A CLASSICAL, PHENOMENOLOGICAL DESCRIPTION OF THE THERMODYNAMICAL PROPERTIES OF UNIAXIAL FERROMAGNETS WITH DOMAIN STRUCTURE

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The classical, phenomenological spin waves theory has been applied to uniaxial ferromagnets with a flat-parallel domain structure. Dispersion laws and normalized wave functions have been derived. By quantization of the results obtained the dependences of magnetization density, energy density and specific heat on temperature have been found. Analytic expressions for these dependences are presented.

1. *Introduction*

The aim of this paper is to determine the influence of the presence of domain structure in ferromagnetic materials on the spin waves energy spectrum and thermodynamical properties of ferromagnets. Classical spin waves formalism has been recently employed in a number of papers dealing both with bulk materials [6, 10, 14], and thin films [1, 2, 10, 11]. As compared with the approximate second quantization method [5, 8, 9, 12, 13], the classical formalism permits one to define precisely the boundary condition imposed on the equation of motion (e.g. for thin films). In the present paper an uniaxial ferromagnet with flat-parallel domain structure is considered. The following types of energies are taken into account: exchange energy, energy of anisotropy, wall quasi-elastic energy and energy of demagnetization. It has been assumed that the sample's magnetic energy can be expressed by a magnetization density vector and spatial derivatives. The set of Euler-Lagrange equations determines the energy minimum conditions. The periodicity conditions, defining the domain structure, are imposed on the solutions of the Euler–Lagrange equations. In the ground state the magnetization density vector is parallel to the direction of the local effective field. Precession of this vector about that direction is described by a classical Landau–Lifshitz equation of motion, without the damping term. The equations of motion have been solved in a local coordinate system (x'_μ),

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where the x_3' axis coincides with the direction of local effective field. Dispersion laws and a normalized wave function have been derived. To obtain mean thermodynamical values for the magnetization density vector, energy density and specific heat, the classical results have been quantized. The finite size of the sample has been taken into account by adopting a boundary condition of the Born–Karman type. Calculations have been carried out in the linearized equations of motion approximation.

2. *Equation of motion*

In analogy to [1, 5, 6] we assume that the magnetic energy F of a ferromagnet has the form

$$F = \int_V f\{M_\alpha, M_{\alpha,\mu}\} d\bar{x}, \tag{1}$$

where $f\{...\}$ is the energy density, $M_\alpha = M_\alpha(\bar{x})$ is the magnetization density vector, $M_{\alpha,\mu} = \partial M_\alpha / \partial x_\mu$. For uniaxial ferromagnets the energy density is

$$f\{M_\alpha, M_{\alpha,\mu}\} = -\tfrac{1}{2} K_1 M_3^2 + \tfrac{1}{2} C_\mu M_{\alpha,\mu} M_{\alpha,\mu} + 2\pi M_1^2 + \tfrac{1}{2} K'(\gamma_3 M_2 - \gamma_2 M_3)^2, \tag{2}$$

where K_1 is the anisotropy constant, C_μ — macroscopic coupling constants, K' — coefficient of quasi-elastic force of the interdomain wall [9, 12]. Summation over repeating indices is understood. The necessary condition for a minimum of the function (1) with the additional condition

$$M_\alpha(\bar{x}) M_\alpha(\bar{x}) = M_0^2 \tag{3}$$

is the set of Euler–Lagrange equations of the form

$$H_\alpha^{\text{eff}} = -\frac{\delta F}{\delta M_\alpha} = \lambda M_\alpha = \lambda \gamma_\alpha M_0, \tag{4}$$

where $\lambda = \lambda(\bar{x})$; $M_\alpha = M_\alpha(\bar{x}) = \gamma_\alpha M_0$. The unit vector $\gamma_\alpha = \gamma_\alpha(\bar{x})$ is directed along the local magnetization vector $M_\alpha(\bar{x})$. In the case of flat-parallel domain structure of the Shirobokov type [3, 4] we have

$$\gamma_\alpha = \gamma_\alpha(x_1); \quad \gamma_\alpha(x_1 + 2n\Delta) = \gamma_\alpha(x_1) \quad n = 0, \pm 1, \pm 2, \ldots, \tag{5}$$

