibility of an experimental confirmation of our calculations. The most important conclusions are drawn and discussed in Section 4.

2. *Model Hamiltonian of magnon-plasmon interaction*

Let us consider a single ion of the magnetic lattice localized in an external magnetic field $\boldsymbol{H}$ and an electric field $\boldsymbol{E}$. The Hamiltonian for this ion is of the following form

$$H_i = H_0 + V, \tag{1}$$

where H_0 contains the intra-ionic interactions $H_{\mathrm{i-i}}$ and the interaction with the crystal field of the lattice $H_{\mathrm{C-F}}$ i.e.

$$H_0 = H_{\mathrm{i-i}} + H_{\mathrm{C-F}}, \tag{2}$$

whereas V is the sum of three components: the spin-orbit coupling, the Zeeman term and the electric dipole interaction

$$V = \lambda \boldsymbol{L}\cdot\boldsymbol{S} + \mu_{\mathrm{B}}(\boldsymbol{L}+2\boldsymbol{S})\cdot\boldsymbol{H} - \boldsymbol{E}\cdot\boldsymbol{D} \tag{3}$$

$\boldsymbol{L}$ ($\boldsymbol{S}$) denoting the orbital (spin) angular momentum operator, λ — a parameter of the spin-orbit coupling, μ_{B} — Bohr magneton, $\boldsymbol{D}$ — an operator of the electric dipole moment of the form $\boldsymbol{D} = \sum_n e\boldsymbol{r}_n$ (e — the electron charge, $\boldsymbol{r}_n$ — the position vector of the n-th electron). It is assumed in (3) that $\boldsymbol{E}$ is constant throughout the region of the atom. The exchange interaction of an ion with the remaining ions of the lattice will be introduced later when considering the magnon system.

Let us consider the case when the ground state $|0\rangle$ of the Hamiltonian H_0 is orbitally non-degenerate and the nearest orbital level is sufficiently well separated so that V can be dealt with by perturbation theory. The last condition is in principle equivalent to the following one:

$$H_{\mathrm{C-F}} \gg \lambda \boldsymbol{L}\cdot\boldsymbol{S} \tag{4}$$

which is fulfilled for ions of the rare-earth metals. The influence of the perturbation V on the level $|0\rangle$ can now be described by a spin Hamiltonian H_S, where we are interested only in those terms which contain the coupling of a spin with the electric field $\boldsymbol{E}$. These terms occur in the third order of the perturbation treatment and, with accuracy up to terms quadratic in $\boldsymbol{E}$ and $\boldsymbol{S}$ and linear in $\boldsymbol{H}$, the Hamiltonian takes the following form

$$H_S = C_{\alpha\beta\gamma}H_\alpha E_\beta S_\gamma + F_{\alpha\beta\gamma}E_\alpha[S_\beta S_\gamma] + G_{\alpha\beta\gamma}[E_\alpha E_\beta]S_\gamma, \tag{5}$$

where $[xy] = xy+yx$, $C_{\alpha\beta\gamma}$, $F_{\alpha\beta\gamma}$ and $G_{\alpha\beta\gamma}$ are components of the so-colled electric field effect tensors [18] and are given as:

$$C_{\alpha\gamma\beta} = -\lambda\mu_B \sum_{a,b} \{P\langle 0|L_\alpha|a\rangle \langle a|D_\beta|b\rangle \langle b|L_\gamma|0\rangle\} (E_0-E_a)^{-1}(E_0-E_b)^{-1} \tag{6a}$$

$$F_{\alpha\beta\gamma} = -\frac{\lambda^2}{2}\sum_{a,b} \{P\langle 0|D_\alpha|a\rangle \langle a|L_\beta|b\rangle \langle b|L_\gamma|0\rangle\} (E_0-E_a)^{-1}(E_0-E_b)^{-1} \tag{6b}$$

$$G_{\alpha\beta\gamma} = \frac{\lambda}{2}\sum_{a,b} \{P\langle 0|D_\alpha|a\rangle \langle a|D_\beta|b\rangle \langle b|L_\gamma|0\rangle\} (E_0-E_a)^{-1}(E_0-E_b)^{-1} \tag{6c}$$

