

CORRELATION FUNCTIONS OF ISOTROPIC HEISENBERG FERROMAGNET WITH AN ARBITRARY SPIN. I

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The correlation functions of the isotropic Heisenberg ferromagnet with arbitrary spin were calculated. To this end the decoupling procedure was applied to Green's retarded commutator functions formed from orthogonal operators being solutions of the Liouvillian $(-\mu h \sum_f S_f^z)^*$ eigenproblem. In this way the procedure of Ishikawa and Oguchi was generalized for the case of a spin S of arbitrary length in every site of the ferromagnetic crystalline lattice. It resulted in the formation of a longitudinal correlation function, which differs from the result of Liu and Siano for the ferromagnetic region. For $T > T_c$ the correlation functions $\langle S_k^- S_{-k}^+ \rangle$ and $\langle S_k^3 S_{-k}^3 \rangle$ assume the same values and satisfy the basic sum rule. In the long-range limit the correlation functions assume the Ornstein-Zernicke form with correlation lengths being functions of temperature. The correlation lengths are convergent at the phase transition point and the critical Fischer index equals zero.

1. Introduction

The properties of correlation functions and especially those of longitudinal correlations of the isotropic Heisenberg ferromagnet have been studied by several authors. Tahir-Kheli and Callen [1, 2] solved the problem of higher order Green's functions and the function of longitudinal correlations computed by this means did not satisfy the principal sum rule. In the paramagnetic region the correlation functions $\langle S_k^- S_{-k}^+ \rangle$ and $\langle S_k^3 S_{-k}^3 \rangle$ assumed various values. Liu [3] and Liu and Siano [4] computed the longitudinal correlation function employing a linear response of the system of mutually interacting spins to a small external time-dependent perturbation. This rather complicated method predicts an isotropic behaviour of the correlation functions for $T > T_c$ and fulfillment of the sum rule

$$\frac{1}{N} \sum_k \langle S_k S_{-k} \rangle = S(S+1).$$

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In the ferromagnetic region a small 5% deviation from the sum rule is observed. In the case of spin $S = \frac{1}{2}$ the specific heat of the simple cubic Heisenberg ferromagnet is negative from the left side of the transition point and the internal energy

$$\langle H \rangle_{T=T_c} < \langle H \rangle_{T=0}.$$

Ishikawa and Oguchi [5] have proposed a new method of calculating the correlation function of a ferromagnet with spin $S = \frac{1}{2}$ based on spin operator algebra. Use was made of the method of two-time commutator Green functions decoupled according to the method by which are neglected the highest cumulants of products of operators forming the Green function being decoupled. The specific heat calculated in this way from the left side of the phase transition point is also negative. This method permits one only to calculate static correlation functions and this fact is one of its disadvantageous features. In this work we made attempts to generalize the Ishikawa and Oguchi method for the case of an isotropic Heisenberg ferromagnet with an arbitrary spin length.

2. Green functions

The Hamiltonian of interacting spins of the Heisenberg isotropic ferromagnet in an external magnetic field directed towards the third axis assumes the form

$$H = -\mu h \sum_f S_f^3 - \sum_{f \neq g} J_{fg} (S_f^+ S_g^- + S_f^3 S_g^3), \quad (2.1)$$

where J_{fg} is the exchange integral between neighbouring sites, $J_{fg} = 0$ for $f = g$. We define the assembly of Green functions for the system as [6]

$$G_{fg}^\alpha(t) = \langle\langle L_{\alpha+1,\alpha}^f(t) | S_g^- \rangle\rangle, \quad (2.2)$$

$$F_{fgh}^\alpha(t) = \langle\langle L_{\alpha+1,\alpha}^f(t) | \hat{S}_g^3 S_h^- \rangle\rangle, \quad (2.3)$$

where operators $L_{\alpha+1,\alpha}^f$ are eigensolutions of the equation

$$[-\mu h \sum_g S_g^3, L_{\alpha+1,\alpha}^f]_- = \mu h L_{\alpha+1,\alpha}^f. \quad (2.4)$$