where Δ is the domain width. The solution (5) for the structure of this type has been extensively discussed in [3, 4], and are of the form

$$\gamma_1(x_1) = 0; \quad \gamma_2(x_1) = \sin \varphi(x_1) = \text{cn}\,(qx_1);$$
$$\gamma_3(x_1) = \cos \varphi(x_1) = \text{sn}\,(qx_1), \tag{6}$$

where $\text{sn}(qx_1)$, $\text{cn}(qx_1)$ are Jacobi elliptic sine and cosine, respectively of the argument qx_1. In (6) the angle between the magnetization density vector and magnetically preferred axis of the crystal (the Ox_3 axis) has been denoted by φ. Moreover

$$q = k^{-1} \sqrt{K_1/C_1}, \tag{7}$$

where k is the elliptic integral modulus, defined by the relation

$$K(k) = \frac{\Delta}{2k}\sqrt{K_1/C_1}\,, \tag{8}$$

where $K(k)$ is the full elliptic integral of the first kind.

To describe the precession movement performed by the vector M_α around the equilibrium position, defined by solutions (6) of Eq. (4), let us introduce the notation

$$M_\alpha(\bar{x}, t) = \gamma_\alpha(x_1)M_0 + m_\alpha(\bar{x}, t) \tag{9}$$

with

$$\gamma_\alpha m_\alpha = 0. \tag{10}$$

The equation of motion has the form

$$\frac{\partial m_\alpha}{\partial t} = g\varepsilon_{\alpha\beta\varrho}M_\beta H_\varrho^{\text{eff}}, \tag{11}$$

where $\varepsilon_{\alpha\beta\varrho}$ — is the Levi–Civita antisymmetric tensor, g — giromagnetic ratio. In the first approximation of the Taylor series [6], the effective field can be expressed as following

$$H_\alpha^{\text{eff}} = \lambda\gamma_\alpha M_0 - \left.\frac{\partial^2 f}{\partial M_\alpha \partial M_\beta}\right|_{M_\varrho=\gamma_\varrho M_0} m_\beta + \left.\frac{\partial^2 f}{\partial M_{\alpha,\mu}\partial M_{\beta,\nu}}\right|_{M_{\varrho,\sigma}=\gamma_{\varrho\sigma}M_0} m_{\beta,\mu\nu}. \tag{12}$$

Let us rewrite vector $m_\alpha(\bar{x}, t)$ in the form

$$m_\alpha(\bar{x}, t) = \int d\omega \int d\kappa \tilde{m}_\alpha \exp\{i(\omega t - \bar{\kappa}\bar{\varrho})\}, \tag{13}$$

where

$$\tilde{m}_\alpha = \tilde{m}_\alpha(x_1; \bar{\kappa}, \omega); \quad \bar{\kappa} = (0, k_2, k_3); \quad \bar{\varrho} = (0, x_2, x_3). \tag{14}$$

Because of the condition (10) components of the vector $\tilde{m}_\alpha$ are no longer linearly independent. To eliminate one of them from the set of equations (11) let us transform to a local coordinate system (x_1', x_2', x_3'), where the Ox_3' axis is parallel to the unit vector $\gamma_\alpha(x_1)$. In the local coordinate system the vector $\tilde{m}_\alpha'$ is defined in the following way

$$\tilde{m}_\alpha' = T_{\alpha\beta}\tilde{m}_\beta; \quad T_{\alpha\beta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \operatorname{sn}(qx_1) & -\operatorname{cn}(qx_1) \\ 0 & \operatorname{cn}(qx_1) & \operatorname{sn}(qx_1) \end{bmatrix}. \tag{15}$$

Introducing (6), (12), (13) into (11) and carrying out the transformation (15) we obtain two equations differing only by a multiplicative constant [6, 9], i.e.,

$$\tilde{m}_1' = S_1 g(y); \quad \tilde{m}_2' = S_2 g(y). \tag{16}$$

Each of these equations has the form of the Lame equation

$$\frac{d^2 g}{dy^2} - \{2k^2 \operatorname{sn}^2 y + A_0\}g = 0, \tag{17}$$

where

$$A_0 = k^2 K_1^{-1}(C_\perp \kappa^2 + 4\pi) - 1 + i\Omega \frac{S_2}{S_1},$$

or

$$A_0 = k^2 K_1^{-1}(C_\perp \kappa^2 + K') - k^2 - i\Omega \frac{S_1}{S_2}, \tag{18}$$

and

$$y = qx_1 = k^{-1}\sqrt{K_1/C_1}\, x_1; \quad \Omega = k^2\omega/gM_0K_1; \quad C_\perp \kappa^2 = C_2k_2^2 + C_3k_3^2. \tag{19}$$