The states $|a\rangle$ and $|b\rangle$ are eigen-states of the Hamiltonian H_0, and E_a and E_b are their respective eigen-values. E_0 is the energy of the ground state $|0\rangle$. Here the operator P stands for summation over all permutations of the operators L_α, D_β and L_γ, etc. In Eq. (5) use is made of the summation convention over repeating indices. For simplicity we have assumed that $\langle 0|\boldsymbol{D}|0\rangle = \boldsymbol{0}$ what is, however, not essential. If the magnetic ion is a centre of inversion, then the tensors given by Eqs (6a)–(6c) vanish identically, and in order to obtain coupling one has to proceed one order higher in the perturbation calculus. In our further considerations we restrict ourselves to the T_d symmetry of the environment of the magnetic ion. The components of the tensors then reduce to $C_{xyz} = C_{xzy} = C_{yxz} = C_{yzx} = C_{zxy} = C_{zyx} = C$, with $C_{\alpha\beta\gamma} = 0$ in the remaining cases. A similar property is also exhibited by the two remaining tensors.

The above considerations are of a general nature i.e. they are valid both for ferro- and antiferromagnets. Because of the higher energy of magnons and for reasons mentioned in the Introduction, we restrict ourselves to antiferromagnetic semiconductors. Let us assume a simple model of an antiferromagnet in which the same ions are attached to the sites $\boldsymbol{R}_p$ and $\boldsymbol{R}_q$ of both sublattices. The Hamiltonian H_{S-E} describing the interaction between the spin system and the electric field $\boldsymbol{E}$ has the following form:

$$H_{S-E} = \sum_p H_S(p) + \sum_q H_S(q), \tag{7}$$

where $H_S(p)$ and $H_S(q)$ denote the Hamiltonians given by Eq. (5) for the ions at the sites $\boldsymbol{R}_p$ and $\boldsymbol{R}_q$, respectively. Let us assume that the external magnetic field $\boldsymbol{H}$ is directed along the negative z-axis. As the field $\boldsymbol{E}$ occurring in Eq. (5) we assume the field originating in the plasmon system and given by the following expression [18]:

$$\boldsymbol{E}(\boldsymbol{R}_l) = -\frac{4\pi ne}{\varepsilon}\left(\frac{\hbar}{2Nm^*}\right)^{\frac{1}{2}} \sum_k \frac{\boldsymbol{k}}{k\omega_k^{\frac{1}{2}}} (a_k e^{i\boldsymbol{k}\cdot\boldsymbol{R}_l} + a_k^+ e^{-i\boldsymbol{k}\cdot\boldsymbol{R}_l}), \tag{8}$$

where l stands for both p and q, $n(N)$ denotes the density (total number) of electrons in the conduction band, ε is the dielectric constant of the semiconductor lattice, m^* — an effective mass of the electron, $a_k(a_k^+)$ — the annihilation (creation) operator of the plasmon with the wave vector $\boldsymbol{k}$ and frequency given by the following relation [19, 20]:

$$\omega_k = \omega_0 + \alpha k^2 \tag{9}$$

where

$$\omega_0 = \left(\frac{4\pi ne^2}{\varepsilon m^*}\right)^{\frac{1}{2}} \tag{10a}$$

$$\alpha = \frac{3\hbar^2(3\pi^2 n)^{\frac{2}{3}}}{10m^{*2}\omega_0}. \tag{10b}$$

In the low temperature region when the spin wave approximation is valid [7], the Hamiltonian (7), on taking into account Eq. (5) and Eq. (8), goes over into the Hamiltonian $H_{\mathrm{m-p}}$ of the magnon-plasmon coupling, which for the case of T_d symmetry takes the following form:

$$H_{\mathrm{m-p}} = \sum_k \{A_{1,k}(a_k\alpha_{\downarrow -k} + a_k^+\alpha_{\downarrow k}^+) + A_{2,k}(a_k\alpha_{\uparrow k}^+ + a_k^+\alpha_{\uparrow -k}^+)\} + \mathrm{H.C.}, \tag{11}$$

where

$$A_{1(2),k} = (-H_z C\eta_k \pm 4FS\zeta_k)\frac{2\pi nei}{\varepsilon}\left(\frac{\hbar SN_0}{Nm^*}\right)^{\frac{1}{2}}\frac{k_x + ik_y}{k\omega_k^{\frac{1}{2}}}. \tag{12}$$