If the vectors $|\alpha\rangle$ are eigensolutions of the Hamiltonian $-\mu h S_f^3$ then the operators $L_{\alpha+1,\alpha}^f$ we may describe as the following form

$$L_{\alpha+1,\alpha}^f = |\alpha+1\rangle \langle \alpha|, \quad \alpha = -S, \dots, S-1; \quad (2.5)$$

and it shall be obey the commutator relations

$$[L_{\alpha,\beta}^f, L_{\gamma,\delta}^g]_- = \delta_{fg} (\delta_{\beta\gamma} L_{\alpha,\delta}^f - \delta_{\delta\alpha} L_{\gamma,\beta}^f). \quad (2.6)$$

The relations between spin operators and the operators $L_{\alpha,\beta}^f$ assume the form

$$\begin{aligned} S_f^+ &= \sum_{\alpha=-S}^S C_\alpha^S L_{\alpha+1,\alpha}^f, & S_f^- &= \sum_{\alpha=-S}^S C_\alpha^S L_{\alpha,\alpha+1}^f, \\ (S_f^3)^N &= \sum_{\alpha=-S}^S \alpha^N L_{\alpha,\alpha}^f, & C_\alpha^S &= \sqrt{S(S+1) - \alpha(\alpha+1)}. \end{aligned} \quad (2.7)$$

The operator \hat{S}_f^3 in the definition (2.3) means $S_f^3 - \langle S_f^3 \rangle$ where

$$\langle \dots \rangle = \frac{1}{Z} \text{Tr} (e^{-\beta H} \dots), \quad \beta = (kT)^{-1}, \quad (2.8)$$

is the mean thermodynamic value with the Hamiltonian H of the isotropic Heisenberg ferromagnet.

Equations of motion [6] for the Green retarded commutator functions $G_{fg}^z(E)$ and $F_{fgh}^z(E)$ assume the form

$$\begin{aligned} EG_{fg}^z(E) &= C_\alpha^S \langle L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^g \rangle \delta_{fg} + \mu h G_{fg}^z \\ &+ 2 \sum_p J_{pf} \sum_{\beta=-S}^S \beta \langle \langle L_{\beta,\beta}^p L_{\alpha+1,\alpha}^f | S_g^- \rangle \rangle_E \\ &- \sum_p J_{pf} \sum_{\gamma=-S}^S C_\alpha^S C_\gamma^S \langle \langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) L_{\gamma+1,\gamma}^p | S_g^- \rangle \rangle_E \\ &- \sum_p J_{pf} \sum_{\beta=-S}^S C_\beta^S \langle \langle L_{\beta,\beta+1}^p (C_{\alpha-1}^S L_{\alpha+1,\alpha-1}^f - C_{\alpha+1}^S L_{\alpha+2,\alpha}^f) | S_g^- \rangle \rangle_E, \end{aligned} \quad (2.9)$$

$$\begin{aligned} EF_{fgh}^z(E) &= C_\alpha^S \langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) \hat{S}_g^3 \rangle \delta_{fh} - \langle L_{\alpha+1,\alpha}^f S_h^- \rangle \delta_{fg} \\ &+ \mu h F_{fgh}^z(E) + 2 \sum_p J_{pf} \sum_{\beta=-S}^S \beta \langle \langle L_{\beta,\beta}^p L_{\alpha+1,\alpha}^f | \hat{S}_g^3 S_h^- \rangle \rangle_E \\ &- \sum_p J_{pf} \sum_{\gamma=-S}^S C_\gamma^S C_\alpha^S \langle \langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) L_{\gamma+1,\gamma}^p | \hat{S}_g^3 S_h^- \rangle \rangle_E \\ &- \sum_p J_{pf} \sum_{\beta=-S}^S C_\beta^S \langle \langle L_{\beta,\beta+1}^p (C_{\alpha-1}^S L_{\alpha+1,\alpha-1}^f - C_{\alpha+1}^S L_{\alpha+2,\alpha}^f) | \hat{S}_g^3 S_h^- \rangle \rangle_E. \end{aligned} \quad (2.10)$$

In (2.9) for the Green function $\langle \langle L_{\beta,\beta}^p L_{\alpha+1,\alpha}^f | S_g^- \rangle \rangle$ and $\langle \langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) L_{\gamma+1,\gamma}^p | S_g^- \rangle \rangle$ we use the decoupling

$$\begin{aligned} \langle \langle L_{\beta,\beta}^p L_{\alpha+1,\alpha}^f | S_g^- \rangle \rangle &\approx \langle L_{\beta,\beta}^p \rangle G_{fg}^z(E), \\ \langle \langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) L_{\gamma+1,\gamma}^p | S_g^- \rangle \rangle &\approx \langle L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f \rangle G_{pg}^z(E), \\ \langle \langle L_{\beta,\beta+1}^p (C_{\alpha-1}^S L_{\alpha+1,\alpha-1}^f - C_{\alpha+1}^S L_{\alpha+2,\alpha}^f) | S_g^- \rangle \rangle &\approx 0. \end{aligned} \quad (2.11)$$