3. Solutions of the equations of motion

Solution of the Lame equation (17) has the form [7]:

$$g(y) = \frac{\vartheta_1\left\{\frac{\pi}{2K}(y \pm y_1)\right\}}{\vartheta_4\left\{\frac{\pi}{2K}\right\}} \exp\{\pm yZ(y_1)\}, \tag{20}$$

where $\vartheta_1(z)$, $\vartheta_4(z)$ are the Jacobi theta functions. The zeta function is defined by

$$Z(y_1) = E(y_1) - \frac{E}{K} y_1, \tag{21}$$

where K, E are full elliptic integrals of the first and second kind, resp., $E(y_1)$ is not a full elliptic integral of the second kind. The parameter y_1 is related to the parameter A_0 by Eq. (17) from which one obtains

$$A_0 = -1 - k^2 \operatorname{cn}^2 y_1 \tag{22}$$

and can be expressed as

$$y_1 = u + iv. \tag{23}$$

Let us introduce a function $\psi(x_1, k_\mu)$ normalized in the (L_1, L_3, L_3) region and defined in the following way

$$\psi(x_1, k_\mu) = \frac{\pm g(x_1)}{\sqrt{\Sigma}\, \|g(x_1, k_1)\|} e^{i\bar{\kappa}\bar{\varrho}}, \tag{24}$$

where $g(x_1, k_\mu)$ is the solution (20) of Eq. (17), $\Sigma = L_2L_3$ and L_i are the edges of the parallelepiped along the $Ox_i (i = 1, 2, 3)$ axis, $L_1 = 2N\Delta$. The solutions (20) of Eq. (17) are single-valued and bounded functions if:

(A) the real part of the parameter y_1, is an even multiplicity of a complete elliptic integral of the first kind, i.e., $u = 2nK$. Then we have

$$\psi_A(x_1, k_\mu) = \frac{1}{\sqrt{\Sigma}} \frac{\operatorname{sn}(qx_1) \pm ik^{-1}q^{-1}k_1}{\{4Nq^{-1}k^{-2}(K - E + q^{-2}Kk_1^2)\}^{1/2}} e^{i(\mp k_1x_1 + \bar{\kappa}\bar{\varrho})}. \tag{24a}$$

Assuming that the modulus k is near 1, $k \approx 1$, which is fulfilled for cases of physical interest [3, 4], the precession frequency $\omega = \omega^{A}$ is given for $Ck_\mu k_\mu \gg K_1+4\pi, K_1+K'$ by the formula

$$\omega^{A} = gM_0\left(K_1+\frac{K'+4\pi}{2}\right)+gM_0Ck_\mu k_\mu = \omega_0^{A}+\omega^{A}(k_\mu). \tag{25a}$$

Frequency ω_0^{A} corresponds to uniform precession, and $\omega^{A}(k_\mu)$ is the frequency for nonuniform precession, i.e., the spin waves frequency.
(B) The real part of the parameter y_1 is an odd multiplicity of K, i.e., $u = (2n+1)K$, then the respective quantities are given by the expressions

$$\psi_{B}(x_1, \bar{\kappa}, v) = \frac{1}{\sqrt{\Sigma}}\frac{\mathrm{cn}\,(qx_1)\,\mathrm{cn}\,(v, k_0)\pm i dn(qx_1)\,\mathrm{sn}\,(v, k_0)}{\{4Nq^{-1}[k^{-2}(E-k_0^2K)\,\mathrm{cn}\,(v, k_0)+E\,\mathrm{sn}^2\,(v, k_0)]\}^{1/2}}\exp\{i\bar{\kappa}\bar{\varrho}\} \tag{24b}$$

and, for $C\kappa^2 \gg 4\pi, K'$

$$\omega^{B} = gM_0\frac{K'+4\pi}{2}+gM_0C\kappa_\mu\kappa_\mu = \omega_0^{B}+\omega^{B}(\kappa_\mu). \tag{25b}$$

The following notation has been introduced in (24b), $k_0^2 = 1-k^2$ and the parameter v takes values from the interval $[0, 2N\pi]$. We can see from the Eq. (24b) that for case (B) the solution of the equations of motion does not have the character of a spin wave.