In Eq. (12), the upper (lower) sign corresponds to the index 1 (2), N_0 is the number of elementary cells of the crystal, $a_{\downarrow k}$ and $a_{\uparrow k}$ ($a_{\downarrow k}^+$ and $a_{\uparrow k}^+$) are annihilation (creation) operators of both kinds of antiferromagnons, η_k and ζ_k are defined as follows:

$$\eta_k = u_k + v_k, \tag{13a}$$

$$\zeta_k = u_k - v_k, \tag{13b}$$

where u_k and v_k are certain coefficients diagonalizing the Hamiltonian of the antiferromagnet and are given by the following formulae [7]:

$$u_k = \cosh\frac{\varrho_k}{2} \tag{14a}$$

$$v_k = \sinh\frac{\varrho_k}{2} \tag{14b}$$

$$\tanh\varrho_k = -\frac{\gamma_k}{1 + \dfrac{H_A}{H_E}}. \tag{14c}$$

H_A and H_E denote the anisotropy field and exchange field, respectively, and γ_k is defined by the following expression

$$\gamma_k = z^{-1} \sum_{\langle q \rangle} e^{ik\cdot(Rp-Rq)}, \tag{15}$$

where the symbol $\langle q \rangle$ at the summation denotes that the sum is limited to the z nearest neighbours of the spin $\boldsymbol{S}_p$ at the site $\boldsymbol{R}_p$. When deriving Eq. (11) we restricted ourselves to one magnon-one plasmon processes only. Terms describing processes of higher orders are neglected since we are interested in the problem of hybridization in which these terms play no essential role.

3. *Raman light scattering from hybridized magnon-plasmon modes*

We start from a brief discussion of the problem of hybridization. Let us begin by the total Hamiltonian H for the system of magnons and plasmons, mutually coupled due to the interaction H_{m-p} given by Eq. (11)

$$H = \sum_{k(k<k_c)} \hbar\omega_k a_k^+ a_k + \sum_k \hbar\Omega_{\uparrow k}\alpha_{\uparrow k}^+\alpha_{\uparrow k} + \sum_k \hbar\Omega_{\downarrow k}\alpha_{\downarrow k}\alpha_{\downarrow k} + H_{m-p}. \tag{16}$$

The frequencies of both kinds of antiferromagnons are given by the expressions:

$$\hbar\Omega_{\uparrow(\downarrow)k} = g\mu_B H_E\left[\left(1+\frac{H_A}{H_E}\right)^2 - \gamma_k^2\right]^{\frac{1}{2}} \mp g\mu_B H_z, \tag{17}$$

where the upper (lower) sign corresponds to the symbol $\uparrow$ ($\downarrow$) and g is Lande's factor. The k_c appearing at the first summation is a cut-off wave vector below which the plasmons are well defined and which is much smaller than any vector from the Brillouin zone boundary [19, 20]. Frow the dispersion law (17) it can be concluded that the magnon branch in the magnetic field H_z is split into two branches, one of which is shifted upwards and the other downwards by an energy equal to $g\mu_B H_z$. This phenomenon allows us, by way of the magnetic field, to bring the higher branch to resonance with the plasmon line. Taking into account that under experimental conditions the investigation of system with carrier concentrations higher than 10^{14} cm^{-3} is simpler and that the plasmon energy is in general larger than that of magnons one can omit in Eq. (16) the interaction of the lower magnon branch, i.e. that of the type $\uparrow$, with the plasmons. One can also neglect the components of the form $a_k\alpha_{\downarrow -k}$ and $a_k^+\alpha_{\downarrow -k}^+$ describing simultaneous annihilation and creation of a magnon-plasmon pair. These terms do not contribute to hybridization in the first order of the perturbation treatment.

Finally, the Hamiltonian of the system takes the following form:

$$H = \sum_{k(k<k_c)} \hbar\omega_k a_k^+ a_k + \sum_k \hbar\Omega_{\uparrow k}\alpha_{\uparrow k}^+\alpha_{\uparrow k} + \sum_k \hbar\Omega_{\downarrow k}\alpha_{\downarrow k}^+\alpha_{\downarrow k}$$
$$+ \sum_{k(k<k_c)} \{A_{1,k}a_k^+\alpha_{\downarrow k} + A_{1,k}^* a_k\alpha_{\downarrow k}^+\}, \tag{18}$$

where the asterisk stands for complex conjugate.