In (2.10) for the Green function $\langle \langle L_{\beta,\beta}^p L_{\alpha+1,\alpha}^f | \hat{S}_g^3 S_h^- \rangle \rangle_E$ and $\langle \langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) L_{\gamma+1,\gamma}^p | \hat{S}_g^3 S_h^- \rangle \rangle$ we shall employ the Ishikawa and Oguchi [5] decoupling procedure in a spirit of the random phase approximation

$$\begin{aligned} \langle \langle L_{\beta,\beta}^p L_{\alpha+1,\alpha}^f | \hat{S}_g^3 S_h^- \rangle \rangle_E &\approx \langle L_{\beta,\beta}^p \hat{S}_g^3 \rangle G_{fh}^z(E) + \langle L_{\beta,\beta}^p \rangle F_{fgh}^z(E), \\ \langle \langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) L_{\gamma+1,\gamma}^p | \hat{S}_g^3 S_h^- \rangle \rangle_E &\approx \langle \langle (L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f) \hat{S}_g^3 \rangle \rangle G_{ph}^z(E) \\ &+ \langle L_{\alpha+1,\alpha+1}^f - L_{\alpha,\alpha}^f \rangle F_{pgh}^z(E), \\ \langle \langle L_{\beta,\beta+1}^p (C_{\alpha-1}^S L_{\alpha+1,\alpha-1}^f - C_{\alpha+1}^S L_{\alpha+2,\alpha}^f) | \hat{S}_g^3 S_h^- \rangle \rangle_E &\approx 0, \end{aligned} \quad (2.12)$$

which is substantiated by a neglect of highest cumulants of product of operators forming the Green function being decoupled. (See Appendix in Ishikawa and Oguchi's work or in [7].)

We shall use the isotropic properties of a ferromagnet assuming that the mean values $\langle L_{\alpha,\alpha}^f \rangle$ are equal to each other at every site of the crystalline lattice. A translational invariance of the Hamiltonian permits one to employ Fourier transformations

$$\begin{aligned} G_{f,g}^{\alpha}(E) &= \sum_k e^{ik(f-g)} G_k^{\alpha}(E), \\ F_{fgh}^{\alpha}(E) &= \sum_{kq} e^{ik(f-g)} e^{iq(f-h)} F_{k,q}^{\alpha}(E), \\ J_{fg} &= \sum_k e^{ik(f-g)} J_k, \quad \langle A_f B_g \rangle = \sum_k e^{ik(f-g)} \langle A_k B_{-k} \rangle, \end{aligned} \quad (2.13)$$

where k is the wave vector belonging to the first Brillouin zone. In the nearest neighbours approximation

$$\sum_k J_{k+q} f(k, q) \approx \frac{J_q}{J_0} \sum_k J_k f(k, q)$$

and after using (2.11), (2.12) and (2.13), the equations (2.9) and (2.10) can be linearized

$$\sum_{\gamma=-S}^{S-1} [(E-\omega_0)\delta_{\alpha,\gamma} + P_{\alpha}^S C_{\gamma}^S J_k] G_k^{\gamma}(E) = P_{\alpha}^S, \quad (2.14)$$

$$\sum_{\gamma=-S}^{S-1} [(E-\omega_0)\delta_{\alpha,\gamma} + P_{\alpha}^S C_{\gamma}^S J_{k+q}] F_{k,q}^{\gamma}(E) = A_{\alpha}^S, \quad (2.15)$$

where

$$\begin{aligned} P_{\gamma}^S &= C_{\gamma}^S \langle L_{\gamma+1,\gamma+1} - L_{\gamma,\gamma} \rangle, \\ \omega_0 &= \mu h + 2\sigma J_0, \quad \sigma = \sum_{\alpha=-S}^S \alpha \langle L_{\alpha,\alpha} \rangle, \end{aligned} \quad (2.16)$$