We impose now the following condition on the constants S_1 and S_2 [2]

$$|S_1|^2+|S_2|^2 = S^2. \tag{26}$$

From the set of equations (18) and condition (26) we get

$$S_2 = -i\frac{D_1}{\Omega}S_1 = -i\frac{\Omega}{D_2}S_2 \tag{27}$$

$$S_1 = \frac{\Omega}{\sqrt{\Omega^2+|D_1|^2}}S; \quad S_2 = -i\frac{D_1}{\sqrt{\Omega^2+|D_1|^2}}S. \tag{28}$$

The following notation has been adopted in Eqs. (26), (27)

$$D_1 = A_0-k^2K_1^{-1}(C_\perp\kappa^2+4\pi)+1; \quad D_2 = A_0-k^2[K_1^{-1}(C_\perp\kappa^2+K')-1];$$
$$\Omega = \sqrt{D_1D_2}. \tag{29}$$

When $k^2 \approx 1$, $k^2 \approx 0$, $C_\perp\kappa^2 \gg 4\pi, K'$ we get from (27) and (29)

$$S_2 = -iS_1 \tag{30}$$

which means that in this case the precession is circular.

4. *Magnons*

To the spin waves we ascribe magnons whose energy and momentum are connected in the following way with the spin waves frequency and wave vector

$$\varepsilon = \hbar\omega; \quad p_\alpha = \hbar k_\alpha, \tag{31}$$

where $\hbar$ is Planck constant divided by 2π. Magnons with zero momentum correspond to a uniform precession. Taking into account the Born–Karman [2] boundary condition we get

$$k_\alpha = \frac{2\pi}{L_\alpha} n_\alpha; \quad \alpha = 1, 2, 3; \quad n_\alpha = 0, 1, 2, \dots \tag{32}$$

For circular precession, and that is the case we are dealing with, the number of magnons $n(k_\mu)$ with momentum $\hbar k_\mu$ can be found from [2]

$$n(k_\mu, x_1) = \frac{\tilde{m}_1'^2 + \tilde{m}_2'^2}{2M_0 g\hbar} = |\psi(x_1)|^2 \frac{|S_1|^2 + |S_2|^2}{2M_0 g\hbar} = |\psi(x_1)|^2 n(k_\mu). \tag{33}$$

Taking into account that every magnon diminished the component M_3', along the local magnetization direction, by $g\hbar$ [2], we get

$$M_3' = M_0 - ng\hbar. \tag{34}$$

As magnons obey the Bose–Einstein statistics we get for the partition function

$$\bar{n}(k_\mu) = \{\exp[\beta\varepsilon(k_\mu)] - 1\}^{-1}, \tag{35}$$

where $\beta = 1/k_B T$, k_B is the Boltzmann constant, T — temperature. Mean value of the magnetization along the local magnetization direction can be expressed, according to (33)—(35) as

$$M_3'(T, x_1) = \langle M_z'(x_1) \rangle = M_0 - g\hbar \int_0^\infty |\psi(x_1)|^2 G(k_\mu)\bar{n}(k_\mu) dk_\mu, \tag{36}$$

where $G(k_\mu) = V/(2\pi)^3$ is the density of states in k_μ-space, $\bar{n}(k_\mu)$ — average number of magnons with momentum k_μ, as given by the distribution (35), V — volume of the sample. Mean value of the energy density is given by

$$\langle E(x_1) \rangle = \int |\psi_A(x_1)|^2 G(k_\mu)\bar{n}(k_\mu)\varepsilon^A(k_\mu) dk_\mu$$
$$+ \int |\psi_B(x_1)|^2 G(\kappa_\mu)\bar{n}(\kappa_\mu)\varepsilon^B(\kappa_\mu) d\kappa_\mu, \tag{37}$$

where the density of states $G(\kappa_\mu)$ is equal to

$$G(\kappa_\mu) = \frac{\Sigma}{(2\pi)^2}. \tag{38}$$

5. *Dependence of the magnetization density, energy density and specific heat on temperature*