Having performed the diagonalization transformation

$$\alpha_{\downarrow k} = e^{-i\varphi_k} \cos\frac{\beta_k}{2}\, c_{1,k} + e^{-i\varphi_k} \sin\frac{\beta_k}{2}\, c_{2,k} \tag{19a}$$

$$a_{\downarrow k}^{+} = e^{i\varphi_k} \cos\frac{\beta_k}{2}\, c_{1,k}^{+} + e^{i\varphi_k} \sin\frac{\beta_k}{2}\, c_{2,k}^{+} \tag{19b}$$

$$a_k = -e^{i\varphi_k} \sin\frac{\beta_k}{2}\, c_{1,k} + e^{i\varphi_k} \cos\frac{\beta_k}{2}\, c_{2,k} \tag{19c}$$

$$a_k^{+} = -e^{-i\varphi_k} \sin\frac{\beta_k}{2}\, c_{1,k}^{+} + e^{-i\varphi_k} \cos\frac{\beta_k}{2}\, c_{2,k}^{+} \tag{19d}$$

we arrive at

$$H = \sum_k \hbar\Omega_{\uparrow k}\alpha_{\uparrow k}^{+}\alpha_{\uparrow k} + \sum_{k(k>k_c)} \hbar\Omega_{\downarrow k}\alpha_{\downarrow k}^{+}\alpha_{\downarrow k} + \sum_{k(k<k_c)} \hbar\omega_{1,k}c_{1,k}^{+}c_{1,k} + \sum_{k(k<k_c)} \hbar\omega_{2,k}c_{2,k}^{+}c_{2,k}. \tag{20}$$

In Eq. (19) φ_k is defined by the expression $A_{1,k} = |A_{1,k}| \exp(2i\varphi_k)$ whereas β_k is determined in the first approximation by the following equation

$$\tan\beta_k = \frac{2|A_{1k}|}{\hbar(\Omega_{\downarrow k} - \omega_k)}. \tag{21}$$

The operators $c_{1,k}\,(c_{1,k}^{+})$ and $c_{2,k}\,(c_{2,k}^{+})$ are annihilation (creation) operators of two hybrids with frequencies, in the first approximation, equal to

$$\hbar\omega_{1(2),k} = \tfrac{1}{2}\,\hbar(\omega_k + \Omega_{\downarrow k}) \mp \tfrac{1}{2}\,[4|A_{1,k}|^2 + \hbar^2(\Omega_{\downarrow k} - \omega_k)^2]^{\frac{1}{2}}. \tag{22}$$

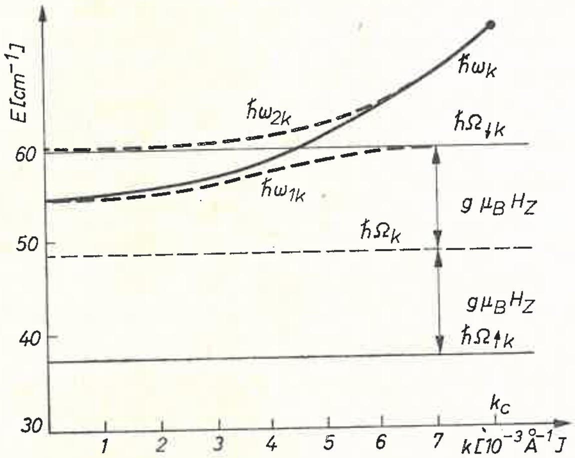

Fig. 1. Schematic picture of magnon-plasmon hybridization in antiferromagnetic semiconductors $\hbar\omega_k$ – the plasmon energy at: $n = 10^{15}$ cm^{-3}, $\varepsilon = 1$, $m^* = m$ ($\hbar\omega_k = \hbar\omega_0 + \hbar\alpha k^2$; $\hbar\omega_0 = 54.8$ cm^{-1}, $\hbar\alpha = 2.8 \times 10^5$ cm^{-1}A^2, $k_c = 8 \times 10^{-3}$ A^{-1}); $h\Omega_k$ – a typical magnon branch (degenerate) without an external magnetic field H ($\hbar\Omega_k = h\Omega_0 + \beta k^2$, $\beta \sim \hbar\Omega_0 \sim 10$ cm^{-1}A^2, $h\Omega_0 = 48$ cm^{-1} – a denotes lattice parameter); $h\Omega_{\downarrow k}$ and $h\Omega_{\uparrow k}$ – the two magnon branches in a field H ($H = 120$ kGs); $h\omega_{1k}$ and $h\omega_{2k}$ – two hybrid modes

The hybridized modes are shown schematically in Fig. 1. From Eqs (19)–(22) it is obvious that one of the branches of hybrids joins the magnon branch $\Omega_{\downarrow k}$ (for $k > k_c$) at the point k_c (on the assumption that the crossing point of the dispersion curves lies sufficiently far below k_c). Light scattering should now be expected to occur by way of excitation of one of the hybrids.