and

$$\begin{aligned} A_{\alpha}^S &= C_{\alpha}^S \langle (L_{\alpha+1,\alpha+1}^k - L_{\alpha,\alpha}^k) \hat{S}_{-k}^3 \rangle - \langle L_{\alpha+1,\alpha}^q S_{-q}^- \rangle \\ &- C_{\alpha}^S \langle (L_{\alpha+1,\alpha+1}^k - L_{\alpha,\alpha}^k) \hat{S}_{-k}^3 \rangle J_q \sum_{\beta=-S}^{s-1} C_{\beta}^S G_q^{\beta}(E) + 2 \sum_{\beta=-S}^S \beta \langle L_{\beta,\beta}^k \hat{S}_{-k}^3 \rangle J_k G_q^{\alpha}(E). \end{aligned} \quad (2.17)$$

Solutions of equations (2.14) and (2.15) take on the form

$$G_k^{\gamma}(E) = \frac{\Delta_{\gamma}^S(E)}{1 + J_k \sum_{\beta=-S}^{s-1} C_{\beta}^S \Delta_{\beta}^S(E)}, \quad (2.18)$$

$$F_{k,q}^{\gamma}(E) = \frac{A_{\gamma}^S + J_{k+q} \sum_{\beta=-S}^{s-1} C_{\beta}^S [A_{\gamma}^S \Delta_{\beta}^S(E) - A_{\beta}^S \Delta_{\gamma}^S(E)]}{(E-\omega_0) (1 + J_{k+q} \sum_{\beta=-S}^{s-1} C_{\beta}^S \Delta_{\beta}^S(E))}, \quad (2.19)$$

where

$$\Delta_{\gamma}^S(E) = \frac{P_{\gamma}^S}{E - \omega_0}, \quad (2.20)$$

We calculate the correlation function $\langle S_k^- S_{-k}^+ \rangle$ from the Green functions $G_k^{\alpha}(E)$ using the spectral theorem [6]. Taking into account

$$\sum C_{\alpha}^S \langle S_k^- L_{\alpha+1, \alpha}^{-k} \rangle = \langle S_k^- S_{-k}^+ \rangle, \quad (2.21)$$

after the summation of the left side of (2.21) we obtain the function of transversal correlations in the form

$$\langle S_k^- S_{-k}^+ \rangle = 2\sigma(e^{\beta \varepsilon_k} - 1)^{-1} = 2\sigma N(\varepsilon_k), \quad (2.22)$$

where

$$\varepsilon_k = \mu h + 2\sigma(J_0 - J_k). \quad (2.23)$$

The function of longitudinal correlations can be calculated by the following means. First we calculate the coordination functions $\langle \hat{S}_k^3 S_{-q}^- L_{\alpha+1, \alpha}^{k+q} \rangle$ from the Green function $F_{k, q}^{\alpha}$ using the spectral theorem. Making use of the identity

$$\frac{1}{N} \sum_q S_{-q}^- L_{\alpha+1, \alpha}^{k+q} = C_{\alpha}^S L_{\alpha, \alpha}^k, \quad (2.24)$$

and summing all the coordination functions over q the following equations can be written down

$$C_{\gamma}^S (1 + \varphi) X_k^{\gamma} - C_{\gamma}^S \varphi X_k^{\delta+1} - \frac{P_{\gamma}^S}{\sigma} \sum_{\beta=-S}^S \beta X_k^{\beta} \Delta_k = -P_{\gamma}^S \eta_k, \quad (2.25)$$

where

$$\begin{aligned} \Delta_k &= \frac{1}{N} \sum_q [1 + N(\varepsilon_q)] N(\varepsilon_{k+q}), \\ \eta_k &= \frac{1}{N} \sum_q \frac{\varepsilon_q - \varepsilon_k}{\varepsilon_q - \varepsilon_{k+q}} [N(\varepsilon_q) - N(\varepsilon_{k+q})], \end{aligned} \quad (2.26)$$

and

$$X_k^{\gamma} = \langle \hat{S}_k^3 L_{\gamma, \gamma}^{-k} \rangle, \quad (2.27)$$

and

$$\varphi = \frac{1}{N} \sum_k N(\varepsilon_k). \quad (2.28)$$

There are $2S+1$ coordination functions of the type X_k^γ and $2S$ equations (2.25). If the set of equations (2.25) is completed with the identity