If the modulus of the elliptic integral is near unity, $k \approx 1$ and $k_0 \approx 0$, then denoting $q^{-1} = \delta$ we get in case (A) that the square of the wave function $\psi_A(x_1; k_\mu)$ modulus given by (24a) will have the form

$$|\psi_A(x_1; k_\mu)|^2 = \frac{1}{V}\left\{1 - \frac{\text{cn}^2(qx_1)}{1+\delta^2 k_1^2}\right\} \tag{39}$$

and for case (B)

$$|\psi_B(x_1, \bar{\kappa})|^2 = \frac{\text{cn}^2(qx_1)}{\delta\Sigma}. \tag{40}$$

In the middle of a domain, i. e., for $x_1 = (n+\frac{1}{2})\Delta$ we get

$$|\psi_A|^2_{\text{dom}} = \frac{1}{V}; \qquad |\psi_B|^2_{\text{dom}} = 0. \tag{41}$$

Whereas in the middle of a wall, i. e., for $x_1 = n\Delta$

$$|\psi_A|^2_{\text{wall}} = \frac{1}{V}\left\{1 - \frac{1}{1+\delta^2 k_1^2}\right\}; \qquad |\psi_B|_{\text{wall}} = \frac{1}{\delta\Sigma}. \tag{42}$$

Magnetization density is given by

$$\langle M_3'(x_1)\rangle = M_0\{1 - \Delta M_0^A - \text{cn}^2(qx_1)[\Delta M^A + \Delta M^B]\}, \tag{43}$$

where

$$\Delta M_0^A = \frac{g\hbar}{M_0}\int_0^\infty \frac{1}{V}\frac{V}{(2\pi)^3}\frac{1}{\exp\{\beta\varepsilon^A(k_\mu)\}-1}\,dk_\mu, \tag{44}$$

$$\Delta M^A = -\frac{g\hbar}{M_0}\int_0^\infty \frac{1}{V(1+\delta^2 k_1^2)}\frac{V}{(2\pi)^3}\frac{1}{\exp\{\beta\varepsilon^A(k_\mu)\}-1}\,dk_\mu, \tag{45}$$

$$\Delta M^B = \frac{g\hbar}{M_0}\int_0^\infty \frac{1}{\delta\Sigma}\frac{\Sigma}{(2\pi)^2}\frac{1}{\exp\{\beta\varepsilon^B(\kappa_\mu)\}-1}\,d\kappa_\mu. \tag{46}$$

In the middle of a domain the magnetization density depends only on the A-type excitations having energy

$$\varepsilon^A(k_\mu) = g\hbar M_0\left(K_1 + \frac{K'+4\pi}{2}\right) + g\hbar M_0 C k_\mu k_\mu,$$

$$\langle M_3'\rangle_{\text{dom}} = M_0\{1 - \Delta M_0^A\}. \tag{47}$$

Because

$$\Delta M_0^A = \frac{k_B^{3/2}\Gamma(3/2)\zeta(3/2)}{4\pi^2 C^{3/2} M_0^{5/2}(g\hbar)^{1/2}} T^{3/2} = R_{dom}^A T^{3/2}, \tag{48}$$

then

$$\langle M_3' \rangle_{dom} = M_0\{1 - R_{dom}^A T^{3/2}\}. \tag{49}$$

The temperature dependence in the middle of a domain agrees with the Bloch 3/2 law. In the middle of a wall we have

$$\langle M_3' \rangle_{wall} = M_0\{1 - \Delta M_0^A - \Delta M^A - \Delta M^B\}. \tag{50}$$

Excitations of the B-type with energies

$$\varepsilon^B(\kappa_\mu) = g\hbar M_0 \frac{K'+4\pi}{2} + g\hbar M_0 C \kappa_\mu \kappa_\mu$$

give the following relation

$$\Delta M^B = \frac{k_B}{4\pi\delta M_0^2 C} T \ln \frac{k_B T}{\hbar\omega_0^B} = R_{wall}^B T \ln \frac{k_B T}{\hbar\omega_0^B}. \tag{51}$$