Further considerations can be simplified essentially by taking into account that the light scattering differential cross-section from plasmons is smaller by several orders of magnitude than that from magnons. One can thus assume that the scattering occurs due to excitation of a magnon and afterwards through energy transfer to the plasmon system by way of magnon-plasmon interaction.

It is worth noting here that, by insertion of the electric field of the light wave into the Hamiltonian (7) instead of $\boldsymbol{E}$, one obtains directly the Hamiltonian $H_{\mathrm{m-r}}$ of coupling between the magnon system and radiation. The most essential part of this Hamiltonian describing Raman scattering from one-magnon excitations is then given by the following formula:

$$H_{\mathrm{m-r}} = \sum_{kk'\sigma\sigma'} M(k\sigma, k'\sigma') \left[(\alpha^{+}_{\uparrow k'-k} + \alpha_{\downarrow k-k'}) b^{+}_{k\sigma} b_{k'\sigma'} + (\alpha_{\downarrow k'-k} + \alpha^{+}_{\uparrow k-k'}) b_{k\sigma} b^{+}_{k'\sigma'}\right] + \mathrm{H.C.}, \tag{23}$$

where

$$M(k\sigma, k'\sigma') = iG\eta_{k'-k} \frac{\pi\hbar}{V} \left(\frac{SN_0}{2} \bar{\omega}_k \bar{\omega}_{k'}\right)^{\frac{1}{2}} \times \left[e^{z}_{k'\sigma'}(e^{x}_{k\sigma} + ie^{y}_{k\sigma}) + e^{z}_{k\sigma}(e^{x}_{k'\sigma'} + ie^{y}_{k'\sigma'})\right]. \tag{24}$$

Above V denotes the volume of the crystal, $e_{k\sigma}$ — a unit vector in the electric field direction of the photon with the wave vector $\boldsymbol{k}$ and polarization σ and $\bar{\omega}_k$ is the frequency of this photon. The Hamiltonian (23) is equivalent to the spin Hamiltonian proposed by Loudon [7] for explaining light scattering from magnetic excitations.

Let us now revert to our Raman scattering study of the hybridized states, which are superpositions of plasmon and magnon states. The Hamiltonian of interaction between the light wave and coupled magnon-plasmon system can be obtained by transition, in Eq. (23), to the operators $c_{1,k}$ and $c_{2,k}$ of Eqs (19a)–(19d). Taking into account that first order Raman scattering originates in excitations with k approximately equal to zero, we obtain the light scattering differential cross section R_1 and R_2 from hybrids $\omega_{1,k}$ and $\omega_{2,k}$ in the following forms:

$$R_1 = \frac{V}{4\pi^3\hbar^2}\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{\frac{1}{2}} \frac{\omega_2}{c^4} \cos^2\left(\frac{\beta_0}{2}\right) |M|^2 (N_1+1)^2 \frac{\Gamma}{\hbar^2(\bar{\omega}_1 - \bar{\omega}_2 - \omega_{1,0})^2 + \Gamma^2}, \tag{25a}$$

$$R_2 = \frac{V}{4\pi^3\hbar^2}\left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{\frac{1}{2}} \frac{\omega_2}{c^4} \sin^2\left(\frac{\beta_0}{2}\right) |M|^2 (N_2+1)^2 \frac{\Gamma}{\hbar^2(\bar{\omega}_1 - \bar{\omega}_2 - \omega_{2,0})^2 + \Gamma^2}, \tag{25b}$$

where $\varepsilon_1 (\varepsilon_2)$ is the incident (scattered) photon dielectric constant, $\bar{\omega}_1 (\bar{\omega}_2)$ — the incident (scattered) photon frequency N_1 and N_2 are the occupation numbers of the two hybridized states, Γ is the half-width of the Raman line, which can be taken from experiment, and

$$M = -iG\pi\hbar V^{-1}\eta_0(2SN_0\bar{\omega}_1\bar{\omega}_2)^{\frac{1}{2}}[e_1^z(e_2^x - ie_2^y) + e_2^z(e_1^x - ie_1^y)] \tag{26}$$

where the factor η_0 can be obtained from Eqs (13a)–(14c).