$$\sum_{\gamma=-S}^S X_k^\gamma = 0, \quad (2.29)$$

then we can find X_k^γ from it. In the last step we make use of the relation

$$\sum_{\gamma=-S}^S \gamma X_k^\gamma = \langle \hat{S}_k^3 S_{-k}^3 \rangle, \quad (2.30)$$

which comes from (2.7). The explicit form of the longitudinal correlation function is

$$\langle \hat{S}_k^3 S_{-k}^3 \rangle = \frac{\sigma M_S A_k}{1 + M_S \eta_k}, \quad (2.31)$$

where

$$M_S = \left\{ \left[1 + \sum_{k=0}^{2S-2} (2S-k) \left(\frac{\varphi}{1+\varphi} \right)^{2S-k-1} \right] \left[1 - \sum_{p=2}^{2S} \sum_{k=0}^{2S-p} \left(\frac{\varphi}{1+\varphi} \right)^{k+p} \right] \right. \\ \left. + \sum_{p=0}^{2S} \sum_{r=2}^{2S} \sum_{k=0}^{2S-r} (2S-k) \left(\frac{\varphi}{1+\varphi} \right)^{2S-1-k+p} \right\} \left[\sum_{k=0}^{2S} \sum_{p=0}^{2S} (2S-k) \left(\frac{\varphi}{1+\varphi} \right)^{k+p} \right]^{-1}, \\ M_{1/2} = \frac{2}{1+2\varphi}. \quad (2.32)$$

The expression (2.31) is the generalization of the result derived in [5] for an arbitrary value of spin S and for the special case of $S = \frac{1}{2}$ reduces to the form obtained by Ishikawa and Oguchi [5].

3. The N -th moment of the z -component of the spin

Let us now derive very useful formulas for the N -th moment of the z -component of the spin. The only assumption we shall accept is that the mean values $\langle L_{\alpha,\alpha}^f \rangle$ are the same at every site of the crystalline lattice. Notice that we can calculate the correlation function $\langle S_k^- L_{\alpha+1,\alpha}^{-k} \rangle$ from Green functions $G_k^\gamma(E)$. Let us define the function φ_α as

$$P_\alpha^S \varphi_\alpha = \frac{1}{N} \sum_k \langle S_k^- L_{\alpha+1,\alpha}^k \rangle. \quad (3.1)$$

From the identity (2.24) and equation (3.1) ensues the set of equations

$$P_\alpha^S \varphi_\alpha = C_\alpha^S \langle L_{\alpha,\alpha} \rangle, \quad (3.2)$$

and

$$\sum_{\alpha=-S}^S \langle L_{\alpha,\alpha} \rangle = 1, \quad (3.3)$$

where

$$P_{\alpha}^S = C_{\alpha}^S \langle L_{\alpha+1, \alpha+1} - L_{\alpha, \alpha} \rangle. \quad (3.4)$$

The set of equations (3.2) and (3.3) have unique solutions

$$\langle L_{S,S} \rangle = \left[1 + \sum_{k=1}^{2S} \prod_{i=1}^k \frac{\varphi_{S-i}}{1 + \varphi_{S-i}} \right]^{-1}, \quad (3.5)$$

$$\langle L_{\alpha, \alpha} \rangle = \langle L_{S,S} \rangle \prod_{i=\alpha}^{S-1} \frac{\varphi_i}{1 + \varphi_i}, \quad \alpha = -S, \dots, S-1. \quad (3.6)$$

From (2.7), (3.5) and (3.6) we get

$$\langle (S^3)^N \rangle = \langle L_{S,S} \rangle \left[S^N + \sum_{\gamma=-S}^{S-1} \gamma^N \prod_{i=\gamma}^{S-1} \frac{\varphi_i}{1 + \varphi_i} \right]. \quad (3.7)$$

When all φ_{α} are equal, as is the case for the isotropic Heisenberg ferromagnet, the equation (3.7) significantly simplifies

$$\langle (S^3)^N \rangle = \langle L_{S,S} \rangle \left[S^N + \sum_{\gamma=-S}^{S-1} \gamma^N \left(\frac{\varphi}{1 + \varphi} \right)^{\gamma} \right]. \quad (3.8)$$

Equation (3.7) stands for the generalization of equations [8] which enables us to calculate the N -th moment S^3 for the case of different φ_{α} i. e. for systems with single-ion anisotropy of various kinds [9].