Whereas the term ΔM^A coming from the A-type excitations in a wall has the form

$$\Delta M^A = -\frac{g\hbar\pi^2}{(2\pi)^3 8\alpha\beta\delta} \sum_{j=1}^{\infty} \frac{1}{j} [1 - \Phi(\sqrt{j\alpha\beta/\delta}), \tag{52}$$

where Φ is a probability integral with the asymptotic representation [15]

$$\Phi(\sqrt{x}) = 1 - \frac{1}{\pi} e^{-x} \sum_{k=0}^{n-1} \frac{(-1)^k \Gamma(k+\frac{1}{2})}{x^{k+1/2}} + \frac{e^{-x}}{\pi} R_n, \tag{53}$$

$$|R_n| < \frac{\Gamma(n+1/2)}{|x|^{n+1/2}\cos\varphi}; \quad x = |x|e^{i\varphi}; \quad \varphi^2 < \pi^2. \tag{54}$$

Expanding ΔM^A for high temperatures, i. e., for $\beta\varepsilon_0^A \approx 0$ we get

$$\Delta M^A = B_{1/2}^A T^{1/2} - B_{3/2}^A T^{3/2} + B_{5/2}^A T^{5/2}, \tag{55}$$

where

$$B_{1/2}^A = \frac{(g\hbar)^{1/2}\pi^{-3/2}\left(K_1 + \dfrac{K'+4\pi}{2}\right)k_B^{1/2}\,2.9}{2^6(M_0 C)^{3/2}},$$

$$B_{3/2}^A = \frac{(g\hbar)^{-1/2}\pi^{-3/2}k_B^{3/2}\,2.9}{2^6 M_0^{5/2} C^{3/2}},$$

$$B_{5/2}^A = \frac{(g\hbar)^{-3/2}\pi^{-3/2}k_B^{5/2}\delta^2\,3.1}{2^7 M_0^{7/2} C^{5/2}}. \tag{56}$$

Eventually for the middle of a wall we have

$$\langle M'_3\rangle_{\text{wall}} = M_0\{1 - B^{\text{A}}_{1/2}T^{1/2} - R^{\text{B}}_{\text{wall}}T - (R^{\text{A}}_{\text{dom}} - B^{\text{A}}_{3/2})T^{3/2} - B^{\text{A}}_{5/2}T^{5/2}\}. \tag{57}$$

The energy density is given by

$$\langle E(x_1)\rangle = \langle E^0\rangle^{\text{A}} + \text{cn}^2\,(qx_1)\,\{\langle E\rangle^{\text{A}} + \langle E\rangle^{\text{B}}\}, \tag{58}$$

where

$$\langle E^0\rangle^{\text{A}} = \int\limits_0^\infty \frac{1}{V}\,\frac{V}{(2\pi)^3}\,\frac{1}{\exp\{\beta\varepsilon^{\text{A}}(k_\mu)\}-1}\,\varepsilon^{\text{A}}(k_\mu)dk_\mu, \tag{59}$$

$$\langle E\rangle^{\text{A}} = -\int\limits_0^\infty \frac{1}{V}\,\frac{V}{(2\pi)^3}\,\frac{1}{1+\delta^2k_1^2}\,\frac{1}{\exp\{\beta\varepsilon^{\text{A}}(k_\mu)\}-1}\,\varepsilon^{\text{A}}(k_\mu)dk_\mu, \tag{60}$$

$$\langle E\rangle^{\text{B}} = \int\limits_0^\infty \frac{1}{\delta\Sigma}\,\frac{\Sigma}{(2\pi)^2}\,\frac{1}{\exp\{\beta\varepsilon^{\text{B}}(\kappa_\mu)\}-1}\,\varepsilon^{\text{B}}(\kappa_\mu)d\kappa_\mu. \tag{61}$$