Since scattering from "pure" plasmons is neglected one should obtain in the absence of hybridization a Raman spectrum line related to excitation of a magnon with the frequency $\Omega_{\downarrow 0}$ (we are not interested in the line at the frequency $\Omega_{\uparrow 0}$). The behaviour of this line becomes particularly interesting when the field H increases attaining the value at which $\Omega_{\downarrow 0}$ approaches ω_0. When the hybridized modes begin to appear, this line corresponds to the frequency $\omega_{1,0}$ and becomes weaker and weaker to the benefit of the line appearing beside it and corresponding to the frequency $\omega_{2,0}$. At the point at which $\Omega_{\downarrow 0} = \omega_0$, both lines should possess the same intensity, equal to one half of that of the initial line. The behaviour described above is a simple consequence of Eqs (19a)–(24b). This provides the possibility of an experimental search for the magnon-plasmon interaction as well as of a confrontation with experiment of the conclusions resulting from the present paper.

4. *Conclusions, and their discussion*

In the present paper we discussed the problem of magnon-plasmon interaction and its influence on Raman light scattering from one-magnon excitations in antiferromagnetic semiconductors. We proposed a model of this coupling based on indirect (via spin-orbit coupling) interaction of the spin system with the plasmon electric field, on the assumption that the ground state of an ion in the crystal field is orbitally non-degenerate. From the nature of the perturbation treatment performed here the conclusion is drawn that this model leads to an insignificant coupling if the ground state of the free ion is of the S-type. If $L \neq 0$, then assuming typical values for parameters occurring in Eqs (6b), (12) i.e. $E_a - E_0 = E_b - E_0 = 10^3\ \text{cm}^{-1}$, $\lambda = 10^2 - 10^3\ \text{cm}^{-1}$, $n = 10^{16}\ \text{cm}^{-3}$, $N_0/N = 10^6$, $\varepsilon = 1$, $m^* = m$, $\langle 0|L_\alpha|a\rangle = i$, $\langle 0|D_\alpha|a\rangle = ea_0 \cong 10^{-18}\ \text{g}^{1/2}\ \text{cm}^{5/2}\ \text{s}^{-1}$ (a_0 — the first Bohr orbit radius of the hydrogen atom), we estimate $A_{1,k}$ to be of the order of $10 - 10^{-2}\ \text{cm}^{-1}$. It is of great interest that the magnon-plasmon interaction is influenced by an external magnetic field H which, if $\lambda > 0$ (C and F have the same sign) causes a weakening of the interaction between plasmons and "↓"-magnons. With $\lambda < 0$ (C and F possess opposite signs), the situation is the inverse. The Hamiltonian obtained in the present paper concerns the case of magnetic ions localized in environments of T_d symmetry, but can be easily derived for other symmetries as well. The T_d symmetry occurs e.g. in antiferromagnetic semiconductors having the spinel structure for the ions localized in the tetrahedral positions. If we omit, in a first approximation, the coupling of the ions in octahedral positions with the field E (the ion is a centre of inversion), the Hamiltonian of the magnon-plasmon interaction in these compounds takes the form given in the present paper or, at least, a similar form.

The possibility of an experimental search for effects of magnon-plasmon coupling is provided by the Raman light scattering from magnetic excitations in the magnon--plasmon hybridization region. The present paper gives the differential cross section for such scattering and its dependence on the external magnetic field.

We have restricted ourselves to the case when the scattering occurs according to the model of Loudon. Even if other scattering mechanisms prove to be more effective, the qualitative behaviour of the Raman spectrum line in the field H will remain unaffected, because it is determined by magnon-plasmon coupling constant, whereas magnon-two--photon coupling constant only determines intensity of this spectral line (Eqs (25a), (25b)). The differential cross section for scattering from the hybrids can then be very easily obtained from that for magnons, performing transformation to the hybrid operators (as in the present paper). This behaviour of the spectral line in the field H provides the clue to the experimental study of the magnon-plasmon coupling.

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