4. Thermodynamic properties

In the temperature region $T < T_c$ the magnetization (3.8) is given approximately by

$$\sigma \approx \frac{2S(S+1)}{3(1+2\varphi)}, \quad (4.1)$$

where

$$1+2\varphi = \frac{1}{N} \sum \operatorname{cth} \frac{\beta \varepsilon_k}{2} \approx \frac{kT}{J_0 \sigma} I(1). \quad (4.2)$$

$I(1)$ is a well known Watson integral [10]. Substituting (4.2) into (4.1) we get

$$kT_c = \frac{2S(S+1)}{3I(1)} J_0. \quad (4.3)$$

Expanding $\text{cth } x$ in (4.2) up to the second term we obtain

$$\sigma \sim \left[1 - \frac{T}{T_c} \right]^{1/2} \quad (4.4)$$

hence the critical index $\beta = \frac{1}{2}$. The magnetic susceptibility is defined by $\lim_{h \rightarrow 0} \frac{\sigma}{\mu h} = \chi$.

From (4.1), (4.2) and (4.3) we get

$$\frac{T}{T_c} = \frac{I(1)}{I(Y)}, \quad (4.5)$$

where

$$\frac{1}{Y} = 1 + \frac{1}{2\chi J_0}. \quad (4.6)$$

For the Watson integral $I(Y)$ the expansion

$$I(Y) = I(1) - A\sqrt{1-Y} + \dots, \quad (4.7)$$

is valid [10] and for simple cubic lattice s. c. $A = \frac{3\sqrt{3}}{\pi\sqrt{2}}$. Hence

$$\frac{1}{\chi} \sim \left[1 - \frac{T}{T_c} \right]^2, \quad (4.8)$$

and the critical index $\gamma = 2$.

For temperature $T > T_c$ one can assume

$$N(\varepsilon_k) \approx \frac{kT}{\varepsilon_k}, \quad (4.9)$$

and

$$M_S \approx \varphi^{-1} + O(\varphi^{-2}). \quad (4.10)$$

Using (4.9) and (4.10) we obtain

$$\langle S_k^- S_{-k}^+ \rangle = \frac{2kT\chi}{1 + 2\chi(J_0 - J_k)}, \quad (4.11)$$

and

$$\langle S_k^3 S_{-k}^3 \rangle = \frac{kT\chi}{1 + 2\chi(J_0 - J_k)}. \quad (4.12)$$

The correlation functions are isotropic in the paramagnetic region

$$2\langle S_k^3 S_{-k}^3 \rangle = \langle S_k^- S_{-k}^+ \rangle, \quad (4.13)$$

and the sum rule is satisfied

$$\frac{1}{N} \sum_k \langle S_k S_{-k} \rangle = S(S+1). \quad (4.14)$$

In the long-range limit, i. e. for small wave vectors

$$a|k| \ll 1, \quad \text{s. c.} \quad (4.15)$$

the transverse and longitudinal correlation functions are of the form

$$\langle S_k^- S_{-k}^+ \rangle = 2v \frac{\kappa_{\perp}^2}{1 + \kappa_{\perp}^2 k^2}, \quad (4.16)$$

$$\langle S_k^z S_{-k}^z \rangle = v \frac{\kappa_{\parallel}^2}{1 + \kappa_{\parallel}^2 k^2}, \quad (4.17)$$

where $v = \frac{6kT}{a^2 J_0}$,

$$\kappa_{\perp}^2 = \kappa_{\parallel}^2 = \frac{J_0 a^2}{3} \chi, \quad (4.18)$$

are the correlations lengths divergent at the point of phase transition. The result of (4.16) and (4.17) agree with those of [11].

5. Discussion

In order to obtain static longitudinal and transversal correlation functions of the isotropic Heisenberg ferromagnet with arbitrary spin, the use was made of the new method of decoupling of the higher order Green functions. The Ishikawa and Oguchi method [5] was generalized. It was found that the specific heat is negative from the left side of the phase transition point for spin $S = \frac{1}{2}$ only. The correlation functions in a long-range limit assume the Ornstein-Zernicke form with correlation lengths κ_{\parallel} , κ_{\perp} divergent at the phase transition point.

We have applied our method for the function of longitudinal correlation (2.31) for an arbitrary spin and the result we have obtained in (4.12) agrees with that [4] in the paramagnetic region. We have also generalized the expression [2] for the N -th moment of the z -component of crystalline lattice spin vector for the case when besides the exchange interaction the single-ion type anisotropy is present.

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