In the middle of a domain, energy density depends only on the A-type excitations and for high temperatures, i. e., $\beta\varepsilon^{\text{A}}_0 \to 0$ we get

$$\langle E\rangle_{\text{dom}} = \langle E^0\rangle^{\text{A}} = \frac{2k_{\text{B}}^{5/2}\Gamma(5/2)\zeta(5/2)}{(2\pi)^2(g\hbar M_0C)^{3/2}}\,T^{5/2}. \tag{62}$$

The specific heat in this approximation is

$$c_{\text{dom}} = \frac{\partial}{\partial T}\langle E\rangle_{\text{dom}} = \frac{5k_{\text{B}}^{5/2}\Gamma(5/2)\zeta(5/2)}{(2\pi)^2(g\hbar M_0C)^{3/2}}\,T^{3/2}. \tag{63}$$

For the middle of a wall

$$\langle E\rangle_{\text{wall}} = \langle E^0\rangle^{\text{A}} + \langle E\rangle^{\text{A}}_{\text{wall}} + \langle E\rangle^{\text{B}}_{\text{wall}}. \tag{64}$$

Again for high temperatures $\beta\varepsilon^{\text{B}}_0 \to 0$ the energy $\langle E\rangle^{\text{B}}_{\text{wall}}$ is

$$\langle E\rangle^{\text{B}}_{\text{wall}} = \frac{k_{\text{B}}^2\Gamma(2)\zeta(2)}{4\pi\delta g\hbar M_0C}\,T^2 \tag{65}$$

and the specific heat due to this type of excitations

$$c^{\text{B}}_{\text{wall}} = \frac{\partial}{\partial T}\langle E\rangle^{\text{B}}_{\text{wall}} = \frac{k_{\text{B}}^2\Gamma(2)\zeta(2)}{2\pi\delta g\hbar M_0C}\,T. \tag{66}$$

The energy $\langle E\rangle^{\mathrm{A}}_{\mathrm{wall}}$ is

$$\langle E\rangle^{\mathrm{A}}_{\mathrm{wall}} = -\frac{1}{64\pi^2}\sum_{j=1}^{\infty} e^{-j\beta\varepsilon_0{}^{\mathrm{A}}}\left\{\sqrt{\frac{\pi}{\alpha}}\,\frac{k_{\mathrm{B}}^{3/2}}{\delta^2 j^{3/2}}\,T^{3/2}\right.$$

$$\left. + \sum_{k=0}^{n-1}(-1)^k\,\frac{\delta^{2k}k_{\mathrm{B}}^{k+5/2}\Gamma(k+1/2)}{j^{k+5/2}\alpha^{k+3/2}}\,T^{k+5/2}\right\}, \tag{67}$$

whereas the specific heat

$$c^{\mathrm{A}}_{\mathrm{wall}} = \frac{\partial}{\partial T}\langle E\rangle^{\mathrm{A}}_{\mathrm{wall}} = -\frac{k_{\mathrm{B}}^{3/2}}{64\pi^{3/2}\delta^2\alpha^{3/2}}\sum_{j=1}^{\infty} j^{-3/2}e^{-j\beta\varepsilon_0{}^{\mathrm{A}}}$$

$$\times\left(\frac{j\varepsilon_0^{\mathrm{A}}}{k_{\mathrm{B}}}\,T^{-1/2}+\tfrac{3}{2}\,T^{1/2}\right) - \frac{1}{64\pi^2}\sum_{j=1}^{\infty}\sum_{k=0}^{n-1}(-1)^k e^{-j\beta\varepsilon_0{}^{\mathrm{A}}}$$

$$\times\,\frac{\delta^{2k}k_{\mathrm{B}}^{k+5/2}\Gamma(k+3/2)}{j^{k+5/2}\alpha^{k+3/2}}\left\{\frac{j\varepsilon_0^{\mathrm{A}}}{k_{\mathrm{B}}}\,T^{k+1/2}+(k+\tfrac{5}{2})T^{k+3/2}\right\}. \tag{68}$$

The term $\langle E^0\rangle^{\mathrm{A}}$ is defined by (59), (62) and the specific heat due to that term is

$$c^{0\mathrm{A}}_{\mathrm{wall}} = \frac{\partial}{\partial T}\langle E^0\rangle^{\mathrm{A}}_{\mathrm{wall}} = c_{\mathrm{dom}} = \frac{5k_{\mathrm{B}}^{5/2}\Gamma(5/2)\zeta(5/2)}{4\pi^2(g\hbar M_0 C)^{3/2}}\,T^{3/2}. \tag{69}$$

The quantities α, $\varepsilon_0^{\mathrm{A}}$, $\varepsilon_0^{\mathrm{B}}$ appearing in Eqs (52)–(68) are defined by

$$\alpha = g\hbar M_0 C; \quad \varepsilon_0^{\mathrm{A}} = g\hbar M_0[K_1+\tfrac{1}{2}(K'+4\pi)];$$

$$\varepsilon_0^{\mathrm{B}} = g\hbar M_0\,\frac{K'+4\pi}{2}.$$

6. *Discussion of the results*

Expressions for wave functions and dependence of the spin wave frequencies on the wave vector have been obtained. These expressions coincide with the respective ones derived by the approximation of second quantization method [9, 12, 13] in the lowest approximation of the Holstein-Primakoff transformation. Average thermodynamical values for magnetization density, energy density and specific heat have been found. All these quantities are local in the sense that they are functions of x_1 (see (43), (58)). For the middle of a domain we have obtained

$$M_0-\langle M_3'\rangle_{\mathrm{dom}} \sim T^{3/2}; \quad \langle E\rangle_{\mathrm{dom}} \sim T^{5/2}; \quad c_{\mathrm{dom}} \sim T^{3/2}.$$

These results are in perfect agreement with those obtained in [12, 13]. Dependence of the average thermodynamical quantities on temperature is of a more complex nature for the middle of a Bloch wall. We have not assumed, as has been done in [12, 13], that for the A-type excitations $k_1 = 0$. For high temperatures, i. e., for $k_B T \gg \hbar\omega_0^A, \hbar\omega_0^B$ in the expression describing the dependence of the magnetization density on temperature, apart from terms proportional to T and $T^{3/2}$ (as in [12, 13]), also terms proportional to $T^{1/2}$ and $T^{5/2}$ appeared.

The dependence of $\langle E \rangle_{\text{wall}}$ and c_{wall} on temperature is of a more complicated and general nature than in [12, 13].

REFERENCES

[1] A. I. Akhiezer, W. G. Baryakhtar, S. W. Peletminski, *Spin Waves*, North Holland, Amsterdam 1968.

[2] A. G. Gurevits, *Magnitnyi rezonans v ferritakh i antiferromagnetikakh*, Nauka, Moskva 1973.

[3] M. J. Shirobokov, *Dokl. Akad. Nauk SSSR* **24**, 426 (1939); *Zh. Eksp. Teor. Fiz.* **11**, 554 (1941); **15**, 57 (1945).

[4] W. J. Ziętek, *Acta Phys. Pol.* **22**, 127 (1962); **23**, 363 (1963); **32**, 21, 93 (1967); *Phys. Status Solidi* **8**, 65 (1965).

[5] W. Wasilewski, *Acta Phys. Pol.* **A41**, 373 (1972).

[6] W. Wasilewski, *Acta Phys. Pol.* **A44**, 401 (1973).

[7] E. T. Whittaker, G. N. Watson, *A Course of Modern Analysis*, Cambridge at the University Press 1927.

[8] M. J. Kurkin, A. P. Tankeev, *Fiz. Met. Metalloved.* **36**, 1149 (1973).

[9] M. M. Farztdinov, E. A. Turov, *Fiz. Met. Mettaloved.* **29**, 458 (1970).

[10] N. M. Salanski, M. S. Eruchimov, *Fizicheskiye svoystva i primenenye plenok*, Nauka, Novosybirsk 1975.

[11] N. M. Salanski, M. S. Eruchimov, *Fiz. Tver. Tela* **14**, 606 (1972).

[12] J. M. Winter, *Phys. Rev.* **124**, 452 (1961).

[13] E. A. Turov, M. J. Kurkin, A. P. Tankeev, IMC Moscow 1973, vol. II, pp. 5–18.

[14] J. A. Gilinski, *Zh. Eksp. Teor. Fiz.* **68**, 1032 (1975).

[15] I. S. Gradstein, I. M. Ryzhik, *Tablitsy integralov, summ, ryadov i proizvedenii*, Moskva 1963.